KERNFORSCHUNGSANLAGE JÜLICH GmbH

Programmgruppe Systemforschung und
Technologische Entwicklung

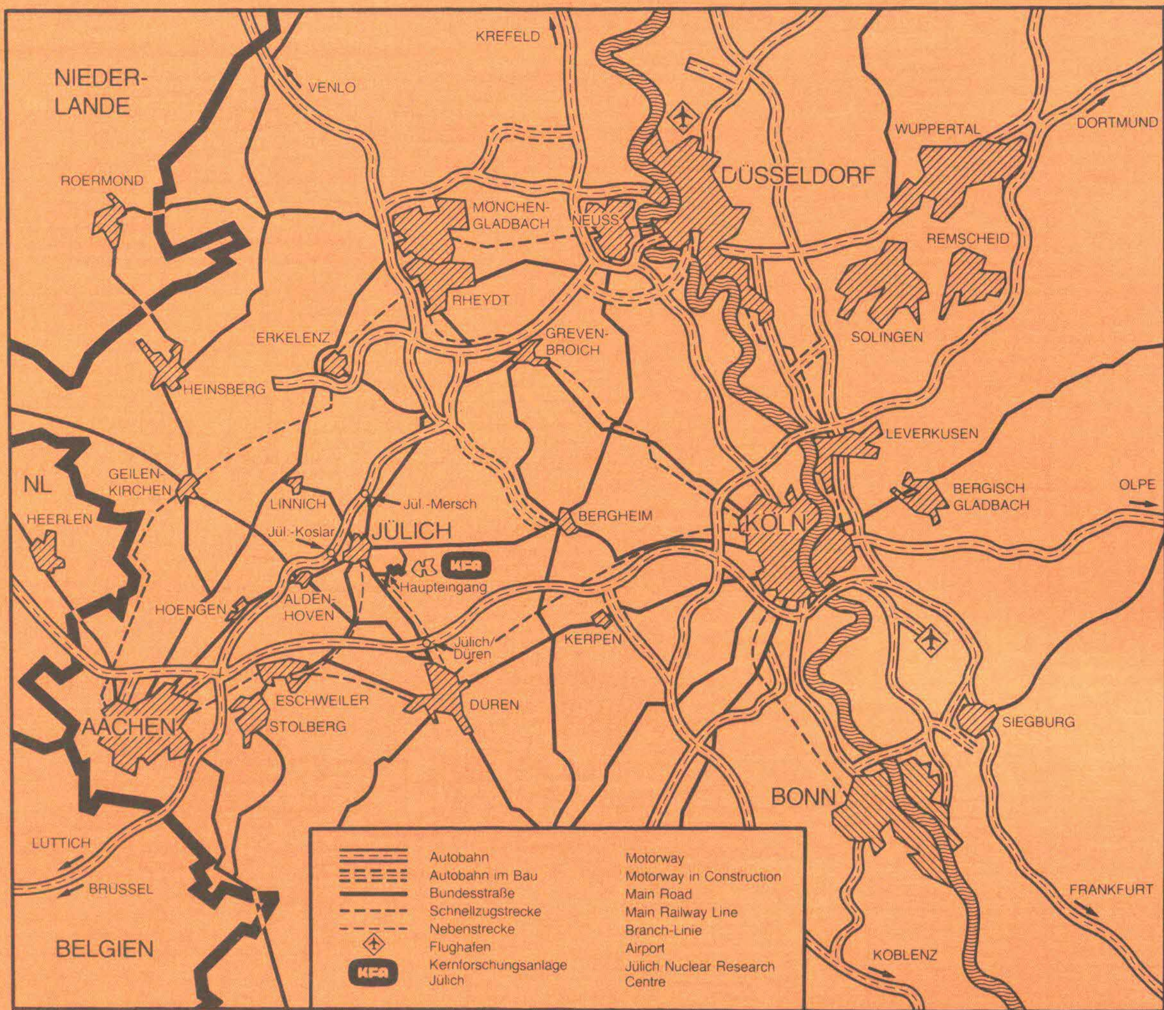
ANGEWANDTE SYSTEMANALYSE
NR. 48

APPLIED SYSTEMS ANALYSIS
NO. 48

MATHEMATICAL PROGRAMMING STUDIES OF SHORT RUN OIL REFINERY RENTS

by
Ana Sánchez

Jül-Spez-377
November 1986
ISSN 0343-7639



Als Manuskript gedruckt

Spezielle Berichte der Kernforschungsanlage Jülich – Nr. 377
Programmgruppe Systemforschung und Technologische Entwicklung Jül-Spez-377

Zu beziehen durch: ZENTRALBIBLIOTHEK der Kernforschungsanlage Jülich GmbH
Postfach 1913 · D-5170 Jülich (Bundesrepublik Deutschland)
Telefon: 02461/610 · Telex: 833556-0 kf d

MATHEMATICAL PROGRAMMING STUDIES OF SHORT RUN OIL REFINERY RENTS

by

Ana Sánchez Buelga

A thesis submitted for the degree of
Doctor of Philosophy of the University of London

Queen Mary College January 1986

ABSTRACT

The purpose of this study is to find general behavioural patterns or tendencies of Oil Refinery Rents in the short term with the objective of constructing functional relationships between them and their determinants.

The theory of rent is examined in general, and in particular in relation to the oil refinery.

Under two main hypotheses, the existence of a competitive equilibrium market for oil, and the Unidimensionality in energy pricing or The First Principle of Energy Substitution, two Linear Programming models are used to conduct experimental work: a Single Refinery Model as a representation of refinery processes and their logistics excluding transport activities; and a more comprehensive one, the 7-area World Energy Model, whereby the world oil refinery system is represented in seven oil refinery areas with their associated refinery technologies and trading links.

With the aid of the Single Refinery, the analytical formulation of rent, and the rent formation within the refinery as changes in the exogenous market conditions occur, are both explored.

The 7-area World Energy Model is used to examine the competitive behaviour of the oil refinery system in satisfying profitably the market products' demands, and the incidence that has on refinery rents. The empirical analysis conducted with the 7-area WEM comprises: (i) a Factorial Design producing refinery rent responses which are used to estimate the first kind of Refinery Rent models, (ii) Parametric Programming on several exogenous parameters producing a substantial number of observations used to estimate the second kind of Refinery Rent models by Linear Regression Analysis.

As a particular application of rent determination, the estimated models in (i) and (ii) are used to determine rents in the area of North West Europe for the period 1972-1983. Numerous time series of relevant data were compiled from the available published sources in order to contrast the estimated Refinery Rent models against reality, in particular, time series of crude and product prices in the spot market of Rotterdam, the one associated with North West Europe. In spite of imperfection of data, reasonably good agreement is found for some models.

Finally, the refinery rents are examined in the presence of a second energy source, coal, which is said to influence the level of rents through the underlying coal/heavy fuel oil price relationship.

Conclusions and guidelines for further research end the thesis.

KURZFASSUNG

Die vorliegende Arbeit bestimmt generelle Verhaltensmuster und Tendenzen in der kurzfristigen Entwicklung der Erlöse von Raffinerien mit dem Ziel, funktionale Beziehungen zwischen den Erlösen und ihren Bestimmungsgrößen herzustellen.

Zu Beginn wird die Erlöstheorie insgesamt und unter dem speziellen Gesichtspunkt "Erdölraffinerie" vorgestellt.

Unter Zugrundelegung der beiden Hypothesen "Vorhandensein eines Gleichgewichtsmarktes für Erdöl" und "Eindimensionalität bei der Energiepreisgestaltung" werden für die rechnergestützten Untersuchungen zwei lineare Programmierungsmodelle verwendet: ein Raffinerie-Modell zur Darstellung der Prozesse und Logistik innerhalb einer einzelnen Raffinerie und ein Weltenergiemodell mit sieben Regionen, in dem die Raffineriesysteme dieser sieben Regionen mit den zugehörigen Technologien und Handelsverbindungen abgebildet sind.

Mit Hilfe des Modells der Einzelraffinerie wird die analytische Formulierung des Erlöses bestimmt und die Bildung des Erlöses innerhalb der Raffinerie bei Veränderungen der exogen vorgegebenen Marktbedingungen untersucht.

Das Weltenergiemodell mit sieben Weltregionen wird benutzt, um das Konkurrenzverhalten des Raffineriesystems bei einer gewinnorientierten Deckung der Nachfrage nach Mineralölprodukten und dessen Einfluß auf die Erlöse der Raffinerie zu untersuchen. Die mit dem Weltenergiemodell durchgeführte Analyse schließt ein: (a) ein Factorial Design zur Erzeugung von Erlös-Responses, die zur Abschätzung der ersten Art von Raffinerie-Erlös-Modellen dienen, (b) die Parametrisierung verschiedener exogener Größen zur Erzeugung von Informationen zur Abschätzung der zweiten Art von Raffinerie-Erlös-Modellen mit Hilfe linearer Regressionsmodelle.

Die Bestimmung der Raffinerieerlöse in Nordwesteuropa für den Zeitraum von 1972 bis 1983 bilden einen speziellen Anwendungsfall der unter (a) und (b) geschätzten Modelle. Zeitreihen der relevanten Daten werden aus den veröffentlichten Quellen zusammengestellt, um die geschätzten Raffinerie-Erlös-Modelle an der Realität zu überprüfen. Diese Überprüfung schließt insbesondere die Zeitreihen der für Nordwesteuropa maßgebenden Spot-Markt-Preise für Rohöl und Ölprodukte in Rotterdam ein. Trotz der Lückenhaftigkeit des Datenmaterials läßt sich für einige Modelle eine gute Übereinstimmung mit der Realität feststellen.

Abschließend werden die Raffinerieerlöse unter Berücksichtigung eines zweiten Energieträgers, Kohle, untersucht, der das Erlösniveau der Raffinerie über die Beziehung zwischen Kohle- und Schwerölpreis beeinflussen soll.

Schlußfolgerungen und Leitlinien für weitergehende Untersuchungen beenden die vorliegende Arbeit.

ACKNOWLEDGEMENTS

The present study was performed as part of a Research Programme that had been initiated by the former Energy Research Unit of Queen Mary College (1970's). In 1978 the Unit became inactive, yet the intention of continuity remained in one of its founders and former director, Professor R.J. Deam. It was in 1982 that I joined Professor Deam's group at Queen Mary College to undertake part of the research on Energy Modelling and Systems Analysis which is now reported.

I am most grateful to Professor Deam for initiating me in the field of Energy Economics, and for the guidance, useful suggestions, comments and assistance received in the development of the work.

I am indebted to my supervisor Professor M.A. Laughton for the encouragement, moral support and help rendered in the course of the study.

I wish to thank Victor Poleo who introduced me to Professor Deam's research group, for his close and beneficial collaboration and shared experience in carrying out theoretical and computational work with the models.

I am also grateful to Heinz Vos for his stimulus, comments and help in the computer graphics aspects of the work.

Thanks are also given to the Petroleum Argus, Europe Oil, and the OPEC Statistical Division for providing me necessary statistical information.

I must also thank Dr. H. Wagner, director of the Programme Group on Systems Research and Technological Development, of the KFA Juelich, FRG, for making possible my joining the Group as a research student to perform work on Energy Modelling. The Programme Group provided the necessary facilities for me to make use of the in house developed Mathematical Programming Systems software and to initiate and complete the editing phase of the thesis.

I wish to express my thanks to the Consejo de Desarrollo Cientifico y Humanistico, Caracas, for the financial backing, and to the Escuela de Computacion of the Facultad de Ciencias, Universidad Central de Venezuela, for their support and allowance to remain abroad during the performance of the work.

While I benefited greatly from Professor Deam's ideas and suggestions, the statements and comments in this work other than citations are those of my own, I alone bear the responsability of any errors of ommission and commission.

CONTENTS

PAGE

Chapter 4: Experimentation at the Local Level: Single Refinery — 115

Chapter 5: Experimentation at the Global Level: 7-area World Energy Model — 161

Appendices

Bibliography 393

LIST OF TABLES

LIST OF FIGURES

ABBREVIATIONS

AHS	Arabian Heavy supply
ALK	Alkylation unit
API	American Petroleum Institute
AD,ATDIST	Atmospheric distillation unit, also crude distillation unit
AL,ALP	Arabian Light, Arabian Light price
BIEE	British Institute of Energy Economics
BIT,bit	Bitumen
bl/d	Barrels per day
bl/mt	Barrels per metric tonne
BLUE	Best Linear Unbiased Estimators
BP	British Petroleum
CC,CCRAK	Catalytic cracking unit
CCC	Catalytic cracking capacity
CCS	Catalytic cracked spirit
CR	Catalytic reforming unit, also platforming unit
COK	Coking unit
CIF	Cost, insurance and freight
CD	Crude distillation unit, also atmospheric distillation unit
CDC	Control Data Corporation
CPDP	Comite Professionnel du Petrole
dwt	Dead weight tonne
DFO,dfo	Distillate fuel oil, gas oil
EFD	Experimental Factorial Design(s)
ETSAP	Energy Technology Systems Analysis Project
FD	Factorial Design
FOB	Free on board
f.o.e.	Fuel oil equivalent
HSFO,hfo	High sulphur fuel oil, heavy fuel oil
HYC,HYCRAK	Hydrocracking unit
HYF,HYDROF	Hydrofining unit
IAEE	International Association of Energy Economists
IBM	International Business Machine Corporation
IEA	International Energy Agency
IIASA	International Institute for

KERO,kero	Kerosine
LSQM	Least Square Method
LFS,lfs	Light feed stock
LP	Linear Programming
LPP	Linear Parametric Programming
LR	Linear Regression
LSFO,lfo	Low sulphur fuel oil, light fuel oil
Max	Maximize
mbl/d	Million barrels per day
mdwt	Million dead weight tonne
MGD	Motor gasoline demand
MSFO	Medium sulphur fuel oil
mt	Metric tonne
mmt	Million metric tonnes
mmt/y	Million metric tonnes per year
Min	Minimize
MIT	Massachusetts Institute of Technology
MP	Multiparametric Programming
MPSX	Mathematical Programming System Extended
o.e.	Oil equivalent
OECD	Organisation for Economic Co-operation and Development
OGJ	Oil & Gas Journal
OPEC	Organization of the Petroleum Exporting Countries
PA	Post-optimal Analysis
PP	Parametric Programming
PLATF	Platforming unit, also catalytic reforming unit
PMS,pms	Premium motor spirit, premium motor gasoline
RF	Refinery fuel
RPU,rpu	Refinery processing unit
RR	Refinery Rent
REF	Reforming
RMS,rms	Regular motor spirit, regular motor gasoline
RD,RESDES	Residue desulphurization unit
SA	Sensitivity Analysis
SR	Single Refinery
SCN,scn	Steam cracked naphtha
SHC	Shipping capacity
srb	Straight-run benzine

SRG,srg	Straight-run gasoline
SPSS	Statistical Package for the Social Sciences
th bl/d	Thousand barrels per day
UAE	United Arab Emirates
US$/bl,$/bl	USA dollar per barrel
$/mt,$/t	USA dollar per metric tonne
VD,VCDIST	Vacuum distillation unit
wfo	Waxy fuel oil
WEM	World Energy Model

INTRODUCTION

The problem of studying the rentability/profitability of a refinery is a complex one. The oil refinery industry being a large corporation of individual and integrated entities, such as the exploration and crude production streams, the transportation and refining sub-systems, and furthermore, given the existence of a market for oil and refined products, the number of factors which necessarily interact within the oil system is large.

Crude oil is an energy carrier available at a finite number of places, fields or ports; a refinery is a transformation unit, the technical link between crude oil and the oil finished products that are to be supplied to a greater number of specific markets, usually distant from the crude oil's sources.

The technological possibilites open to the refiner in processing the crude, and the also diverse transport routes available to reach particular areas, pose a problem for the refiner, and for the analyst in general, of bringing such an amount of refining and transportation options into an adequate framework of analysis aimed at finding the best options technically, economically and competitively.

A system of such a magnitude is best represented in an integrated, programmable and computable model. A comprehensive model of the oil system must necessarily be universal: crude is a worldwide traded commodity, finished products are worldwide demanded commodities.

On the other hand, crude oil is not the unique energy carrier, the various available energy sources are substitutable subject to different technological and economic constraints. Whereas at present crude is the leading energy source as regards both, energy supply and pricing, a secondary energy carrier will in time take the leadership: crude is exhaustible, its price has already set the pace for the development of competitive alternative energy sources, and this process will continue well into the future. A comprehensive model of the energy system should of course include all *marketable* forms of energy, but consider crude oil as the leading energy carrier.

On these grounds, it is sensible to concentrate on the oil system and explore thouroughly its physical and economic complexities. The World Energy Model (WEM), the one used throughout the thesis, has been constructed to study the outlined interactions. It is primarily a World Oil Model based on the crude's leadership notion. Extensions to more than one energy form can be suitably done for the sake of studying the mechanisms of energy substitution.

The conceptual fundamentals rest on the belief of the crude oil's leadership of the energy scene formulated as the *Principle of Unidimensionality* in energy pricing which states that the crude oil price being dictated (more specifically Saudi Arabia's crude price), all other crudes, oil products and forms of energy's prices in the international markets are uniquely related to that of crude.

A question now arises, where is the refinery placed within this conceptual framework?. As referred above, the refinery is the manufacturing establishment of finished oil products. For the refiner the *price (cost) of crude* is of prime importance, refining costs can be controlled and product prices determined according to *his* refinery's facilities and *market conditions*: thus a refinery in the energy world is not an isolated 'factory'. Its rentability/profitability is highly dependent on whatever exogenous changes may occur, e.g., in the market place, in governmental policies (taxes, duties, royalties, etc.), in the world energy structure. The effects of these changes on its rents are noticeable and can be measured. Moreover, a change in the *pricing energy source* will have a tremendous economic (and structural) effect on the international oil industry. In other words, the refinery as part of the world oil system is in turn, part of the world energy system, in this context is viewed in the present study.

The literature on the subject of the oil refinery/market and energy modelling in general is vast. While much of the literature has been devoted to economic, social and political examinations of the implications of particular producer's and consumer's oil strategies/behaviour through time on current and future market expectations, those works have been carried out on predominantly qualitative basis.[1] They are of course of great value, they give us insights into the world of energy (the world of oil) providing us with the theoretical and analytical elements whereby our more 'deterministic' analyses can be developed.

Numerous quantitative studies have been also undertaken, specially as a result of the 1973 oil embargo. Only then seems to have arisen the concern for developing computerized oil/energy models of a local or

---

1 Of relevance, to cite but some, are: Adelman, M.A., *The World Petroleum Market*, (USA: The Johns Hopkins University Press, 1972); Bending, R. et al., *Energy Economics*, (Cambridge: Cambridge University Press, 1981); Chevalier, J-M., *The New Oil Stakes*, (London: Penguin Books Ltd., 1975); Odell, P.R., *Oil and World Power*, (Great Britain: Penguin Books, 7th. edition, 1983); Schneider, S.A., *The Oil Price Revolution*, (USA: The Johns Hopkins University Press, 1983); Slesser, M., *Energy in the Economy*, (Hong Kong: The Macmillann Press Ltd., 1978).

global nature mainly delineated to measure the implications of various hypothetical OPEC positions on the consumer countries's energy policies on energy's technological and economic development and to study interfuel substitution.

Energy models are of a great variety. They can be grouped into various classifications depending on distinct features, namely, the time horizon, short or long run models; the methodological approach: linear, non-linear, optimizing, simulation, econometric; the application and scope: energy policy assessment, industrial, energy physical accounting, energy-economy interaction; and the geographical boundaries, i.e., sectoral, regional, global.

It is not intended to give an exhaustive review of the energy models developed and/or applied. Comprehensive surveys are found elsewhere.[2] We would like however to make a brief comparison between the 7-area WEM and the energy models reviewed.

The survey presented by Manne[2] reviews some seven energy models: six have regional scope, they are for different purpose applications to the USA. A particular one, that presented by Keneddy,[3] bears similarity with the 7-area WEM: it is also a global model, it comprises six world areas with transportation links, and assumes the existence of a 'Cartel-determined, Persian Gulf crude oil price', and price determination elsewhere through competition. Unlike the 7-area WEM, it optimizes a non-linear welfare function, and aims at testing projections of OPEC pricing policies.

Rath-Nagel and Voss[2] present 13 energy models from which eight are applied to the USA, three to the European Community, only two having a global scope. Most of those models are focused on the energy-economy

---

[2] Choucri, N., 'Analytical Specifications of the World Oil Market, A review and comparison of twelve models', *Journal of Conflict Resolution*, XXIII, 2, (June 1979), pp.346-372; Manne, A., et al., 'Energy Policy Modeling: A Survey', *Operations Research, Journal of ORSA*, XXVII, 1, (Jan-Feb 1979), Feature Article; Charpentier, J.P., *A review of energy models No. 1*, (Laxenburg, Austria: IIASA, 1974), *A review of energy models No. 2*, (Laxenburg, Austria: IIASA, 1975), and Charpentier J.P., and Beaujean, J.M., *A review of energy models No. 3*, (Laxenburg, Austria: IIASA, 1976); Rath-Nagel, S., and Voss, A., 'Energy models for planning and policy assessment', *European Journal of Operational Research*, VIII, (1981), Invited Review, pp. 99-114.

[3] Keneddy, M., 'A world oil model', in Jorgenson, D.W., (ed.), *Econometric Studies of U.S. Energy Policy*, Chapter 3., (Amsterdam: North-Holland Publishing Company, 1976).

interactions, i.e., on the economic impact of different energy policies in a particular country or world region. Since also most of them are long-term models (the time horizon usually goes far into the next century), uncertainty clearly represents a problem. Assumptions made as regards future energy prices, in particular the price of crude oil, and future energy supply and demand tendencies introduce an inevitable noise in results, in many cases making the latter dependent on particular *interests*, e.g., of the planning, political side.

Choucri[2] presents an extensive review focusing on oil models only. He provides relevant comparisons and criticisms of available world oil models. Twelve distinct models are examined and compared. All models have a marked tendency to concentrate the problematic of the world oil system in OPEC pricing strategy and power, a point which is strongly critized by Choucri and to which we also add. As a conclusion he points out,[4]

> [...] there is generally an explicit formulation of an adversarial situation in which only producers and consumers interact and in which the emphasis is generally on the concerns and priorities of the consumer countries or the constraints and optimal prices for producer countries. World oil models seldom adopt or appreciate a systemwide or broader perspective on the overall exchanges linking these countries.

The 7-area WEM differs among most world oil models in several aspects: firstly, the model came up from the oil industry itself,[5] thus its construction embeds all the oil industry's experience and wide view of the international oil trade. Secondly, the global nature of the model does not assign an explicit role to OPEC, except that of determining the price of Saudi Arabia's crude (an action which takes place at the OPEC Conference table). This is the fixed price in the model. Likewise, any other energy price could be the 'fixed price' were the price leadership in the energy world transferred. OPEC countries are part of distinct regions in the model, their crudes and products are, as all the rest, in international trade. The 7-area WEM is a short run sup-

---

[4] Choucri, 'Analytical Specifications of the World Oil Market, A review and comparison of twelve models', op.cit., p.368.

[5] Prof. R.J. Deam was a pioneer in the application of Linear Programming techniques to the oil refinery logistic problems; his incursions on the subject can be traced back to the fifties in Australia at British Petroleum, and then the initiation and construction of oil refinery models from an international perspective in the early sixties at the British Petroleum, U.K.

ply model comprising a disaggregate representation of the oil refinery industry with wide incorporation of refining processes and transportation links, all available crudes worldwide and crude domestic production. The supply and demand balance resolves itself (given the price of crude) in a competitive market assumed to exist at each area associated spot market. The short run nature of the 7-area WEM makes it a suitable representation of the world oil market with reference to the spot market behaviour.

The layout of the thesis is as follows:

Chapter 1 includes the general conceptual platform upon which the models are built. The modelling hypotheses are put forward, in particular, the *Principles* or *Laws of Energy Substitution* are considered. They are extensively treated as part of a separate research finally reported in a Ph.D thesis.[6] Therefore they are only discussed and the Principles applied throughout without demonstration. Also introduced in Chapter 1 are the two models used in the thesis, a Single Refinery (a one-area refinery model) and the 7-area WEM (a 7-area refinery model).

Chapter 2 presents in detail the Single Refinery model. It describes the refining sub-system, and formulates its representation in a Linear Programming manner. The differences between a one-area refinery model and a n-area refinery model are the transportation links: the transport sub-system of the 7-area WEM is described as an extension to the Single Refinery model. Some economic implications of the Linear Programming formulation of the models are also presented in relation to the modelling hypotheses posed in Chapter 1. From a mathematical programming point of view, some further aspects are discussed.

Chapter 3 is a theoretical chapter: it reviews literature on Rent Theory, and establishes the Concept of Rent in the Oil Refinery from the viewpoint of this thesis. Further, by comparison to the Rent of Land, a topic which developed early within classical economic theory, a similar analysis is carried out to demonstrate the parallelism between the determination of the rent of land and the determination of the oil refinery rent in the short run when the refinery equipment is assumed to be fixed.

Chapters 4 and 5 report the experimental work conducted on the basis of the previous theoretical background. Chapter 4 presents all empri-

---

[6] Giesecke, C., 'World computer model of oil markets: OPEC pricing strategy model in the short and long term', Ph.D thesis, (Queen Mary College, University of London, December 1984).

cal exercises carried out with the Single Refinery. The refiner and his refinery operations programme, and economics under given market conditions are analysed. Parametric Programming is used as experimental methodology.

Chapter 5 presents the experimentation done with the 7-area WEM. This is the *main* experimental phase inasmuch as it provides the information used in the estimation of the Refinery Rent models. Two methodological approaches are used, namely, Factorial Experimental Designs (a 2^5 Factorial Design), and Parametric Programming. A presentation of the fundamentals of Factorial Design and its application to the 7-area WEM is done. Two kinds of Refinery Rent models are estimated: the first kind, based on the Factorail Design's rent responses, and the second kind, based on the Parametric Programming's rent responses.

Chapter 6 aims at relating the estimated models to the real refinery world. The Refinery Rent models are validated after constructing *refinery rent proxies* for the North West Europe region for the period 1972-1983. The proxies being the *average annual capital charges* of refinery technology. The second kind of Refinery Rent models provide better approximations to rent trends and values confirming the connexion between spot prices and refinery rents.

Chapter 7 puts forward the case of refinery rents under conditions of a two-tier price structure. It discusses the behaviour of rents when an energy substitution process takes place in the system. The particular case is heavy fuel oil/coal substitution and its implications in crude price (leadership) and refinery rents are discussed. Chapter 7 ends with the conclusions of this exercise.

Finally, the general conclusions of the thesis, and future research guidelines are put forward.

The author feels this thesis may be further developed in many aspects and has been far developed in many others. In view of the multidisciplinary nature of the work it seemed necessary to put forward theory which otherwise would not have been required. Accordingly, the reader may find the preliminaries in statistical, economic, chemical engineering and operational research aspects *excursus* of the main subject, especially Chapter 3 and the first four sections of Chapter 5. That reader familiar with them, can without losing generality skip the sections. Contrariwise, the interested reader may find it useful to enquire of this theory and find further reading guidelines.

CHAPTER 1: WORLD ENERGY MODELLING

In this chapter it is the intention to put forward the background considerations on energy modelling with particular reference to the World Oil System and its representation on the basis of a competitive equilibrium market for oil.

1.1 INTRODUCTION

The World Energy System (WES) is deemed as a representation of the world available energy forms, their physical and economic relationships and the underlying mechanisms determining their behavioural responses to external forces.

The energy forms are oil and gas, hydroelecticity, nuclear and coal, wind an solar, and a particular energy sub-system is the World Oil Sub-System, a system in itself.

A formal description of the WES is a World Energy Model (WEM). It is also a tool of analysis in the sense of Energy Systems Analysis (ESA), namely, a process aimed at understanding the structural behaviour of a given Energy System.

The nature of the WES is such that all the energy forms are interconnected through price relationships and technological coefficients in appropriate equivalent units. The WES's technological and economic relationships are static during short periods of time. In the long term they are however dynamic. It is sustained here that pricing all forms of energy at any time is determined by the economic dominance of a *leading energy source*. Knowing its price, all other energy sources' prices can readily be determined through the identified technological and economic energy links.

The position of leading energy source also changes in time, but its relationships to the other energy sources are in the short run fixed. Crude oil has been for over forty years and still remains the leading energy carrier. It is then essential to understand this energy sub-system in order to make any assumption about next energy structural changes. The concentration hereafter will be placed on the World Oil System Analysis.

1.2 MODELLING HYPOTHESES OF THE WORLD OIL SYSTEM

At the Energy Research Unit, Queen Mary College (1972), the attempt was made to understand the World Energy System through a World Energy Model. On the assumption that oil is the leading form of energy pricing, the focus in modelling the energy system was centred in the Oil System. At a first stage it was developed a short run model aimed at integrating the four oil sub-systems, as outlined in the diagram below, through a mathematical framework of Energy Analysis.

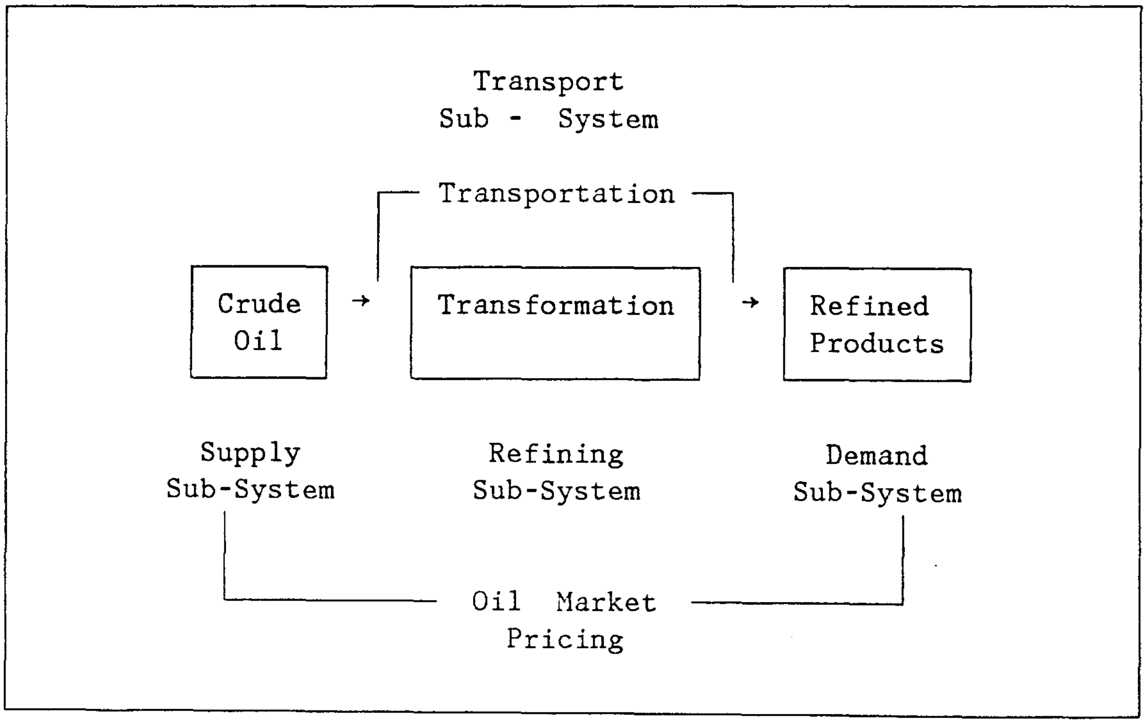


Figure 1. Oil System diagramatic representation.

A set of generic[7] variables are defined next in order to set out the hypotheses and assumptions upon which the model is constructed.

---

[7] They are broad classes of variables that in due course will be either disaggregated for a detailed representation of activities or reformulated to fulfil modelling requirements. For instance, vector b next does not include here the inherent refinery constraints on capacity and represents supply of crude with no disaggregation by crude producer.

The existence of s crude oil suppliers, m oil refined products (commodities) of final demand each of which can be produced by all or some of the n refiners (oil refined producers) and c consumers, final users of oil products is assumed.

The variables are then defined as:

$b = (b_{cs})$, cs=1,s vector of crude supplies, every supplier produces a non-negative amount of crude oil,

$X = (x_{ij})$, i=1,m, j=1,n matrix of refiner-j production of oil product-i of final demand,

$D = (d_{ih})$, i=1,m, h=1,c consumer-h demand of commodity-i, oil products are internationally traded so that consumer-h can import and/or export a particular product-i;

$E = (e_{ih})$, i=1,m, h=1,c consumer-h exports of product-i,

$M = (m_{ih})$, i=1,m, h=1,c consumer-h imports of product-i,

$p = (p_l)$, l=1,s+m vector price, prices of crudes and oil refined products,

$\Theta = (\Theta_{hj})$[8], h=1,c, j=1,n share of profits of refiner-j attributed to consumer-h, and,

$U_h(d_{ih})$, i=1,m, h=1,c is a utility function by which a consumer-h shows his preferences with respect to product-i consumption.[9]

The short run modelling hypotheses H.1 to H.4 of the 7-area WEM are then stated as follows:

H.1 The World Oil System is a competitive equilibrium market. This is to say, there exists a single price, *p*, the equilibrium price, that clears the market, i.e., brings supply and demand into balance. The allocation of supplies-b, oil products-X, exports-E, imports-M and

---

[8] The definition of profit matrix Θ was taken directly from Ginsburgh, V.A., and Waelbroeck, J.L., *Activity Analysis and General Equilibrium Modelling*, (The Netherlands: North-Holland Publishing Co., 1981).

[9] As in Ginsburgh, V.A., and Waelbroeck, J.L., Ibid., the utility function $U_h(d_{ih})$ has particular properties: a) if consumer-h prefers d_{ih}' to d_{ih}'', then $U_h(d_{ih}') > U_h(d_{ih}'')$ b) if consumer-h is indifferent between d_{ih}' and d_{ih}'', then $U_h(d_{ih}') = U_h(d_{ih}'')$.

demands-D is such that excess demand is non-positive and oil products in excess supply have zero-price, for all j, for all h:

$$p\,(\Sigma\, b_{cs} - \Sigma\, x_{ij}) \leq 0 \tag{1.1}$$

$$p\,(\Sigma\, x_{ij} + \Sigma\, m_{ih} - \Sigma\, e_{ih} - \Sigma\, d_{ih}) \leq 0 \text{[10]} \tag{1.2}$$

Equations (1.1) and (1.2) represent respectively the balances between the supply and refining sub-systems, and between the refining and demand sub-systems. Both equations can be combined to form the general equilibrium equation,

$$p\,(\Sigma b_{cs} + \Sigma\, m_{ih} - \Sigma\, e_{ih} - \Sigma\, d_{ih}) \leq 0 \tag{1.3}$$

If the market's conditions move away from equilibrium, i.e., shortages or surpluses develop, crude oil suppliers and oil product consumers are assumed to react freely to reaccomodate the system to a new equilibrium position. Thus the market adjusts rapidly to discrepancies in supply and demand through the price mechanism, *p*, by which it is further assumed suppliers and consumers make decisions so as to:

- maximize profits (while minimizing costs) for the crude oil supplier and the refiner; additionally, the refiner minimizes crude supply costs, thus the interests of both crude producers and refiners are opposed:

$$\operatorname*{Max}_{b}\ p_b b, \qquad \operatorname*{Max/Min}_{x\quad x}\ p_x X \tag{1.4}$$

Later it will be seen how the refiner's maximization criterion becomes a minimization criterion in the short run because product demands and refinery's resources are assumed fixed, hence the only interest of the refiner is the minimization of total costs, namely crude, transportation and refining costs;

- maximize the utility function U(D) for consumers subject to their budget constraint,

---

[10] It is assumed that the consumer does not possess initial stock of product-i. The opposite would be to substract a term, y_{ih}, to equation (1.2).

$$\max_{D} U_h(D) \tag{1.5}$$
$$\text{subject to } p_d D \leq \Sigma\, \Theta_{ij} p_d D$$

i.e., the consumer tries to obtain his maximum benefit subject to constraints of minimal profit for the producer.[11]

In optimization problems *Marginal Analysis* (i.e., the analysis of the behaviour of particular economic variables as small changes in related variables occur) provides a rule of decision from the point of view of Microeconomics. Marginal Analysis has its mathematical equivalence in the differential calculus. Profit maximization (cost minimization) is reached when marginal conditions are met, i.e., when marginal revenues are equal to marginal costs:

$$P = p_x X \ - \ p_b b \tag{1.6}$$

This implies that the function (1.6) must be differentiable with respect to the quantity produced/demanded (i.e., it must be at least linear):

$$\partial P/\partial D \ = \ \partial p_x X/\partial D \ - \ \partial p_b b/\partial D = 0 \quad \text{at the optimum, and from here,}$$
$$\partial p_x X/\partial D \ = \ \partial p_b b/\partial D,$$

so that the revenue from an additional unit (or small increment) of demand (output produced) is equal to the cost of producing an additional unit (or small increment) of the product demanded.

---

[11] For a discussion of the formulation and solution of an equilibrium problem see Ginsburgh, V.A., and Waelbroeck, J.L., *Activity Analysis and General Equilibrium Modelling*, op.cit., three equilibrium searching approaches are put forward, namely, searching for equilibrium prices, searching for equilibrium utilities and searching for equilibrium welfare. Throughout, the present study considers the problem from the point of view of producers rather than consumers. The refiners are assumed to pay for the inputs and satisfy demand at the equilibrium market price-p. The market is assumed to be in equilibrium also from the point of view of the consumer. Prices in this context are *signals* of the market conditions without external interferences, so that the signal received by both sides is the unique price-p.

H.2 The total product demand in the competitive market for oil is almost price inelastic, this is to say, a percentage change in quantity demanded/sold is smaller than a percentage change in price, as shown in Figure 2. On the other hand, small changes in the pattern of supply/demand may result in the same unique equilibrium price that clears the market. In the same figure, for instance, for the demand interval (d_2, d_3), p_2 is the equilibrium price. With no change in price-p then, the demand curve for the individual firm becomes horizontal, meaning that the price of an additional unit of output will be also the marginal revenue in turn equal to the average revenue, hence price equals marginal cost. From **H.1** and this latter:

marginal revenue = price = marginal cost.

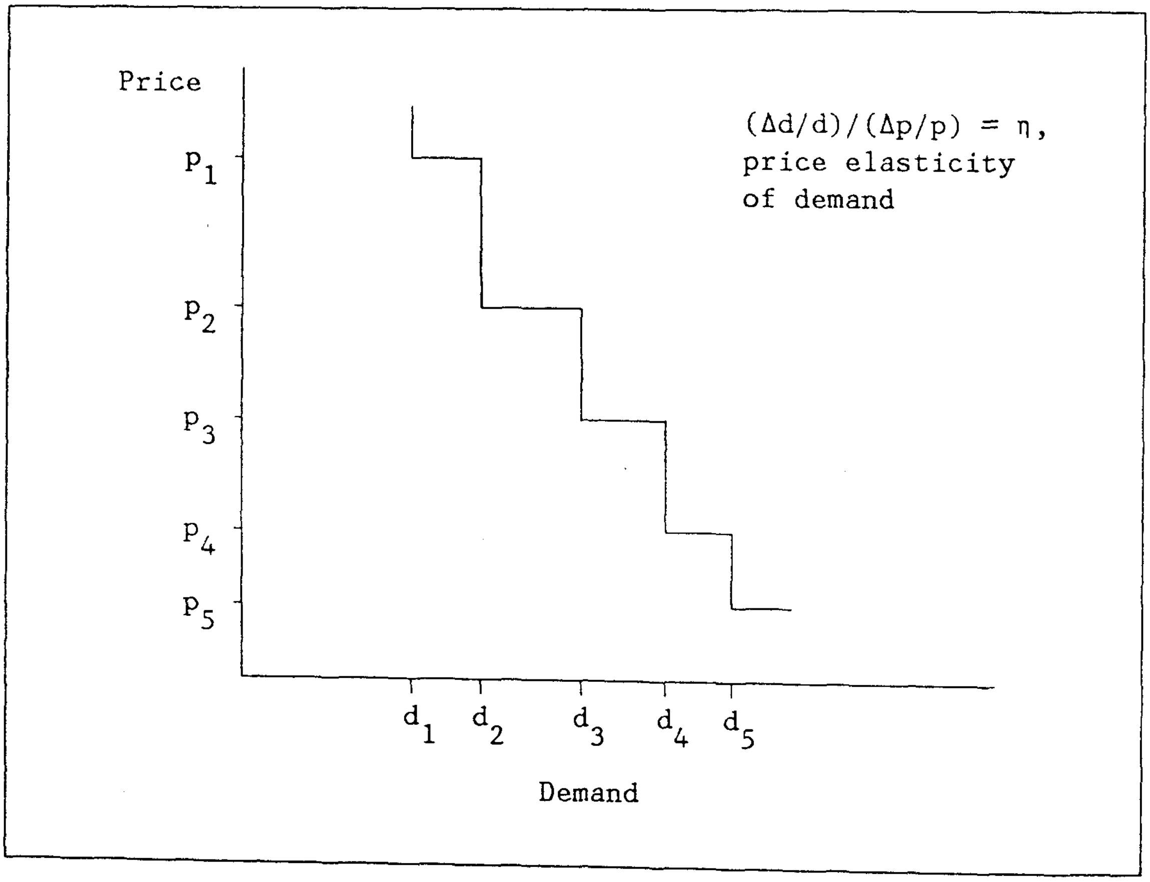


Figure 2. Equilibrium price-p and product demand relationships.

That the unique same equilibrium price prevails, is a long run equilibrium condition. Whereas that price may realize in the short run, it is more likely that supply and demand unbalances develop bringing about price changes thus changes in the level of revenues for the crude producer and the refiner. And that, in turn, occurs when the marginal cost of producing a small incremental quantity is greater than its average cost. Because the price is equal to the marginal cost, the difference between price (marginal cost) and average cost is the per unit revenue (greater than zero). Contrariwise, it may also occur that the average cost is greater than the marginal cost, the difference being then the per unit loss. Thus in the short run, to which the present study confines, the oil refinery can exhibit normal profits, no profits or losses depending upon the product price and average cost relationships of its refined products. Of course the oil refinery does not produce a single but various products of distinct qualities in joint production. The marginal value of such production is the weighted value of the final composite barrel or tonne of crude after processing (i.e., in terms of output). The average cost is the average variable cost of the inputs required to produce that output, namely, crude, transportation and refining costs.

Also in the short run, the industry's manufacturing resources are fixed, i.e., they have an inelastic supply curve. Crude oil supplies-b (except *the marginal crude source*, as it will be seen later) and oil product demands-D are assumed fixed too. The model shall determine the equilibrium vector-p, production levels-X, exports-E amd imports-M satisfying the given pattern of supply/demand.

H.3 Worldwide the oil spot markets are free competitive markets, thus the oil spot markets are the realizations of the world oil system in a competitive market.

Spot markets refer to spot sales of crude and/or oil products, usually surpluses of major oil companies and even of crude producers. Whereas the conditions for a perfect competitive market of the oil system to exist need not to be met generally, because of government intervention in the oil companies' operations through taxation, equities, royalties and the like, it has been realized that:

- the spot markets function as balancing elements in the crude oil system since it is not the case that a country or company reaches a perfect equilibrium between the crude supply and its local oil product demand;

- there exists a large number of sellers (the majors, and traditional and new crude producers) and buyers (mainly the independents), and information is equally obtained by all sellers and buyers through the specialized *spot market traders*;

- the quantities sold in the spot are estimated to be only 2 to 10% (maximum) of the world total crude trade.[12]

Short term (spot) price movements may not be a reflection of real market conditions because there is a distorting day to day fluctuation at the spot, but the spot long term generalized trend is reflecting market developments. The significance of the spots in the world oil market is put in these lines,[13]

> [...] little happens in the oil industry without reference to what is generally called the spot market, particularly the Rotterdam spot market.

Rotterdam is a well-known spot market for oil as a large part of West Europe's trade passes through there. As regards other spot markets such as Singapore, USA East Coast, Italy and the Caribbean, the prices found there are in line with Rotterdam's, the differenttials accounting for freight rates and other transportation costs.

H.4 The *price mechanism* whereby the energy crude supply and product demand balance in the competitive oil (spot) market, emerges from the *First Principle* or *Law of Energy Substitution*, also termed *Unidimensionality* in price. There exists in the world energy market an energy carrier whose price determines the equilibrium price vector-p, i.e, the prices of all other forms of energy. This is at present the price of the Arabian Light crude, which is dictated: OPEC sets the price of the leading energy source.

On the basis of hypotheses **H.1** and **H.2**, optimizing, static partial equilibrium models have been widely used[14] in an attempt to formalize the WES interactions. Linear Programming Models, for their very nature of optimizing and static models, and their fundamental functioning being (economically) explained by the concepts of marginal analysis have proved,[15] to be the 'ad hoc' modelling tool.

---

[12] In OPEC, *Basic Oil Industry Information*, (Vienna 1983), p.41.

[13] Ibid., p.40.

[14] See, for instance, Manne, A., et al., 'Energy Policy Modelling: A Survey', *Operations Research, Journal of ORSA*, op.cit.

[15] Deam, R.J., et al., 'World Energy Modelling: Part I. Concepts and Methods', *A Special Energy Policty Publication on Energy Modelling* (UK, 1974).

Applicability of LP techniques are widely known among economists and operational researchers in general. Ginsburgh and Waelbroeck devote two chapters of their work to the treatment of LP and general equilibrium problems. They point out, "...the linear programming framework is a more flexible tool than is often thought, and can be used to represent equilibrium problems of very general structure".[16]

Cites to the usefulness of LP in process analysis are duly encountered in the literature. Manne states,[17]

> Linear Programming allows both for the possibility that a single item may be produced by more than one process, and for the possibility that a single process may produce more than one item.

And in Dorfman,[18]

> [...] the linear-programming analysis provides more information than the marginal approach; it not only defines a goal in terms of optimal quantities of inputs and outputs, but it also gives specific directions for achieving this goal in terms of the various activities available to the firm.

While the assumptions of linearity and fixed technology backed by these models might not appear justifiable in the long run, they are, however, valid ones here, on the grounds of H.2 and the fact that the analysis is confined to the short run.

It has been pointed out[19] that one of advantages of using LP techniques upon other analytical tools such as the non-linear programming methods and marginalism, is the relative ease in adapting the information available for estimating the functions' parameters to the LP general constraints on linearity:

---

16 Ginsburgh, V.A., and Waelbroeck, J.L., *Activity Analysis and General Equilibrium Modelling*, op.cit., p.19.

17 Manne, A.S., 'A Linear Programming Model of the U.S. Refining Petroleum Industry', in: Manne, A.S. and Markowitz, H.M., *Studies in Process Analysis: ECONOMY-WIDE PRODUCTION CAPABILITIES*, (USA: John Wiley & Sons, Inc., 1963)., p.51.

18 Dorfman, R., Samuelson, P., and Solow, R., *Linear Programming and Economic Analysis*, (Tokyo: McGraw-Hill Kogakusha Ltd., 1958), p.141.

19 See Boulding, K.E., and Spivey, W.A., *Linear Programming and the Theory of the Firm*, (New York: The Macmillan Company, 1960).

> Although the basic principle behind linear programming and related techniques on the one hand and the marginal analysis is essentially the same, the greater value of the linear techniques lies in the fact that the basic limiting inequalities, as well as the utility or maximand function, may be more accesible to the information system.To some extent this simply follows from their being linear, a linear relationship being the next simplest to a plainly observable number.

However, gathering of information should not be neglected and assumed as a straightforward task. To cite Manne again,[20]

> This is not to say that the implementation of such models is a routine mechanical procedure.[...] The gathering of data is not just a problem of filling out numerical coefficients within previously designed boxes.

Also the static nature of the LP models model appears to be valid in the world energy context nowadays. Since so much uncertainty is involved in predicting future world energy supplies and demands a model aimed at explaining the Energy System's structural behaviour with particular emphasis in the oil system seems to be desired.

It is noted that hypotheses **H.1** and **H.3** are closely connected: hypothesis **H.1** is a general formulation while hypothesis **H.3** is a particularization. From the point of view of the present theory, if **H.3** is realized in the real world, **H.1** will be confirmed.

Hypotheses **H.3** and **H.4** have been already confirmed for the case of pricing oil products at Rotterdam (this is referred to later). Further verification (full confirmation) of hypothesis **H.3** hence of **H.1**, would be provided by contrasting the remaining Rotterdam spot prices, the crude and product prices at all other spot markets, and the worldwide refinery rents and freight rates against real values. Then and then only would hypothesis **H.4** be globaly confirmed and *Undimensionality* become a *demonstrated Principle*. The fundamentals of hypothesis **H.4** are treated separately in sections 1.3 and 1.4 following.

Hypothesis **H.2** would be confirmed as a *by-product* of verifying the remaining ones. In fact hypothesis **H.2** is rather an assumption arising from observations of oil market events and usual refining practices.

---

[20] Ibid., p.51.

1.3 THE MARKER PRICE AND UNIDIMENSIONALITY IN PRICING

In the WES there exists at any time an energy carrier or enery source that leads the determination of the energy equilibrium prices. Such an energy source is called *the marker*, and *the marker price* its price, to be conveniently dictated. Additionally, the marker is usually the *marginal* energy source inasmuch as can adapt production to bring energy supply and demand into balance.

The position of marker in the WES is basically driven by changes in political, economic/technological and market's demand forces, thus dynamic in time. The marker has been crude oil since the 1940's: it is Saudi Arabian Light crude at present, it can be traced back to the Kuwait's crude during the sixties, the Iranian Agha Jari during the fifties and the Venezuela's light crude during the forties.

For a crude oil to be in the position of marker two conditions are to be satisfied:

- its costs of production must be relatively low, it must also be higher priced than other competing energy sources;

- the marker's production must adjust *instantaneously* to surpluses or shortages in demand, its availability must be almost *unlimited* over a long period.

The marker price has been historically determined by the oil companies (the Majors) that controlled both downstream and upstream operations. After OPEC creation (1960), the control of prices has been progressively passed on to OPEC, its major effect in the world oil market has been noticed during the seventies when most OPEC countries nationalized the oil companies. The marker price determination is in fact dependent on Saudi Arabia that fixes the price of Arabian Light, all other OPEC and non-OPEC producers are *price takers*, their crude prices are aligned with Arabian Light's. The difference in prices reflects quality and locational imparities, and raises the ever *controversial* debate of the *crude oil price differentials*, this is to say, what the actual differentials in $/mt or $/bl between Arabian Light (official) and the other crudes should be.

On the notions of price leader and price taker rest the rationale behind the price mechanism determining energy equilibrium prices in the models used here. The price mechanism has been termed *Unidimensionality*, only one degree of freedom in energy pricing. Giving but one external condition, the price of the marker, all other prices in the world energy market are accordingly determined.

This concept is the key assumption of the 7-area WEM, it has been formulated as the *First Law* or *Principle of Energy Substitution*,[21] as follows:

> At equilibrium the *international* prices of all forms of energy at all locations are uniquely related to the *one* price, that of the marker crude. These unique prices are those found in the market place after time lags. A two or multiple tier price structure is unstable and, in time, reverts to a single-tier price structure.

A price mechanism in such a way conceptualized provides the links between energy technology and energy economics:[22] from a mathematical/technical point of view it is feasible to construct a physical linear relationship (production function) between a crude oil and its product yields. Similarly, the price of an oil product can be linearly related to the price of crude, freight rate and cost of refining. The price of a non-marker crude can be also mathematically related to the marker price. It is possible to represent the physical and economic interactions of the oil system by constructing a system of linear equations whereby the number of equations matches the number of unknowns minus one. The system is solved by fixing a price, namely, the marker crude price. The SR and 7-area WEM are then modelled on the assumption of Unidimensionality in pricing where LP permits the linear formulation and ensures the uniqueness of the oil price system solution given the price of the Arabian Light crude.[23]

---

[21] See Deam, R.J., and Giesecke, C., 'Towards Three 'Laws' of Energy Substitution', in: Tempest, P. (ed.), *Energy Economics in Britain*, (London: Graham & Trotman, 1983), Chapter 16, p.253; and Giesecke, C., 'World computer model of oil markets: OPEC pricing strategy in the short and long term', Ph.D thesis, op.cit.

[22] See Deam, R. J., 'World Energy Modelling: Part I. Concepts and Methods', op.cit.

[23] In most of the economic price systems a price known as the *numeraire* is fixed in order to solve for the unknowns. The marker crude resembles the *standard commodity* of Sraffa's *standard system* of production insofar as the standard commodity (being there the numeraire) is also fixed in price and a linear system of prices solved for the other commodities' prices. Sraffa's system is further referenced in this thesis, Chapter 3. Sraffa's work is found in Sraffa, P., *Production of Commodities by means of Commodities*, (Cambridge: Cambridge University Press, 1960).

The non-crude energy sources can also enter the system. Petroleum in itself is not *used* or *substitutable*, its refined products are. A non-crude energy source can be made physically equivalent to crude oil (in terms of oil equivalent units) by identifying the part or parts of the crude oil it can actually substitute. In this way there exists a physical hence economic relationship between crude and each of the forms of energy whereby a *single* price can be left free: fixing it the physical-economic energy interconnections will determine simultaneously (when suitably formalized, e.g. in a LP model) the prices of all other energy sources in line with the marker price.

Unidimensionality has been properly tested for the case of oil product prices in North West Europe[24] with reference to the Rotterdam spot prices.

Although the main objective of this work is not to test once again Unidimensionality in Rotterdam, the construction of models of refinery rents by experimenting with the 7-area WEM, and their further verification against the oil spot market, is an indirect way of confirming or rejecting the hypothesis inasmuch as the 7-area WEM main pricing assumption is Unidimensionality.

---

24 See O'Carroll, F.M., 'Price Determination and Economic Mechanisms in the European Oil Industry 1964-71', Ph.D thesis, (Queen Mary College, University of London, May 1974); and Deam, R.J., 'Understanding Energy - A Rational Basis for Planning Alternative Strategies', in: Dunkerley, J., (ed.), *International Energy Strategies, Proceedings of the 1979 IAEE/RFF Conference* (Cambridge, Massachussetts: Oelgeschlager, Glunn & Hain, Publishers, Inc., 1980), Chapter 22.

1.4 THE MARKER PRICE AND ALTERNATIVE ENERGY SOURCES

In the short run a particular crude may remain the marker, the production of alternative energy may be considered fixed; however, in the long run crude will cease to be so. Alternative marginal energy sources will eventually develop; the time in the future when this occurs will depend on the political and socio-economic forces dominating the then world energy picture.

Although the marker price is dictated, it has upper and lower limits: the lower limit given by its cost of production, the upper limit given by the cost of the alternative energy source. Marker price increases will induce the research and development of new energy sources, eventually bringing about economically competitive substitutes for the current marginal source. Low prices will prevent inmediate development of alternative energy sources and a fast depletion of the present marginal source for the marginal source producer will increase production so to make a substantial profit. A price exists between those limits which optimizes both, the producer's revenue (maximization). and the consumer's energy bill (minimization).[25]

At some point in time, a disequilibrium in the system may arise, and a two or more tier price structures develop. In order to restore equilibrium, the marker price needs then adjustment if its position is to be maintained. Failure to do that in the short run will lead the consumer to assume future increases in oil prices thus to develop alternative energy substitutes. The refined barrel of crude can be almost fully substituted: coal and uranium substitute for fuel oil, natural gas (LNG and methanol) for gas oil, kerosine and fuel oil, and alcohols and synthetic gasoline for gasoline, kerosine and gas oil. The economics involved in building the appropriate plants and the price of the marker will set the pace for the development of alternative energy sources, which indeed has already started.

The short and long term price strategies of the marker crude have been also formulated in line with the First Principle of Energy Substitution.[26]

---

[25] Giesecke, C., 'World computer model of oil markets: OPEC pricing strategy in the short and long term', Ph.D thesis, op.cit., which includes a case study on interfuel substitution based on energy prices, namely, the methanol/motor gasoline substitution.

[26] See Deam, R.J., and Giesecke, C., 'Towards Three 'Laws' of Energy Substitution', in: Tempest, P., (ed.), *Energy Economics in Britain*, op.cit., pp.253-259.

Accordingly, *The Second Law* or *Principle of Energy Substitution* states the short run pricing strategy of the marker producer:

> To maximise the present net worth of an oil producer's resource, its ratio of production to that of world production is equal to the modulus of the price elasticity with respect to world oil production, suitably corrected for present net worth, i.e.,

$$q_i/(q_o+q_i) \quad = \quad |\Sigma| \ (P_M-P_N)/P_M$$

where

q_i = production of the ith producer

q_o = sum of production of all other producers

$|\Sigma|$ = the modulus of the price elasticity with respect to q_o and q_i

P_N = present net worth of oil i

P_M = price of marker crude

And, *The Third Law* or *Principle of Energy Substitution* states the long run pricing strategy of the marker producer:

> There is an optimum price for Saudi crude which maximises its asset value in the long term and which minimises the long term cost of all energy to consumers.

It goes beyond the scope of this thesis to give full evidence of the Three Principles of Energy Substitution. Giesecke's thesis is devoted to do so. Nor is this thesis concerned with the Saudi Arabian pricing strategies, the crude price (official) FOB Ras Tanura, the marker in the model, is assumed and experimental hypotheses of market behaviour under special conditions formulated and tested. The Second Principle is applicable in the short run given that there is only a marker, currently Saudi Arabian Light crude, and provides a conceptualization of the Saudi Arbia's pricing strategy in the short run as a maximizer of its own wealth.

The Third Principle of Energy Substitution is referred to in Chapter 7 of this thesis. There the case for a transition period in which coal is taking the energy price leadership through the coal/heavy fuel oil substitution is introduced and the Arabian Light price range maintaining Saudi Arabia's leadership is determined.

1.5 ON THE WORLD AND LOCAL ENERGY MODELS

The WEM and the SR (Single Refinery) models are the two oil system models used throughout the study. Both are Linear Programming models built on the basis of hypotheses **H.1 - H.4** put forward in previous sections.

The WEM is currently representing the worldwide petroleum and gas industries within a seven refinery area aggregation (which is presented in Table C-1 of Appendix C). Its objective is to minimize the total account of costs, *refiner's bill*, derived from the crude oil and oil products transportation, oil refining and marketing operations. The profit-maximization objective implied by **H.1** becomes then a cost-minimization objective in the WEM fomulation. Indeed,[27]

> The 7-area Short-Term Model is set up to do this [to maximize the overall profit], except that in the short run demands are fixed, nothing in the choice of strategy will change the product prices, hence revenue is fixed and maximization of profit becomes minimization of cost [for the refiner].

Because of hypothesis **H.2**, non-marker crude supplies, refining capacities and oil product demands, along with the per unit refining and shipping costs, are exogenously fixed. The free variable, the marker crude price, is also an exogenous fixed value, the single price degree of freedom.

Under the natural constraints on refining and shipping operations, quality control on processes and quality specifications on final products, the model *will endogenously determine* the levels of refining, blending and transport activities, and the marginal values of all non-marker crudes and oil products at all locations, the marginal values of the refinery processing unit capacities at each of the seven refineries, and the marginal value of the available aggregated shipping capacities.

This duality of LP is summarized as,[28]

> One of the remarkable properties of the linear-programming solution is that the value of the fixed resources emerge in the

---

[27] World Energy Models Ltd., *Seven Area Short Term World Energy Model, A Technical Description*. (London, 1976).

[28] Dorfman, R., Samuelson, P., and Solow, R., *Linear Programming and Economic Analysis*, op.cit., p.167.

course of determining the optimal program and do not require additional computation.

Their interpretation in the context of oil refinery economics as is understood here, namely,

- crude oil marginal values: the crude price for the producer and hence the crude cost for the refiner;

- oil product marginal values: the marginal cost, intermediate product price at the intermediate refinery level, and market price at the marketing level of finished product;

- refinery processing unit capacities marginal values: they are the marginal rents, i.e., the scarcity values of fixed resources, and key factors in analysing the short run refinery profitability and the long run capacity expansion decisions;

  and,

- shipping availability marginal values: they correspond to the marginal rents of transporting crude oil and oil refined products via oil tankers, i.e., they are the shipowners' rents, components of the crude oil and product prices,

has a twofold significance.

Firstly, because the model is optimizing, those prices represent equilibrium prices when at the optimum. In particular, the crude oil and oil product prices (at the marketing level) can be compared to the real world prices, in turn, to the prices at the Rotterdam spot market, or at another spot, so that **H.1** of the model can be verified. It has been reported elsewhere that the equilibrium prices determined by the 7-area WEM are in correspondence with the prices occurring at the Rotterdam spot market. There, the model was used to generate the equilibrium prices at Rotterdam for the period 1964-1971 given the Kuwait's crude price, the marker for the period. A comparison between some oil product prices, namely, heavy fuel oil, gas oil and regular motor spirit generated by the model and the Rotterdam spot prices at the time, evidenced,[29] "...there is a lag ('friction' in the market)

---

[29] O'Carroll, F.M., 'Price Determination and Economic Mechanisms in the European Oil Industry 1964-71', Ph.D theis, op.cit.; results of the latter are reported in Deam, 'Understanding Energy - A Rational Basis for Planning Alternative Strategies', in: Dunkerley, J., (ed.), *International Energy Strategies*, op.cit.

before actual prices correspond to these determined prices but the trends are clearly comparable."

It is also suggested that prices will always tend to this equilibrium position; disequilibrium positions are temporal since the energy system will be ultimately driven to a stability.

Secondly, the interconnection between the equilibrium prices allows for the determination of functional relationships between a given number of price variables through linear regression. This is done in this thesis for the case of marginal values on refinery processing units capacities.

Crude oil and oil product prices are either FOB (free on board) or CIF (cost, insurance and freight rate) as they refer either to the price at their own production area or at an import location. The Single Refinery model, a one-area refinery model (representing an oil refinery as depicted in Figure 3 on page 27), is a simplification of the 7-area WEM where transport activities have been omitted, thus transportation costs do not enter directly in the cost-minimizing objective function. The freight rate is already accounted for in the CIF cost of crudes and product prices are assumed to be FOB at the refinery area.

Neglecting transport activities would mean the Single Refinery has not limitations in the amount of crude it can receive, which is not the real case. In the model, an indirect way of implying transport constraints is done by fixing the amount of non-marker crude supplied and of products demanded in the area. When more crude is required, the model balances by increasing liftings of the AL crude. This latter appears to be a realistic approach of accounting for crude shortages in most of the areas studied since AL is routed to almost all of the world areas. Since no limitations, in principle, exist for the marker liftings and there is excess tanker capacity nowadays, the ommission of the transport sub-system in the SR should not matter for experimental purposes at the local level.

However, results might be considerably altered when a transport subsystem is integrated in the model. This requires a regionalization of the space under study, be it a country, a particular world region or the world in itself. The usefulness of a single refinery model lies within the limits of the particular refiner and its operations program. The SR is a useful, easy to manage tool for the integrated study of the refinery's logistics and economic performance.

The refining and transport sub-systems are dealt with separately in Chapter 2 next.

CHAPTER 2: SINGLE REFINERY: THE REFINING SUB-SYSTEM

The Single Refinery is described as the basic linking unit between the supply sub-system (crude oil) and the demand sub-system or refined products of final demand. The Single Refinery's LP representation and the underlying relationships between processes and refinery processing units are considered. The main features of LP modelling in general and its usefulness in modelling the refinery operations are also discussed.

2.1 SINGLE REFINERY: GENERALITIES

The refining sub-system is that part of the oil system performing the petroleum downstream operations. An oil refinery is a factory where both oil purification by simple distillation processes and more complex upgrading processes are combined in order to transform the crude oil into finished marketable products; constraints on oil product quantities and qualities required by the market determine the upgrading facilities the refinery should be endowed with if to satisfy a particular market at a particular time.

An oil refinery also takes consideration of receiving the crude oil supply and distributing the final products to the market areas. The refinery location in relation to the consuming areas plays then an important role in the economics of the whole refinery performance.

For the sake of further discussions hereinafter the terms *crude supply and demand composition*, *supply and demand structure*, *supply and demand patterns* and *chemistry of supply and demand* will be used interchangeably.

From the point of view of complexity of operations, two type of oil refineries are distinguished:

- A *hydroskimming* refinery: it is the simplest commercial refinery that can be operated (built) yielding straight-run products and motor gasoline but with no ability to transform residues to motor gasoline.

- A *complex* refinery: that refinery having installed highly complex conversion processes that cause the chemical change of petroleum compounds. The principle of these processes is usually either to convert the low priced residue components (high viscosity and sulphur content) such as vacuum and waxy fuel oils into higher priced light ends to blend further to gasoline; or to reconvert the gases resulting from various processes into light ends such as alkylates, that will also blend to gasoline.

Distillation processing units, namely, crude atmospheric and vacuum distillation units and the catalytic reforming unit comprise a hydroskimming refinery, usually including also a hydrofining unit.

Refinery processing units of a complex refinery are: catalytic cracking, hydrocracking, alkylation, visbreaking and flexicoking. The more refinery processing units a refinery possesses, the more room for manoeuvre it has to adapt the crude supply chemistry to the product demand chemistry; and depending upon capacity utilization, the greater the chance for the refiner to make a profitable refinery performance.

Figure 3 on page 27 depicts the oil flow diagram of the Single Refinery. The presence of the catalytic cracking unit makes it a complex refinery, resembling an average British Petroleum refinery of Europe as can be seen from Figure 4 on page 28 and Figure 5 on page 29 respectively.[30]

The Single Refinery is meant to represent one single area of the seven refinery areas in the 7-area WEM, namely, North West Europe, but has been reduced in the number of refinery processing units modelled. The alkylation, residue desulphurization, hydrocracking and coking units have been omitted.

From an economic point of view the refiner is interested in the internal allocation of resources in order to meet at *his* minimum cost the refinery programming operations. Before the actual oil product sale occurs the refiner wants to estimate the profitability of the refinery under given market conditions. In doing so, an evaluation of every product at the intermediate refinery level is needed. Economically, the refinery may be split into three separate units:[31]

Crude Supply Production	→	Processing Refining	→	Marketing Distribution

[30] The diagrams have been drawn with slight modifications from The British Petroleum Company Ltd., *Our Industry Petroleum*, (London, 1977).

[31] The following diagram is consistent with that of section 1.1.

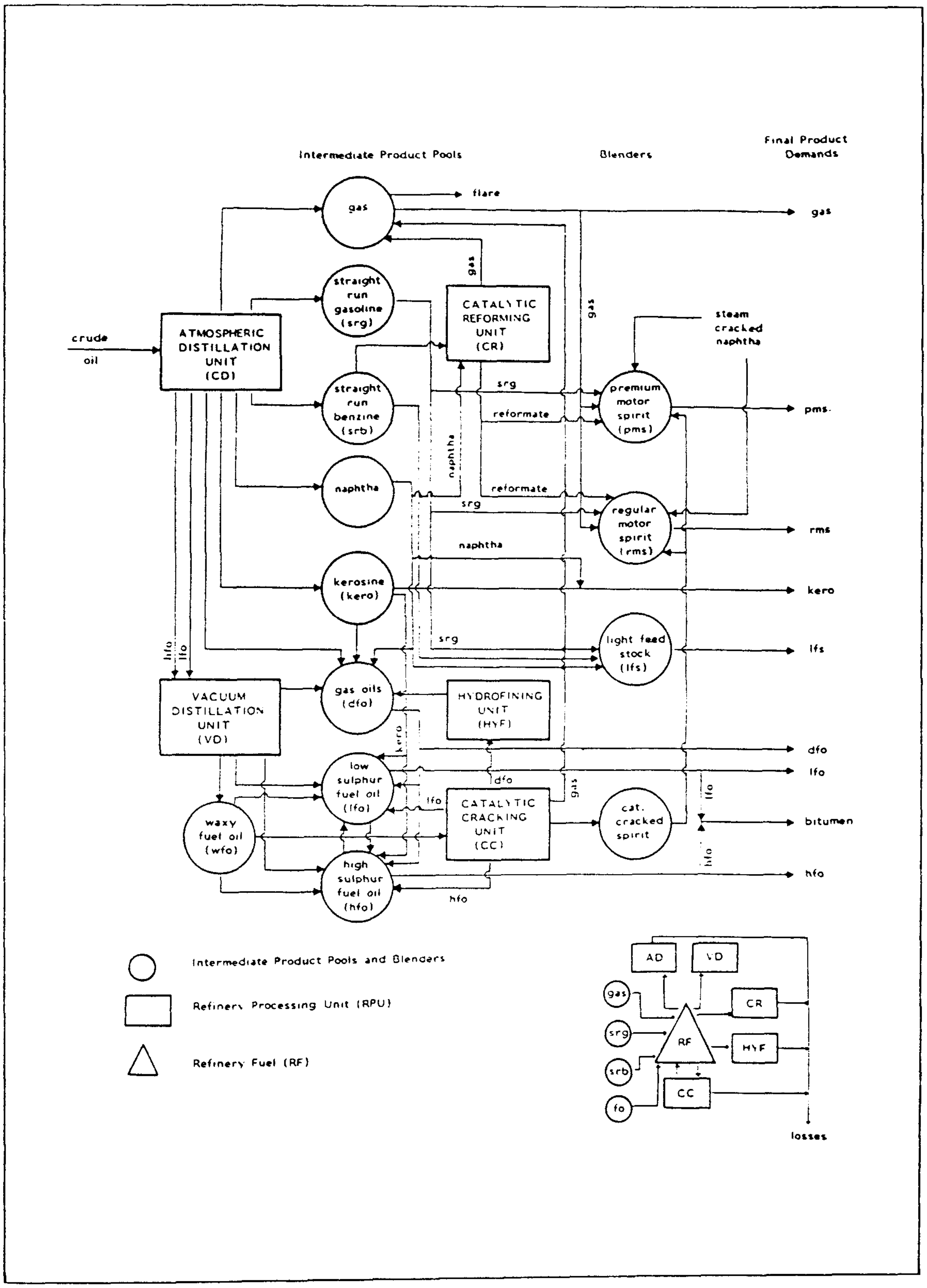


Figure 3. Single Refinery Oil Flow Diagram.

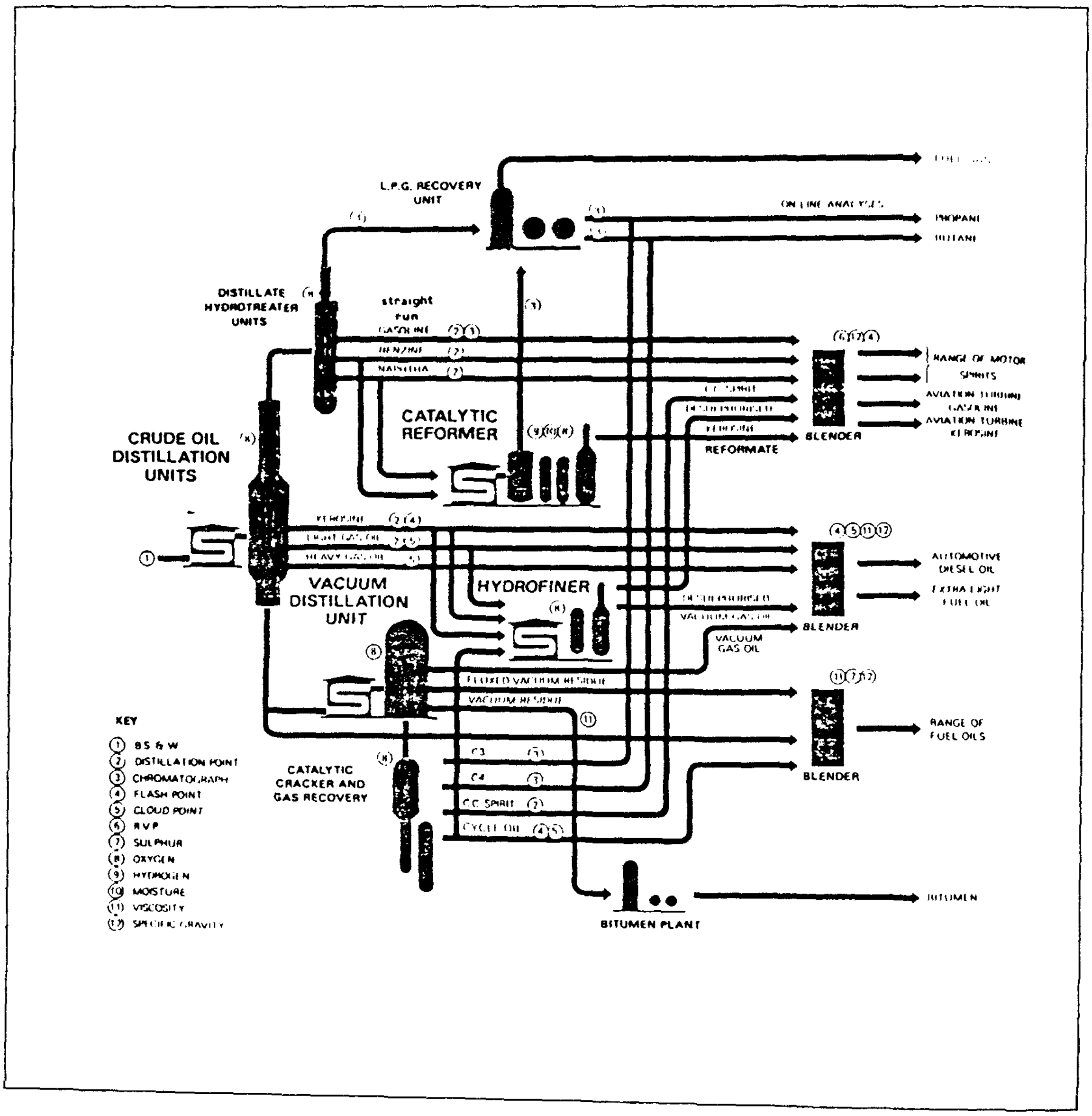


Figure 4. Oil diagram of the Bavarian BP Refinery.

At every point a *transfer price* is associated with the by-product and an economic balance then worked out. A transfer price is a unique value developed for internal sales. It represents the economic link between two refinery processing units, and therefore it can be both a *cost* and a *price*, if referred respectively to an input or output product to or from the particular refinery processing unit. Revenues and costs are then worked out for profit analysis on the basis of transfer prices. As Beckenstein points out,[32] "The sum of these values serves

[32] See Beckenstein, A.R., et al., *Performance Measurement of the Petroleum Industry*, (USA: Lexington Books, D.C. Heath & Company, 1979), p.31.

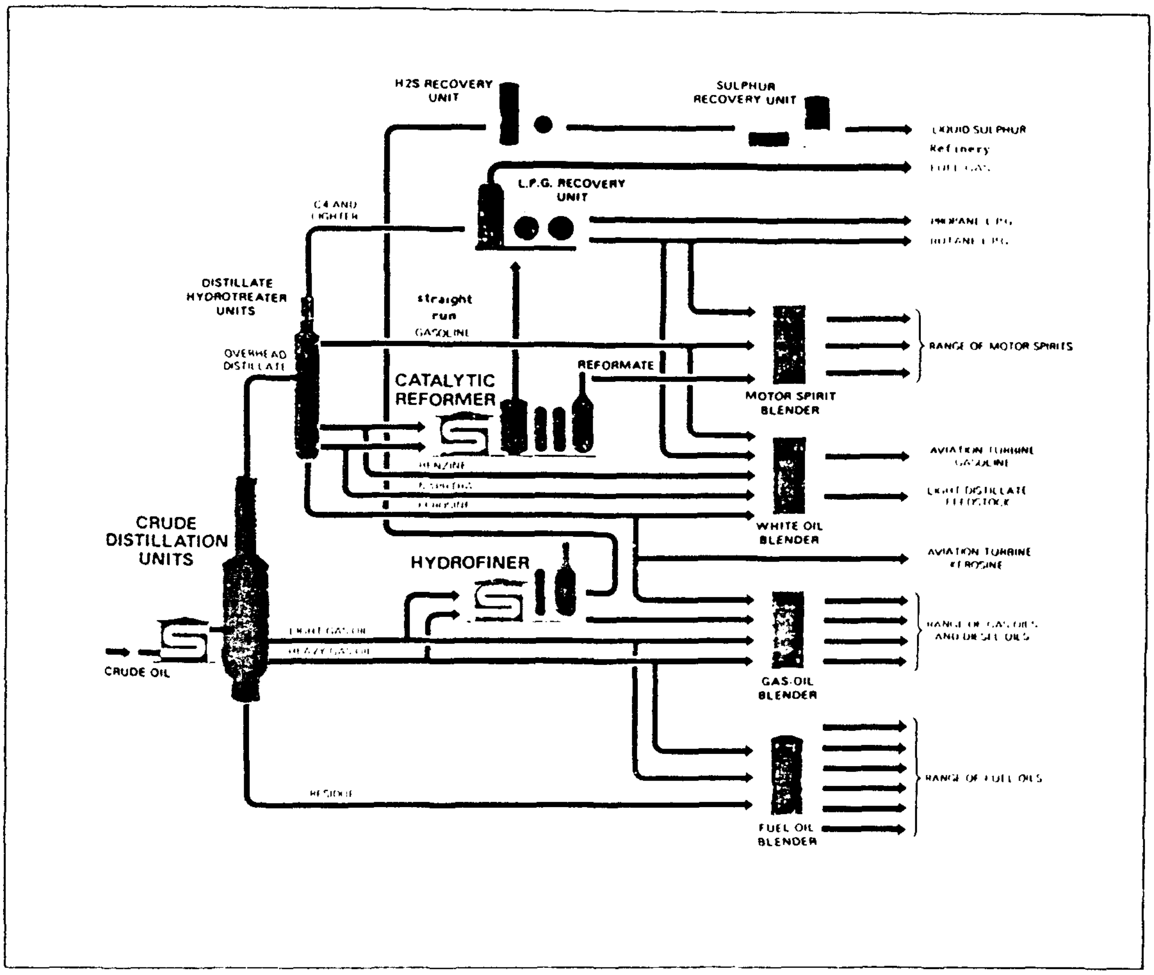


Figure 5. Oil diagram of the Rotterdam BP Refinery.

as both the revenues for the refining function and the product costs for the marketing function when profitability calculations are made."

Several methods of deriving transfer prices are available to the refiner, namely, *cost-based*, *marginal cost*, *two-step approach*, *market based*, *negotiation* and *linear programming*.[33]

As discussed in section 1.5, LP is the operational research technique used here to model both the technology of an oil refinery and the whole integrated world oil market. Linear Programming, in turn, is a

[33] The interested reader can refer to Beckenstein, A.R., Ibid., Chapter 4, pp.41-56, for a description of the transfer price calculation methods.

widely used technique in the oil industry; instances of early reports are found in Manne and Marschack[34] , and Deam.[35]

It should be noted that no contradiction arises between equilibrium prices, marginal values, and transfer prices for they are referred to a *unique* price, price vector. The marginal values calculated via LP are the transfer prices in the profit analysis context. Transfer prices at the crude supply or finished product level are assumed to be the crude and product *spot* market prices, the prices of market clearance. Transfer prices at the intermediate level (i.e., at the level of product transference within the refinery units) are the intermediate product prices. Like in the oil spot market, intermediate product prices *clear* the refinery physical transference, in other words, they allow for mass and economic balances between supply and demand at any processing unit in the refinery. References to these prices are continuously made in the present work for the economic significance they entail in the refinery context.

Next some of the common refinery processes (hydroskimming and complex) are briefly presented (section 2.1.1), and an algebraic representation of the Single Refinery in order to set out the terminology and variable definitions referred to in following chapters is also given (section 2.2).

---

[34] See Manne, A.S., 'A Linear Programming Model of the U.S. Petroleum Refining Industry' and Marschack, T.A., 'A Spatial Model of U.S. Petroleum Refining', in: Manne, A.S., and Markowitz, H.M., *Studies in Process Analysis: ECONOMY-WIDE PRODUCTION CAPABILITIES*, op.cit.

[35] Deam, R.J., 'The Integrated Programming of Refinery Supply and Procurement', mimeograph, (April 1958).

2.1.1 Refinery Processes

This section aims at giving an introductory description of some refining technologies, basically those modelled in the SR or the 7-area WEM, within the categories of hydroskimming and complex processes.

Hydroskimming Processes

Atmospheric and vacuum distillations: these processes represent the basic refinery technology consisting in the separation through temperature of the crude oil into several components or cuts according to their specific boiling-point characteristics. The process is done continuously and produces distinct distillate products within certain boiling ranges. The cuts are variable, depending on the type of crude, the particular process employed and the refining requirements.

There are six distillation processes modelled in the SR and 7-area WEM which represent the following crude cuts:

Boiling range °C	Cut	Disposal
up to 4	Isobutane	Alkylation feed
5 - 70	Straight-run gasoline	Motor spirit pools
70 - 150	Straight-run benzine	Catalytic reforming feed, motor spirit pools.
150 - 185	Naphtha	Catalytic reforming feed, kerosine and gas oil pools.
185 - 235	Kerosine	Hydrocracking feed, fuel oils and kerosine pools.
235 - 343	Gas oil	Hydrocracking feed, fuel oils and gas oil pools.
343 +	Atmospheric residue	Vacuum distillation feed, fuel oils and bitumen pools.
343 - 370	Vacuum gas oil	Combined with gas oil form long-gas oil, the feedstock of some cracking processes.
370 - 525	Waxy distillate	Cracking feedstock and fuel oils pools.
525 +	Vacuum residue	Fuel oils and bitumen pools.

As can be seen from the cuts above, some atmospheric residues are sent to the vacuum distillation unit where they boil at a vacuum pressure to avoid the decomposition of some of their high-boiling compounds which occurs when heated at their boiling points at atmospheric pressure.

All above cuts have different qualities associated with the crude origin that change when blended in the product pool with other cuts from crudes of diverse compositions.

Catalytic reforming or platforming: catalytic reforming is a refinery process of great importance in the making of motor gasolines as its main function is to increase the gasoline octane number by 'rearranging' the hydrocarbon compounds of straight-run benzine and/or naphthas (70 - 185^{0}C) in the presence of a catalyst, however, no chemical change is caused to the compounds. The reaction causes the high concentration of paraffins and naphthenes in those cuts to be converted to aromatics which have much higher octane number: the octane number is usually raised from 40 to 95-100 depending on quality requirements (see section 2.2.1.a for a description of the Research Octane Number specification).

The models include eight reforming processes, four for every kind of gasoline modelled, i.e., regular and premium: three operational severities measured as the required octane number of 85, 95 and 100, and two feedstock combinations for each severity, namely 100% straight-run benzine and 50% straight-run benzine/50% naphtha. Gases are also produced in the reforming processes.

Desulphurization: desulphurization of petroleum compounds is a refinery process used in the removal of impurities such as sulphurs that appear chemically in the components. Such contaminating molecules have a corrosive and poisonous nature that may destroy equipment and catalysts thus acting in detriment of the finished products qualities. The process is also called **hydrofining** and is used to remove all forms of sulphur compounds of any part of the crude. Hydrogen is essential as is mixed with the feed in the reactor and heated to appropriate temperature and pressure, sulphur is removed on the catalyst. Some light ends are produced in the reaction.

In the models, the hydrofining unit treats the gas oil distillates, and the **residue desulphurization** unit the atmospheric residues. The residue is desulphurized to 1.05% sulphur content. The following components are produced: gas, straight-run gasoline, straight-run benzine, naphtha, kerosine, gas oils (.03% sulphur) and residues (1.05% sulphur).

Complex Processes

Alkylation: alkylation is a refinery process whereby the lighter ends of petroleum are compounded to larger molecules called *alkylates*. Chemically, alkylates (iso-paraffins) result from the reaction of propylene and butylene with isobutane in the presence of a catalyst. The reactions are represented in the 7-area WEM by two processes which take the isobutane and unsaturated gases to aggregate them into compounds in the range of gasolines and kerosines. One process produces alkylate to blend to premium motor spirit, the other to regular motor spirit. Both yield kerosine in the same proportions.

Catalytic cracking: catalytic cracking is one of the most important refining technologies as processes the waxy fuel oils and long-gas oils by cracking of their molecules in the presence of a catalyst and heat, to produce a range of compounds of higher quality such as gases, catalytic cracked spirits (blending to motor gasolines), and gas oils; fuel oils are also produced as bottom ends which are partly recycled and partly blended to the fuel oil pools.

There are eight cracking processes represented in the models which require waxy fuel oils or long-gas oils, and work at different operational severities, i.e., low, medium and high conversion and recycle severities. The yield is higher for cracked spirits (light and heavy) than for gas oil, thus the crackers represented are motor spirit production oriented rather than gas oil oriented. The streams produced with the cracking processess are:

Stream	Disposal
Isobutane	Alkylation feed
Unsaturated gases	Alkylation feed
Light catalytic cracked spirit Heavy catalytic cracked spirit	Regular + premium motor spirits.
Catalytic cracked gas oil	Hydrofining feed
Catalytic cracked light and heavy fuel oils	Fuel oils pools, recycle bottoms.

Hydrocracking: it is a cracking process which in the presence of hydrogen converts middle distillates into light end compounds of a very good quality. There are six activities represented in the models with three possible feedstocks: kerosine, low sulphur and high sulphur gas oils. The streams produced and disposals are as follows:

Stream	Disposal
Isobutane + other gases	Gas pool
Light hydrocrackate	Regular + premium motor spirits
Heavy hydrocrackate	Blend to naphtha.

Coking: coking is a severe thermal cracking which upgrades residues to lighter hydrocarbon fractions and manufactures petroleum coke used in the steel and aluminum industries. The input in the models' processes is low sulphur fuel oil which is upgraded to gas, straight-run gasoline, straight-run benzine, naphtha, coker gas oil and coke. One process sends the coker gas oil to the catalytic cracker, the other to the low sulphur fuel oil pools.

The hydroskimming and some of the complex processes are represented in the Single Refinery diagram, Figure 3 on page 27.

2.2 LP FORMULATION OF THE SINGLE REFINERY MODEL

The Single Refinery Model follows the *economist* approach to formulate a LP problem, for inputs and outputs are all listed together with the whole set of activities comprising the refinery transformation processes. It conveniently treats, for a cost-minimization LP problem, inputs and outputs as negative and positive values respectively.

The standard cost-minimization LP problem, let it be the primal, is stated as,

Primal

$$\begin{aligned} \text{Min} \quad z &= cx \\ \text{s.t.} \quad Ax &\le b \\ x &\ge 0 \end{aligned} \tag{2.1}$$

where,

- x vector of activity variables,
- c vector of costs attached to activity vector-x,
- b vector of available factors (resource vector),
- A matrix of technological coefficients linking activity vector-x to available factors-b by a set of linear constraints on the activity vector-x;

and its associated dual problem, a profit-maximization LP problem, becomes,

Dual

$$\begin{aligned} \text{Max} \quad Z &= yb \\ \text{s.t.} \quad A'y &\ge c \\ y &\ge 0 \end{aligned} \tag{2.2}$$

where,

- y vector of marginal values imputed to available factors,
- A' transpose of matrix A.

The constraints $Ax \le b$ and $x \ge 0$ define the primal space of feasible solutions for the cost-minimization problem. Likewise, the constraints $A'y \ge c$ and $y \ge 0$ define the dual feasible space for the profit-maximization problem.

By the Duality Theorem of Linear Programming, for x^o and y^o feasible and optimal solutions to (2.1) and (2.2) respectively, it follows:

$$cx^o = y^o b \tag{2.3}$$

meaning that at the optimum, total costs must equal total revenues. This is to say, at the optimum, basic activities (corresponding to active variables in the primal problem) are zero profitable, i.e., their marginal values by their usage of factors (revenue) equals their costs; the following condition holds for every x_j becoming an active variable (basic), summing up (Σ) on all factors-i:

$$\Sigma \; a_{ij} y_i^o - c_j = 0, \tag{2.4}$$

where a_{ij} is an element of A, and y_i^o is the marginal value associated with factor-i.

Conditions (2.3) and (2.4) are recognized to be the equilibrium conditions of marginal analysis.

It is possible to relax one (or a set of) type constraint(s) in (2.1) and (2.2) as for obtaining a mixed system consisting of equalities, inequalities and non-negative variables and variables unrestricted in sign (as is the case in the SR and 7-area WEM).

In order to give a compact formulation of the SR problem in terms of primal and dual, activities, factors, technological coefficients and marginal values are identified and presented in diagramatic form in Figure 6 on page 38. The coding of variables and constraints is presented in Appendix C and the SR Technology Matrix in Appendix E.

2.2.1 The SR primal/dual feasible spaces: refining operations vector-x, cost function vector-c, refinery resource vector-b, technology matrix-A and equilibrium price vector-p

x - Crude supply and refinery process activities.

$x = (x_m, x_u, x_{fr}, x_{lr}, x_{np}, x_{pd})$, $\quad x \geq 0$,

unit = million metric tonnes/year,

x_m marker crude supply at refinery,

x_u activity level at the refinery processing unit-u, where,

u = crude distillation, vacuum distillation, catalytic cracking, catalytic reforming and hydrofining units;

x_{fr} amount of fuel transferred from the product pools to the refinery fuel pool,

x_{lr} amount of lead content reduced from the gasoline pools,

x_{np} amount of intermediate product-n blending to finished product-p,

x_{pd} amount of finished product-p extracted from the finished product-p pool to meet product-p demand.

Each refining process (element of vector-x), comprises a set of different components representing all its refining modes. For some processes a disaggregation at crude level is done, as for instance the crude distillation operations.

There is an upper bounded blending activity, that of blending kerosine and naphtha in certain proportions to the final kerosine pool,

$$0 \leq x_{bv} \leq u_{bv}, \tag{2.5}$$

where u_{bv} is the maximum amount of kerosine and naphtha blended in the latter proportions.

c - Marginal cost vector: variable costs and cost of marginal crude.

$c = (P_m, c_u, 0, -c_{lr}, 0, c_{pd})$, unit = \$/metric tonne,

p_m marker crude price, \$ per metric tonne FOB Ras Tanura of Arabian Light lifted,

c_u marginal/average operating cost per unit of tonne processed at the refinery processing unit-u, where,

u = catalytic reforming, catalytic cracking and hydrofining processing units,

c_{lr} lead content reduction cost per unit of lead reduced from the motor spirit pools,

c_{pd} extraction cost per tonne of finished product-p sent to final demand.

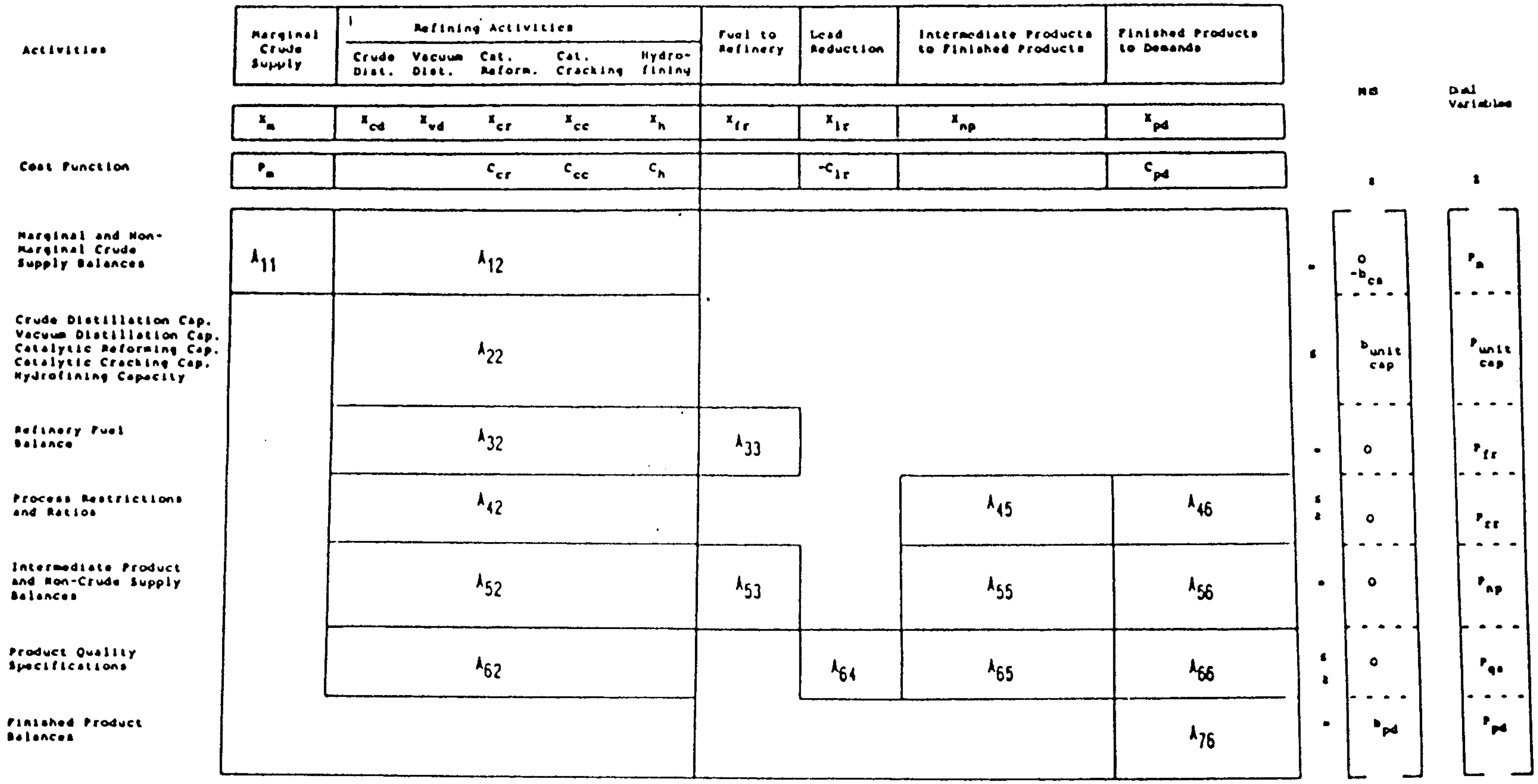


Figure 6. Single Refinery Model in matrix form.

Some refining processes have zero cost, namely, the crude distillation and blending processes. This is so in line with the almost negligible costs of distillation, and the fact that this is a basic thus always necessary process.

b - Availability of resources: crude supply, refinery processing unit capacities and product demands.

$b = (0, -b_{cs}, b_u, 0, 0, 0, -b_{ncs}, 0, b_{pd})$, unit = million metric tonnes/year,

$-b_{cs}$ non-marginal crude supplies at refinery area,

$b_u = b_{unit} = (b_{cd}, b_{vd}, b_{cr}, b_{cc}, b_h)$, where any element of b_u is the available area aggreagate capacity of refinery processing unit-u;

b_{ncs} non-crude supply at refinery (steam cracked naphtha),

b_{pd} finished products, area aggregate demand.

$A = (A_{ij})$ - Matrix of technological coefficients, where each submatrix A_{ij} of A consists of the input-output refinery process coefficients linking constraint class-i to refinery process class-j.

A_{11} this sub-matrix reduces in the SR to a single coefficient with value equals to unity, the intersection of the Arabian Light lifting activity (column) with the Arabian Light balance row.

A_{12} crude supply balance coefficients,

a_{12} = -1.005, metric tonnes of crude oil supply at the refinery per metric tonne of crude processed by distillation; .005 is the handling loss,

a_{12} = 0.0, for processes other than distillation.

A_{22} refinery processing unit capacity usage coefficients,

a_{22} = the capacity of any refinery processing unit required per metric tonne of processing charge to that unit.

A_{32}, A_{33}, A_{53} coefficients on refinery fuel requirements,

$-a_{32}$ = metric tonnes of fuel required from the refinery fuel pool per metric tonne of processing charge to a refinery unit,

a_{33} = metric tonnes of refinery fuel going to the refinery fuel pool per metric tonne of product contributing to the refinery fuel pool,

$-a_{53}$ = metric tonnes of refinery fuel going out of the pool of intermediate products and contributing to the refinery fuel pool.

A_{42}, A_{45}, A_{46} coefficients on product restrictions and ratios,

a_{42} =

- metric tonnes of gas oil of greater than 1% sulphur produced per metric tonne in distillation,
- metric tonnes of high/low sulphur bitumen in a metric tonne of high/low vacuum residue produced,

a_{45} =

- metric tonnes of gas oil of greater than 1% sulphur in a metric tonne of gas oil blended to high/low sulphur fuel oil,
- metric tonnes of high/low sulphur bitumen used per metric tonne of bitumen extracted from the bitumen blending pool.

A_{52} product yield coefficients, where the product yield coefficients are the marginal productivities of inputs to the refinery processing units, namely the marginal productivity of either a metric tonne of crude oil, if looking at the crude distillation unit, or of a metric tonne of an intermediate product, if looking at any other refinery processing unit; for instance, the waxy fuel oil as input to the catalytic cracking unit.

a_{52} = metric tonnes of a (particular) product yielded per metric tonne of refinery unit processing charge.

A_{55} blending finished product coefficients,

a_{55} = metric tonnes of intermediate product blended to the finished product pool per metric tonne of product of final demand.

A_{56} finished product extraction coefficients,

a_{56} = metric tonnes of finished product going out of the finished product pool per metric tonne of product extracted to final demand.

A_{62}, A_{64}, A_{65}, A_{66} quality specification coefficients, where the quality specification coefficients are attached to quality control constraints on some finished products in order to test their specific blending qualities against the market requirements. The term *unit* below depends on the quality specification and the product blended as explained further in section 2.2.1.a. The coefficients in the quality specification constraint rows have the following meaning:

a_{62} = *unit* contribution to motor spirit lead content per metric tonne of product produced,

a_{64} = *unit* reduction of the product specification per metric tonne of gasoline produced,

a_{65} = *unit* contribution to the final product specification per metric tonne of intermediate product blended to the finished product pool,

a_{66} = *unit* contribution to the final product specification per metric tonne of finished product sent to final demand.

A_{76} final product demand coefficients,

a_{76} = metric tonnes of final product going to final demand; these coefficients are equal to unity.

p - Price vector: marginal values imputed to crude and non-crude supply, intermediate products and finished products and refinery processing units.

$p = (p_m, p_{cs}, p_u, p_{fr}, p_{rr}, p_{np}, p_{ncs}, p_{qs}, p_{pd})$, p's unrestricted in sign,

p_{cs} marginal value of crude-cs at the refinery area-a, $ CIF per metric tonne of crude-cs,

$p_u = p_{unit}$ marginal values attached to refinery units capacities.

Hereafter the notation for the refinery processing unit values will be changed to ρ_u to mean the rent at the refinery processing unit-u;

ρ_u $ per mt of processing feed in refinery processing unit-u,

p_{fr} marginal value of refinery fuel: refinery fuel internal cost,

p_{rr} marginal value of constraints on restrictions and ratios,

p_{np} marginal value, transfer price or marginal cost of intermediate product-p,

p_{ncs} marginal value of steam cracked naphtha, (non-crude supply),

p_{qs} marginal values on product quality specifications,

p_{pd} marginal values on final products, prices FOB refinery area.

Except otherwise indicated all prices are in US$ per metric tonne.

From an economic point of view, marginal values on chemical restrictions and ratios, and on product quality specifications can be used to predict the effect on total refining cots/revenue of relaxing or tightening up a particular constraint or quality specification. If relaxing, (meaning in a LP context increasing a right hand side coefficient on a less than constraint) it would be expected at least not to worsen the optimal solution. On the other hand, if tightening up (increasing the right hand side coefficient in a greater than constraint) it would be expected to worsen the optimal solution: further severities would be imposed to the refinery in meeting required product specifications at higher costs.

2.2.1.a Oil Product Quality Control Specifications

Quality control constraints are impossed to some intermediate (blending components) and finished products in order to test their specific blending qualities against the market requirements. The requirements are at times a reflection of governmental restrictions to the industry for environmental reasons, as the cases of sulphur content in fuel oils and lead content in motor spirits. They vary with the area.

Table 2-1 presents the product quality specifications for which a product quality constraint is modelled in both the SR and the 7-area WEM.

A generic quality specification constraint will be as follows:

$$\Sigma\, a_{ij}x_j - a_{ip}x_p \quad \begin{matrix} \leq 0 \\ > 0 \end{matrix} \tag{2.6}$$

where,

x_j is a blending activity bringing component-j into the pool, element of sub-vector x_{np} defined before,

x_p is a finished product extraction activity taking component-j out of the pool, element of sub-vector x_{pd} defined before,

and,

a_{ij}, a_{ip} are the technological coefficients a_{62} to a_{66} defined before and constructed in the models as follows:

$$a_{ij},\ a_{ip} = \frac{(q_{ij} - q_{itp})}{k_j} \cdot a_{jp} \tag{2.7}$$

where,

p product for which a quality specification-i is required,

q_{ij} the contribution to specifiation-i of blending component-j, for an activity blending component-j into the pool; and the requirement of specification-i from the activity taking component-j out of the pool,

q_{itp} a product-p target specification on quality specification-i,

a_{jp} yield of blending actitivity, i.e., the amount (in metric tonnes) of component-j which each blending activity adds to or substracts from the product-p pool.

The q_{itp}'s are a priori fixed values in the specification (they can be zero), which are introduced in some cases to simplify the quality control computations: when q_{ij} coincides with q_{itp} the corresponding a_{ij} coefficient vanishes.

Qualities differ in their blending basis, namely, molar, weight or volume basis; for qualities not blending by weight, technological coefficients must be divided by a factor k_j which represents:

- SG_j (weight of component-j/weight of water), the specific gravity of component-j for quality blending by volumen; and,
- MW_j (moles component-j/weight component-j), the molecular weight of component-j for quality blending by moles.

In addition, some of the qualities do not blend linearly: for them specially divised indices are applied that do blend linearly.

Knowing the target values q_{itp}, the yields a_{jp}, and the constants k_j, the user is able to calculate the actual blending component specifications q_{ij} backwards from the matrix technological coefficients a_{ij} using expression (2.7).

Table 2-1. SR and 7-area WEM: oil product quality specifications.

Quality Specification-i	Blending Basis	Product-p	Constraint Technological coefficient-a_{ij}
Flash Point (FPI) The temperature at which the gas oil vapour will momentarily flash; test for product handling without danger	Molar (Index)	Fuel Oils	(FPI-j/MG-j) - (FPI-tp/MG-tp) target value-tp: 16 index at 136°C
Pour Point (PPI) Test to estimate the relative amount of wax in distillates and fuel oils. It measures the ability of an oil product to flow at low temperaure	Weight (Index)	Gas Oil Fuel Oils	(PPI-j - PPI-tp) target values-tp: index at 0°F 11.22 gas oil 120 fuel oils
Sulphur Content Percentage by weight of sulphur content in the blend; test for environmental reasons	Weight (%)	Gas Oil Fuel Oils	%(mt-sulph/mt-j) target value-tp: 0% sulphur content
Viscosity (VI) It is a measure of the fuel resistance to internal flow; a test for pumpability	Weight (Index)	Fuel Oils	VI-j target value-tp: 0 index at .8 centistokes and 210°F
Lead Content (Pb) Test to control the amount of lead in the gasoline blend	Volume	Motor Spirit	(gPb/lt-j)/SG-j target value-tp: local (area) specification

Table 2-1. (Cont.)

Distillation + Loss at 100°C (D+L) It measures cold starting performance	Volume (%)	Motor Spirit	(D+L-j - D+L-tp)/ SG-j target value-tp: local (area) specification
Olephin Content (OC) Limit on the amount of cracked spirit in the blend in order to control oxidation	Volume (%)	Motor Spirit	(OC-j - OC-tp)/ SG-j target value-tp: local (area) specification
Temperature for 36:1 vapour/liquid ratio (VL) The vapour/liquid ratio represents the surface pressure it takes to keep a liquid from vapourizing; test for measuring volatility	Volume	Motor Spirit	(VL-j - VL-tp)/ SG-j target value-tp: local (area) specification
Research Octane Number (RON) Test for measuring antiknock tendency of motor spirits. The unity of knock intensity is the octane number, which is the percentage by volume of iso-octane mixed to heptane to meet the blend knock intensity	Volume (Index)	Motor Spirit	(RON-j - RON-tp)/ SG-j target value-tp: local (area) specification

2.2.2 The Objective Function

The Single Refinery problem is that of allocating the available resources at minimum cost while maximizing the revenue obtained from the sales of finished products to meet demand.

What is involved in fulfilling these two dual objectives is a series of economic entities; in the primal cost-minimization function:

- crude supply costs: costs of the marker crude supply, i.e., the price of Arabian Light (exogenously fixed) times its marginal lifting,

- refining operating costs: these costs include costs of catalysts and chemicals and bonus on lead content reduction,

- procurement costs: costs incurred to the refiner when shortages of oil products arise,

that give rise to the prices in the dual profit-maximization objective function:

- costs on non-marker crude and non-crude supplies,

- the imputed values to the refinery processing units,

- the prices of oil products,

- imputed costs on non-optimal operations, i.e., those activities to which the refinery may eventually make use however deviating the overall refinery performance from optimum (usually referred to as the value of bounded vectors in LP terms).

Note marginal values arise in the dual objective, i.e., all crude supply costs but the crude marker cost/price, product prices and refinery rents, inasmuch as their values are not fixed but endogenously worked out. The costs exogenously fixed (the marker crude and operating costs) are accounted for in the cost-minimization problem. To work out the total refinery variable costs an *economic balance* is to be performed on the basis of the primal/dual equivalence at the optimal solution as in relation (2.3), page 35.

The 7-area WEM is additionally accounting for transport and bunker costs as well as for shipowners' and port rents as derived in the dual objective function.

The fixed costs, namely, depreciation, allowances, machinery and building maintenance and salaries, are not included in the model since as long as any throughput is being maintained they are invariant costs. The labour costs, generally included in the variable costs of most other firms, are in the refinery invariable with output thus entering the account of the fixed costs. It is then implicitly assumed that in the short run, labour, depreciation and maintenance charges are independent of the utilization of the equipment.

The rents on the refinery processing units generated by the models are an indication for the refiner of the short run levels of profitability in operationally running the refinery. This point is further dealt with in Chapter 3.

In terms of the variables and parameter definitions of section 2.2.1, the primal and dual objectives appear respectively as in (2.8) and (2.9):

$$\text{Min } z = p_m x_m + c_u x_u - c_{lr} x_{lr} + c_{pd} x_{pd} \qquad (2.8)$$

$$\text{Max } Z = - b_{cs} p_{cs} - b_u \rho_u - b_{ncs} p_{ncs} + b_{pd} p_{pd} \qquad (2.9)$$

The objective function (2.8) is entering the problem's solution algorithm, the dual objective function solution is automatically generated with the primal.

2.3 SR EXTENSIONS: THE TRANSPORT SUB-SYSTEM

The transport sub-system is the link between the SR, one area model, and any n-area model, in paticular the 7-area WEM (see Figure 1 on page 8). An n-area model comprises n-single refineries as that previously described plus the activities and constraints associated with the transportation of crude oil and finished products inter-area.

Figure 7 on page 50 depicts the transportation sub-matrix of the 7-area WEM as an extension to the SR matrix of Figure 6 on page 38. The additional activities-x, resource vector components-b, technological coefficients-A, objective function coefficients-c and the new marginal values now generated are described below. The seven areas of WEM are in turn grouped into five world regions-r. The description of areas-a and regions-r, and the coding of the transport variables and constraints as in the 7-area WEM are found in Appendix C.

x - Transport activities.

$x = (x_{rcj}, x_{ract}, x_{rect}, x_{racp}, x_{recp}, x_{eact}, x_{abpt}, x_{afb})$, $\quad x \geq 0$,

unit = million metric tonnes/year,

x_{rcj} amount of crude-c liftings for export at jetty terminal in region-r,

x_{ract} amount of crude-c transported from region-r to area-a via tanker size-t,

x_{rect} amount of crude-c transported from region-r to entrepot-e via tanker size-t,

x_{racp} amount of crude-c transported from region-r to area-a via pipeline,

x_{recp} amount of crude-c transported from region-r to entrepot-e via pipeline,

x_{eact} amount of crude-c transported from entrepot-e to area-a via tanker size-t,

x_{abpt} amount of product-p transported from area-a (exporting area) to area-b (importing area) via tanker size-t,

Transport activities	Jetty	Crude Oil Supply: Tanker Area	Crude Oil Supply: Tanker Entrepot	Crude Oil Supply: Pipeline Area	Crude Oil Supply: Pipeline Entrepot	Transfer Entrepot to Area	Product Shipment	Fuel to Bunkers	Crude Dist.	RHS	Dual Variables
	x_{rcJ}	x_{ract}	x_{rect}	x_{racp}	x_{recp}	x_{eact}	x_{abpt}	x_{afb}	x_{acd}		
Cost function	0	c_{ract}	c_{rect}	c_{racp}	c_{recp}	c_{eact}	c_{abpt}	0			
Crude oil supply at Jetty	1	-1	-1	-1	-1	-1	-1			0	p_{acJ}
Crude oil supply at Field	1									b_{acF}	p_{acF}
Limit on local crude oil supply		1		1						b_{aL}	p_{aL}
Crude balance at refinery		1		1		1			-1.005	0	p_{ac}
Crude balance at entrepot			1		1	-1				0	p_{ec}
Crude oil port restrictions		1				1				b_{ap}	p_{ap}
Crude oil pipeline capacity				1	1					b_t	p_t
World tanker availability		a_{82}	a_{83}			a_{86}	a_{87}			b_{apl}	p_{apl}
Refinery fuel balance								-1		0	p_{fr}
Bunker fuel balance		$-a_{102}$	$-a_{103}$			$-a_{106}$	$-a_{107}$	1		0	p_{fb}
Finished product balance area-a							-1			b_{apd}	p_{apd}
Finished product balance area-b							1			b_{bpd}	p_{bpd}

Figure 7. 7-area WEM Transport Sub-System in matrix form.

x_{afb} amount of fuel oil transferred from the refinery fuel pool to the bunker fuel pool in area-a;

c - Marginal transport costs.

$c = (0, c_{ract}, c_{rect}, c_{racp} c_{recp}, c_{eact}, c_{abpt}, 0),$

unit = \$ / metric tonne,

c_{ract} transport unit cost of crude-c from region-r to area-a via tanker size-t; it consists of stores, maintanance, and port dues,

c_{rect} transport unit cost of crude-c from region-r to entreport-e via tanker size-t,

c_{racp} transport unit cost of crude-c from region-r to area-a via pipeline,

c_{recp} transport unit cost of crude-c from region-r to entrepot-e via pipeline, this is pipeline dues,

c_{react} transport unit cost of crude-c from entrepot-e to area-a via tanker size-t, which is as c_{ract} plus entrepot dues,

c_{abpt} transport unit cost of product-p transported from area-a to area-b via tanker size-t;

b - Crude supply, tanker supply, pipeline capacity, port constraints, fuel requirements and product demands.

$b = (0, b_{acF}, b_{aL}, 0, 0, b_{ap}, b_{apl}, b_t, 0, 0, b_{apd})$

unit = million metric tonnes/year, million dead weight tonnes/year;

b_{acF} maximum crude-c supply at field area-a,

b_{aL} maximum or minimum crude supply of one or more crudes at area-a,

b_{ap} maximum or minimum capacity limits on crude supply at port area-a,

b_{apl} maximum crude oil pipeline capacity in area-a,

b_t worldwide maximum availability of tanker size-t,

b_{apd} finished product-p aggregate demand of exporting area-a,

b_{bpd} finished product-p aggregate demand of importing area-b;

A - Matrix of transport coefficients.

a_{8j}, j=2,3,5 dead weight tonnes of capacity of tanker size-t required per metric tonne of crude-c transported from region-r to area-a, from region-r to entrepot-e or from entrepot-e to area-a via tanker size-t (respectively with j);

a_{87} dead weight tonnes of capacity of tanker size-t required per metric tonne of product-p transported from area-a to area-b via tanker size-t,

a_{10j}, j=2,3,6,7 bunker fuel requirements per metric tonne of crude or product transported; there are two entries in the bunker fuel rows for every transport activity, namely, in the bunker fuel at source (region, area or entrepot source) and in the bunker fuel at destination (area or entrepot). It is then assumed that bunker requirements are shared between exporting and importing areas.

p - Price vector: marginal values imputed to crude supplies at jetty, and at refinery area (FOB and CIF prices), and ports, shipowners' and pipeline rents.

$p = (p_{acj}, p_{acF}, p_{aL}, p_{ac}, p_{ec}, \rho_{ap}, \rho_{apl}, \rho_t, p_{afr}, p_{afb})$

p_{acj} FOB price of crude-c at jetty area-a, \$/mt of crude,

p_{acF} FOB price of crude-c at field area-a, \$/mt crude,

p_{aL} FOB price of crude on limited supply at area-a, some areas have restrictions in some but not all crudes supplied hence the FOB price there may not refer to one particular crude but to a kind of *pool* of crudes;

p_{ac} CIF price of crude-c at refinery area-a, \$/mt crude,

ρ_{ap} port rent in refinery area-a, \$/mt of crude/product discharged,

ρ_{apl} pipeline rent in refinery area-a,

ρ_{t} shipowners' rent, ship hiring cost of tanker size-t, \$/dwt/day,

p_{afr} marginal value of refinery fuel, area-a refinery fuel internal cost, \$/mt fuel oil produced at the refinery area-a,

p_{afb} marginal value on bunker, area-a bunker fuel cost, \$/mt heavy fuel oil used for bunkers.

2.3.1 The Extended Objective Function

The cost-minimization objective function of the 7-area WEM includes additionally to the refining costs (over all crudes and areas) and the marginal crude supply cost, the costs of all transportation activities as described in the previous section. Thus the term below adds to equation (2.8), the four summation signs being per region-r, per area-a, per crude-c and per ship size-t respectively:

$$\Sigma\ \Sigma\ \Sigma\ \Sigma\ \ (c_{ract}x_{ract} + c_{rect}x_{rect} + c_{racp}x_{racp} +$$

$$c_{recp}x_{recp} + c_{eact}x_{eact} + c_{abpt}x_{abpt})$$

This extension generates, in turn, other terms in the dual maximization objective function, namely, per area-a and per tanker size-t:

$- \Sigma\ b_{ap}\rho_{ap}$	$- \ \Sigma\ b_{apl}\rho_{apl}$	$- \ \Sigma\ b_t\rho_t$	$+ \ \Sigma\ b_t\rho_t$
Port rent	Pipeline rent	Shipowners'rent ($\rho_t > 0$)	Shipowners' loss ($\rho_t < 0$)

The tanker availability constraints are constructed in such a way that the shipowner must pay the laid-up cost for every ship of size-t. If the capacity available of ship size-t is not fully used, a negative value (equal to the ship size-t laid-up cost) will be associated with the corresponding constraint, and this latter is accounted as a loss for the shipowner. If the capacity of ship size-t is fully used, a less negative or positive marginal value will be associated with it: if positive, it will be accounted for as the shipowners' rent on ship size-t, if negative, as the hiring cost for the exporter of crude or product.

The port constraints refer to the minimum/maximum amount of crude which can be brought into the port area in the different tanker sizes. The port capacities are associated with the area available crude distillation capacity on the grounds that crude distillation units are part of the transport system. Since tankage is expensive, and a crude unit is relatively cheap, saving is made by transporting crude in very large vessels rather than in smaller tankers, hence the crude unit capacity is *a* maximum limit for the amount of crude it could be discharged at the port refinery area. On the other hand, the maximum tanker size allowed to the port is given by the local (area) port physical capacity.

If the port capacities result in excess, explicit slacks associated with both the area crude distillation and port capacities will have positive values (basic activities) indicating that the available port capacity must be reduced by their values giving the effective port capacity. From this effective capacity some slack (other than explicit slack variables) may still occur, the optimal solution then generates a zero marginal value on the port constraint. If however, a zero (non explicit) slack variable is generated then a positive marginal value will appear for the corresponding port constraint. It is clear that idle crude distillation capacity will not generate a crude distillation rent but there may exist a positive port rent since the refinery capacity associated with the port may be fully used.

Port rent is the *marginal cost of the port* and represents in the 7-area WEM, the relative advantage of a port receiving tankers of higher size with respect to those of smaller size in the area.

2.4 WEM AND SR BALANCES

2.4.1 Mass Balance

Mass balance is the material input/output balance (crude supply-product demand) in metric tonnes performed at the oil system world level, at the refinery area-a level and/or at the refinery processing unit level. Whatever the case, equality between inputs and outputs must hold to represent the equilibrium in the oil market and the correctness of the LP model. The following diagram depicts the oil system inputs and outputs,

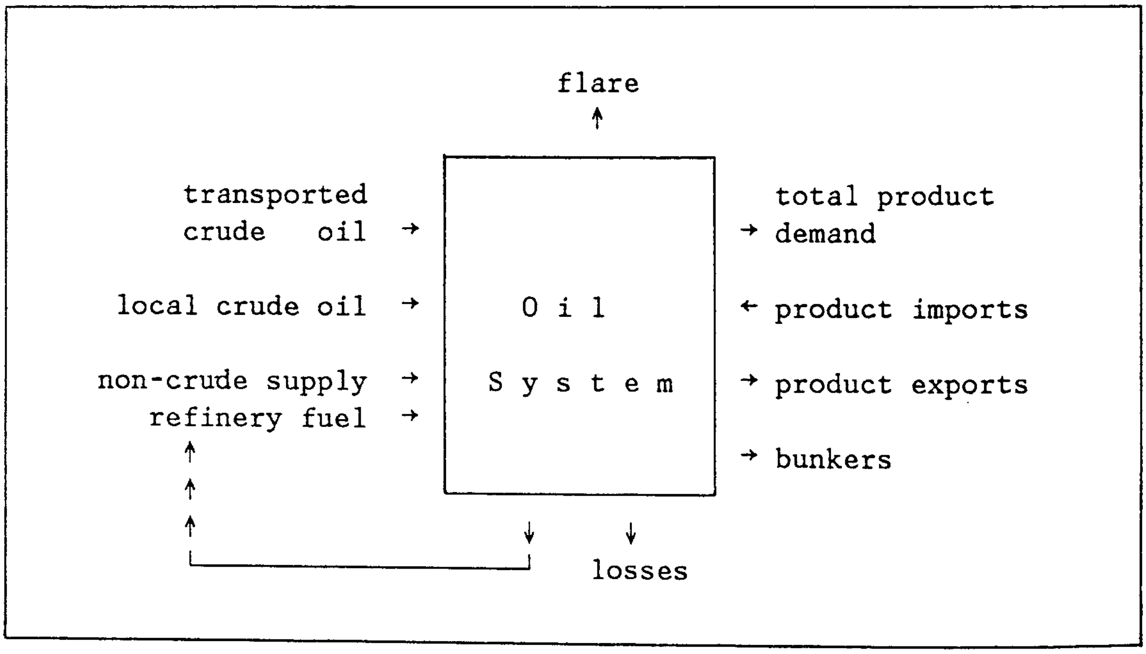


Figure 8. Oil System Mass Balance diagram.

For the 7-area WEM, total input and output are accounted for as:

Total input = transported crude oil + local crude oil + non-crude oil supply

Total output = product demand + refinery fuel + bunkers + product imports + product exports + flare + losses

In the absence of transport activities, operations related to transportation of crude, bunkers and product imports and exports do not occur in the Single Refinery model. Energy supply reduces to crude supply plus certain amount of steam cracked naphtha (this being the only non-crude supply) for blending to regular and premium motor gasoline.

In terms of the notation put forward in the previous section, the mass balance relation reads as:

$$x_m + b_{cs} + b_{ncs} = b_{pd} + x_{fr} + \text{losses} + \text{flare} \qquad (2.10)$$

2.4.2 Economic Summaries

Economic summaries are derived from the models optimal solutions on the basis of the primal/dual relationship (2.3). Economic balances can be performed as the mass balance for the whole model and for its sub-components. The main economic summaries are following:

(i) An overall 7-area WEM economic balance will take account of the value of inputs plus total refinery operating costs and transport costs and the total value of finished products (including imports and exports) minus total rents (i.e., refinery processing units' rents, shipowners' and pipeline rents). That is,

> marginal crude revenue + total refinery operating costs + total crude and product transportation costs + costs on procurements = finished product value - non-marginal crude value - indigenous crude value - non-crude suppliers' revenue - refiners' rent - port rent - pipeline rent - shipowners' rent + shipowners' loss - total cost on all bounded vectors

(ii) A 7-area WEM refinery area-a economic balance will only consider refining costs since the crude is priced CIF refinery area-a except for local crude which is FOB priced. The value of products takes account of the value of finished products and exports (FOB, area-a) plus bunkers and refinery fuel minus the value of imports (CIF area-a) and penalties due to procurements and the like (total cost on bounded vectors, i.e., on constrained recipe blending of finished products). There is eventually a refiners' rent for the making of finished products, however no port or pipeline rent is valued at the refinery level within the 7-area WEM:

FOB value of finished products (local product demand) + FOB value of product exports + value of refinery fuel + value of bunkers = CIF value of product imports + CIF value of transported crude + FOB value of local crude + non-crude supply revenue + value of indigenous crude + refinery rent + total operating costs area-a + total cost on all bounded vectors

(iii) The Single Refinery economic balance will account for the costs and values as in the refinery area-a of the 7-area WEM except that bunkers, exports and imports values are omitted.

marginal crude revenue + refinery operating costs = value of finished products - non-marginal crude rent - non-crude suppliers' revenue - refinery processing units rents - total cost on all bounded vectors

For x^o, y^o, optimal solutions of (2.1) and (2.2), and keeping the previously defined notation, the overall economic balance is read as:

$$p_m x_m^o + c_u x_u^o - c_{lr} x_{lr}^o + c_{pd} x_{pd}^o =$$

$$b_{pd} p_{pd} - b_{cs} p_{cs} - b_{ncs} p_{ncs} - b_u \rho_u - c_{bv} x_{bv} \qquad (2.11)$$

(Unit: million US$/year)

The last term of all above equations (and of following ones), the cost on bounded vectors or cost on non-optimal refinery operations, must be substracted from the dual objective function. If this variable, x_{bv} is either a basic variable, therefore having a zèro reduced cost, or a non-basic variable set at its lower bound of zero, no penalty will be imposed on the objective function. Only when x_{bv} is a non-basic variable set at its upper bound there exists a non-zero cost, c_{bv} penalizing the total revenue (this is explained further in section 2.5.2). The reduced cost c_{bv} is also seen as an index of refinery's departure from optimal operating conditions.

(iv) Rearrangement of terms in the SR and 7-area WEM balance equations allows for other economic summaries.

• The *world energy bill*, also *world refiner's bill* accounts for all the consumer's expenses, namely, crude and non-crude supplies costs plus shipping costs, and refinery, ship and port owner's rents:

energy bill = total crude value (CIF) + refinery variable operating costs + refiner's rent + non-crude supplier's revenue + port rent + pipeline rent + shipowners' rent + CIF value of imports + cost on bounded vectors

In the SR port, pipeline and shipowners' rents, and value of imports are omitted thus:

$$\text{energy bill} = p_m x_m^o + c_{cs} x_{cs}^o + c_u x_u^o - c_{lr} x_{lr}^o + b_u \rho_u + b_{ncs} p_{ncs} \tag{2.12}$$

• The *refiner's rent* is the difference between the finished product value and total variable costs, i.e., operating costs and crude supply costs.

refiner's rent = value of finished products - total crude CIF value - non-crude suppliers' revenue - total operating costs - cost on bounded vectors

In the SR,

$$\text{refiner's rent} = b_{pd} p_{pd} - p_m x_m^o - b_{cs} p_{cs} - c_u x_u^o + c_{lr} x_{lr}^o - c_{pd} x_{pd}^o - b_{ncs} p_{ncs} \tag{2.13}$$

• The *producer's revenues* is the total revenue accruing to the producer of crude, also called *crude rent*,

producer's revenue = marginal crude value (FOB in WEM, CIF in SR) + non-marginal crude value (FOB in WEM, CIF in SR) + indigenous FOB crude value

In the SR,

$$\text{producer's revenue} = p_m x_m^o + b_{cs} p_{cs} \tag{2.14}$$

From relations (2.11), (2.12), (2.13) and (2.14), the following relations arise:

energy bill = value of finished products,

refiner's rents = value of refinery processing units = $b_u \rho_u$,

producer's revenue = energy bill - refiner's rent - non-crude supply - refinery operarting costs.

• Additionally, the crude spot freight rates from an exporting area to any importing area are easily derived in the WEM (a relation per area, per crude, per tanker size) to read:

freight rate = CIF crude value (at importing area) - FOB crude value (at exporting area)

and, the right hand side of this equality balances as follows,

CIF crude value - FOB crude value = transport operating costs + port rents + bunkers value (at source and destination) + bare boat charge (ship hiring cost or shipowners' rent)

As mentioned in Chapter 1, additionally to verifying crude and product prices, and refinery processing unit rents against the corresponding Rotterdam spot values, a work on the verification of spot freight rates generated by the 7-area WEM against those prevailing in the oil transport sub-system worldwide would be necessary to fully support the hypothesis of Unidimensionality in energy pricing implicit in the model.

2.5 LINEAR PROGRAMMING ADDITIONAL NOTES

2.5.1 On the LP solution software: APEX-III and MPSX

Two LP solution software packages have been used to solve the SR and 7-area WEM experimental cases, namely, APEX-III (from Control Data Corporation) and MPSX (from IBM).[36] Both are provided basically with the same programming features, i.e., the Revised Simplex Algorithm for solving the prima/dual problem, Linear Parametric Programming and Mixed Integer Programming. Additionally, MPSX provides the Separable Programming option for approximating non-linear functions. The software is programmed to solve a standard minimization problem (as in problem (2.1)), by a priori converting all restrictions into equalities by adding the appropriate slack variables.

The packages include also Matrix Generator programs which facilitate the presentation of the LP problem to the computer algorithm. The user does not need to formulate the constraints which in real practical applications may become considerably large: the SR model comprises 60 rows (constraints) and 80 columns (variables) whereas the 7-area WEM includes around 800 rows and 3250 columns in its original version. The matrix generator does require the preparation of an input data deck in a special format which the user can construct manually.

A further computer package, OMNI (from Haverley Systems, Inc.), facilitates greatly the generation of the LP technology matrix and provides report writing options. It can create the input data deck when supplying it with the necessary primary data. OMNI is then an intermediate programming phase between the models raw data and the real a_{ij} technological coefficients required by APEX-III and MPSX. An OMNI software or similar is obviously of great utility for it reduces further the errors in data handling on top of allowing for every piece of intermediate information to be tabulated and accessed directly when changes of parameters are required. However, the cost of maintaining OMNI is high, there is then a trade-off between its use and the cost of an error when using cheaper less accurate data handling methods.

OMNI versions of the SR and the 7-area WEM are now implemented.

---

[36] In this thesis APEX-III was used at the University of London Computer Centre, and MPSX at the Programme Group on Systems Research and Technological Development (STE), KFA, Juelich, FR Germany, while the author was performing fieldwork on Energy Modelling there.

2.5.2 Bounded Vectors

The LP problem variable vector-x is normally restricted to be non-negative, i.e., $x \geq 0$. In many problems further restrictions on variables are imposed. In the models there are some upper bounded variables so that problem (2.1), page 35, becomes:

$$\text{Min} \quad z = c_{nb}x_{nb} + c_{bv}x_{bv} \tag{2.15}$$

$$\begin{aligned} \text{s.t.} \quad A_{nb}x_{nb} + A_{bv}x_{bv} &\leq b \\ 0 \leq x_{bv} &\leq u_{bv} \\ x_{nb} &\geq 0 \end{aligned}$$

where, $x = (x_{nb}, x_{bv})$, $c = (c_{nb}, c_{bv})$, and $A = (A_{nb}, A_{bv})$ are respectively the variable vector-x, objective function parameters-c and technology matrix-A conveniently partitioned into the non-bounded and bounded vector classes.

To solve problem (2.15) by the Simplex Algorithm the so-called Upper Bounding Technique[37] is applied: it requires slight modifications to the LP theory with respect to the feasibility and optimality conditions and the algorithm itself. Accordingly, problem (2.15) is converted first into problem (2.16) by adding the slack variable s_{bv} to the upper bounded variable,

$$\text{Min} \quad z = c_{nb}x_{nb} + c_{bv}x_{bv} \tag{2.16}$$

$$\begin{aligned} \text{s.t.} \quad A_{nb}x_{nb} + A_{bv}x_{bv} &\leq b \\ x_{bv} + s_{bv} &= u_{bv} \\ x_{nb}, s_{bv} &\geq 0 \end{aligned}$$

In this problem, a *feasible* solution will be one assigning to each variable x_{bv} a non-negative value not exceeding its upper bound u_{bv} (and, of course, satisfying the constraints); it should then be noticed that the upper bound condition must be satisfied by both basic

---

[37] For details, see Dantzig, G.B., *Linear Programming and Extensions*, (New Jersey: Princeton University Press, 1963), pp.368-375; and Taha, H.A., *Operations Research, An Introduction*, (New York: Macmillan Publishing Co., Inc., 1971), pp.267-278.

and non-basic variables in order to prevent any non-basic variable to become infeasible when entering the basis. In so doing, some non-basic variables are fixed at their lower limit of zero and some at their upper limit of u_{bv}.

A *basic feasible* solution will be one in which the non-basic variables take values either zero or u_{bv}.

From (2.16) a non-basic variable set at its upper bound of u_{bv} is further transformed in $x_{bv} = u_{bv} - s_{bv}$ in order to make the linear system one in which all non-basic variables are set at zero. In this way, if x_{bv} is set at its lower bound of zero, it will be left invariable in the system, otherwise, the expression $u_{bv} - s_{bv}$, where s_{bv} is zero, will substitute for it. The problem will then appear as,

$$\text{Min} \quad z = c_{nb}x_{nb} + c_{bv}[\ x_{bv} \ \text{ or } \ (u_{bv} - s_{bv})\ ] \tag{2.17}$$

$$\text{s.t.} \quad A_{nb}x_{nb} + A_{bv}[\ x_{bv} \ \text{ or } \ (u_{bv} - s_{bv})\ ] \leq b$$

$$x_{bv} = u_{bv} - s_{bv}$$

$$x_{nb}, s_{bv} \geq 0$$

For a non-basic variable x_{bv} set at its lower bound of zero the normal optimality conditions apply since the coefficient of x_{bv} in z is non-altered. If x_{bv} is a non-basic variable set at its upper bound of u_{bv} then,

a) the variable s_{bv} becomes non-basic at its lower bound of zero and its coefficient in (2.17) is of the opposite sign of that of x_{bv} hence reverting the optimality condition;

b) an additional term, namely, $c_{bv}u_{bv}$ is added to the cost row: the coefficient c_{bv} attached to a non-basic variable set at its upper bound is the *reduced cost* per unit of activity level x_{bv} and $c_{bv}u_{bv}$ the total cost implied by keeping the non-basic variable x_{bv} at a non-zero level;

c) additional values $A_{bv}u_{bv}$ are added to the original constraints and the corresponding x_{nb} (basic variables) values expressed as,

$x_{nb} = (A_{nb})^{-1}$ $[b - A_{bv}\ u_{bv} - s_{nb}]$, for $s_{nb} \geq 0$ slack variables.

The Upper Bounding Technique is implemented in the APEX-III and MPSX solution algorithms. The non-basic variables set at their upper bounds and their corresponding reduced costs are shown in the solution printout of the problem. They are required when the analyst is performing mass or economic balances in the LP model.

2.5.3 Sign Convention on Marginal Values

A precise definition of a price-p as understood in the LP context is stated as:[38]

> The *price* of an item is its exchange ratio relative to some standard item. If the item measured by the objective function is taken as standard, then the *price*-Π_i of item i is the change it induces in the objective Z per change of b_i, for small changes of b_i.

The price Π_i, p_i in this work, is then the derivative of the objective function Z with respect to factor b_i, i.e., $p_i = \partial Z/\partial b_i$, and a marginal value from the point of view of marginal analysis. The mathematics of LP in adopting a sign convention for prices-p_i may stress either the positive or the negative side of p_i, i.e., the latter is either a right hand side (increases in b_i) or left hand side (decreases in b_i) partial derivative.

If the standard being measured is profit, then conventionally, an asset or input factor to a firm (refinery) have associated negative prices for an increase in their availabilities will produce a decrease in the objective function Z; a product output will have a positive

[38] Dantzig, G.B., *Linear Programming and Extensions*, op.cit., p.264.

price as an increase in production (demand) will expectedly produce an increase in profit.

SR and 7-area WEM dual objectives are profit maximization. Expression (2.9), page 48, comprises the dual objective for the SR:

$$\text{Max } Z = -b_{cs}p_{cs} - b_u\rho_u - b_{ncs}p_{ncs} + b_{pd}p_{pd}$$

The former sign conventions are applied in working out the general economic balance for the SR (expression (2.11) on page 58). Crude and non-crude supplies, and refinery unit capacities have associated negative prices for small increases in their availabilities will produce a decrease in total revenue *given product demands and prices remain constant*; and product demands have positive prices.

Similarly it could have been possible to value input and output prices as positive and negative respectively. In this case the primal coefficient row would have been inverted in sign and type (minimization would change to maximization) so to make the economic balance possible.

2.5.4 Stability of the Solution of a LP Problem

In order to investigate the stability of the optimal solution of the LP problems, methodological approaches using basically the primal-dual relationships have been developed, namely, Sensitivity Analysis (SA) or Post-optimality Analysis (PA), and Linear Parametric Programming (LPP).

While in carrying out the analysis both methods work from an existing optimal solution, SA is concerned with the ranges of the technological, cost and/or resource vector coefficients which keep the same solution feasible and optimal; and LPP investigates the series of optimal solutions as one or more coefficients (technological, cost and/or resource) are changed at a time.

SA is useful in investigating the stability of determined coefficients of an optimal solution. This implies the calculation of the coefficients ranges (r_{min}, r_{max}) in which that solution will remain optimal. If an optimal solution is very sensitive to determined coefficient values, i.e., the range of variation of the coefficient is small, it would be necessary an effort in finding accurate estimates of the coefficient (see section 2.5.5).

The LPP goes beyond the ranges of an optimal solution and analyses the different optimums when a coefficient is set to vary linearly with a parameter λ.

LPP is the post-optimal methodological approach carried out in this work for the interest here is that of tracing the behaviour of the system, i.e., of the crude and product prices, refinery units rents, etc., under particular market conditions. Every market condition is then represented by a particular setting up of a parameter or parameters λ_i with which an optimal performance is associated.

In general, the LPP method is formulated as:

$$\text{Min} \quad z = (c + D\lambda)x \tag{2.18}$$

$$\text{s.t.} \quad Ax \leq b + B\theta$$

$$x \geq 0$$

$$\lambda_{min} \leq \lambda \leq \lambda_{max}$$

$$\theta_{min} \leq \theta \leq \theta_{max}$$

where, x, c, b and A are as in (2.1), page 35; parameters λ and θ vary in the given ranges, and D and B are known vector constants (usually equal to unitary vectors). As mentioned earlier, the LPP method starts with the optimal solution (that at wich λ and θ are zero) and chooses consecutively λ and θ values producing optimal solutions until the parameters limits are reached, or not further change of any optimal solution occurs within the parameters limits.

In performing right hand side LPP the feasibility of the problem may be affected so checks on this line are necessary every time in calculating the next optimal solution.

The LPP simultaneously applied to the right hand side and objective function coefficients is known as Multiparametric Programming (MP) where the solution algorithms become complicated indeed.[39] In this particular thesis, only one coefficient at a time is linearly varied.

The LP solution packages assist the analyst in performing SA and LPP. They are provided with a so-called RANGE option by which the ranges

---

[39] See Gal, T., *Postoptimal Analysis, Parametric Programming and Related Topics*, (Great Britain: McGraw-Hill Inc., 1973), and Taha, H.A., *Operations Research*, op.cit.

r_{min} and r_{max} of coefficients-c and vector-b are generated. Similarly, LPP can be performed by giving as input the minimum and maximum values of parameters λ and θ and the D and B values. Using APEX-III the user can obtain the optimal solutions for variations of one or more than one of the coefficients *either* in the objective function coefficients-c *or* in the resource vector-b. MPSX allows additionally for simultaneous paremetirizing vectors c' and b, i.e., it has MP facilities available.

2.5.5 Insensitivity of the SR Technology Matrix

An alternative matrix representation of a LP probrem may be done by partitioning the technological coefficient matrix A in four sub-matrices A_{ij}, $i,j=1,2$ in order to group constraints and activities together according to their status (at any current iteration tableau, including the optimal solution's) of slack/binding constraints and basic/non-basic activities. The resultant four-element partition and its associated primal/dual objectives can be algebraically presented as follows:

$$x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}$$

$$\begin{array}{cccc} & & \text{Min} & \text{Max} \\ c = & \begin{bmatrix} c_1 & c_2 \end{bmatrix} & \begin{bmatrix} z \end{bmatrix} & \begin{bmatrix} Z \end{bmatrix} \end{array}$$

$$A = \begin{bmatrix} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{bmatrix} \quad \begin{bmatrix} b_1 \\ \hline b_2 \end{bmatrix} \quad \begin{bmatrix} p_1 \\ \hline p_2 \end{bmatrix}$$

$$c = \begin{bmatrix} c_1 & c_2 \end{bmatrix}$$

where,

$x = (x_1, x_2)$ crude supply and refinery process activities vector,

x_1 active processes of vector x, $x_1 \neq 0$,

x_2 unused activities (non-negativities set equal to zero);

$c = (c_1, c_2)$ unitary cost vector,

c_1 costs on active processes x_1,

c_2 costs on unused activities x_2;

$b = (b_1, b_2)$ resource availability vector,

b_1 available factors attached to binding constraints, i.e., constraints on crude and non-crude supply, product demands and those refinery unit capacities and quality specification constraints becoming binding,

b_2 available factors attached to slack constraints, i.e., refinery unit capacities and quality specification constraints remaining slack;

$p = (p_1, p_2)$ price vector,

p_1 vector of marginal values attached to binding (effective) constraints,

p_2 vector of marginal values attached to slack (non-effective) constraints, it is equal to zero;

$A = (A_{ij})$, $i,j=1,2$ technology matrix,

A_{11} sub-matrix of technological coefficients linking binding constraints (effective factors) to active processes,

A_{12} sub-matrix of technological coefficients linking binding constraints to unused activities,

A_{21} sub-matrix of technological coefficients linking slack constraints (non-effective factors) to active processes,

A_{22} sub-matrix of technological coefficients linking non-effective constraints to unused activities.

The conditions for a feasible and optimal solution (of a primal minimization problem) in terms of the above partition are given by:

$$x_1 = (A_{11})^{-1} b_1 \quad \text{and} \quad A_{21} x_1 \le b_2 \qquad \text{(Feasibility)}$$

$$p_1 = c_1 (A_{11})^{-1} \ge 0 \quad \text{and} \quad p_1 A_{12} \le c_2 \qquad \text{(Optimality)}$$

It follows from here that sub-matrix A_{11} is crucial to the solution whereas coefficients of sub-matrices A_{12} and A_{21} could be changed between $-\infty$ and their respective upper limits of c_2 and b_2 without affecting the existing optimal and feasible basis.

According to the LP theory, every LP problem comprises an *insensitive* technology sub-matrix, that is, sub-matrix A_{22}, whose coefficients play no role in determining the problems's solution; they could be changed in the interval $(-\infty,+\infty)$ having no effect whatsoever on the optimal solution therefore needing not to be accurately obtained. Efforts can be then placed on the accuracy of the other sub-matrices coefficients, specially of sub-matrix A_{11}. The inmediate checks should be done on the thecnological coefficients of those basic activities on original equality constraints.

Some experimental work was performed in this line with the SR: from about one hundred LPP cases which included changes in the chemistry of supply/demand and refinery unit capacities (resource vector), and changes in the Arabian Light price (objective function coefficient), matrix A_{22} was identified. For every case the intersections between non-basic variables and slack rows (constraints) in the optimal solution were recorded. The intersections presented below occurred in every case, they form then sub-matrix A_{22} of the Single Refinery. The coding of processes and constraints is in Appendix C.

Single Refinery Activities

Constraints		Distillate A3DTJ	A3ETJ	A3VHM	A3WHM	Platform A3PR3	A3PR6	A3PR5	Blending A44TH	A44TL	A44KH	A44KL	A44YR	A3TR1
Refinery	1B3BCD	•	•	•	•									
Units	1B3BCV			•	•									
Capacities	1B3BCP					•	•	•						
	1B4HFP		•		•				•		•			
	1B4HFS		•		•				•		•			
Quality	1B4HFV		•		•				•					
Control	1B4LFP	•		•						•				
Spec.	1B4RMD					•	•	•					•	
	1B4RME					•	•	•					•	
	1B4RMB													•

These technological coefficients were then varied within the range of (-9000,+9000) which is obviously unreasonable in terms of the refinery constraints, and further runs performed for the same variant cases: *the same sub-matrix A_{22} above resulted again in every case.*

At the refinery level, matrix A_{22} represents those activities the refinery is not currently pursuing on particularly currently non-effective factors. Being for the SR:

- Some distillation processes, namely, atmospheric cuts of the heaviest crude in SR, Tia Juana (Venezuela), and vacuum cuts of the lightest crude in SR, Hassi Messaoud (Algeria) by certain operating modes.
- Some catalytic reforming processes of regular motor spirit.
- Blending activities: blending low sulphur waxy distillate and kerosine to the low sulphur fuel oil pool; blending kerosine to the high sulphur fuel oil pool; and blending catalytic cracked spirit to regular motor gasoline.
- Reduction of lead content of regular motor spirit.

These processes intersect with the non-effective factors:

- Atmospheric and vacuum distillation and catalytic reforming capacities, i.e., these units are always in surplus.
- High sulphur fuel oil quality specificiations on pour point, sulphur content and viscosity.
- Low sulphur fuel oil quality specification on pour point.
- Regular motor spirit quality specifications on Distillation + Loss @ $100^{o}C$, olephin content and lead content.

For the quality specification constraints being non-effective factors means they meet appropriately the products market requirements with no need for them to be binding (as they are $\leq$ constraints) and with no contribution from their associated non-pursued processes.

2.6 CONCLUDING REMARKS

The assumption of a competitive equilibrium market in the world oil system is best represented in a Linear Programming model. Minor non-linear functions inherent to the oil refining sub-system can be linearized without causing drastic effects on the real refining functioning.

Hypothesis **H.1** of section 1.2, Chapter 1, can be easily checked in the model through mass and economic balances. Equilibrium in mass and economic balances support **H.1** in the model: a mass balance at the refinery level is a representation of a non-excess supply/demand pattern. Imbalances are duly accounted for by introducing additional disposal and procurement variables at high penalty costs for the refinery. They are an indication of infeasibilities in the system, i.e., of excess or deficit in supply and/or demand (but not necessarily indications of infeasibilities in the LP context) and of non-optimality in terms of economic performance.

An economic balance at any level follows by using the corresponding vector of marginal values, equilibrium price-p, and the activity flows at that level. Numerous mass and economic balances as expressed by equations (2.10) - (2.14) have been carried out for different SR parametric programming cases. Neglecting minor computational errors, the equalities prevailed in every case. The Arabian Light acts invariably as the swinger (marginal crude source) in the provision of marginal supplies of crude.

The economic balance is continuously used in this thesis, in particular in Chapter 3 when studying the formation of refinery rent at the refinery processing unit level. The LP primal/dual relationships provide a useful mean of analysis at the disaggreagated level.

The step-wise LP models solutions allow also for a good representation of hypothesis **H.2** inasmuch as marginal conditions are met and parametric programming of varying supplies/demands for fixed equilibrium prices, and vice versa, varying prices for fixed demands (this latter by variations of the marker crude price and any other a priori fixed input price) are plausible to perform.

That hypotheses **H.1** and **H.2** hold in the models shows the right kind of model has been selected to represent them and not that the hypotheses are indeed met in reality. The 7-area WEM validation process requires the verification of hypotheses **H.1** and **H.2** against the real world to which it is necessary hypotheses **H.3** and **H.4** are verified for the models' outcome, as discussed already at the end of section 1.2, Chapter 1.

CHAPTER 3: THEORETICAL CONSIDERATIONS IN THE STUDY OF REFINERY RENTS

Without pretending to be exhaustive, a review of literature on the subject of rent theory is first introduced. It is later intended to place the conceptual study of refinery rents in the framework found: it is shown that the traditional analysis applied to the case of the rent of land is, with slight modifications, applicable to the refinery as a particular case of the rent on fixed capital.

3.1 INTRODUCTION

In the attempt to give a theoretical background to the study of Refinery Rents is found necessary to review available literature on rent theory. Various theoretical groups[40] have conceptualized the subject of rent from different points of view, namely, the classicals (D. Ricardo, A. Smith), the neo-recardians (P. Sraffa, L. Pasinetti),[41] the neo-classicals (W. Marshall, L. Walras) and the paretians (J. Robinson, K.E. Boulding, H.D. Henderson). Equivalence of concepts seems to prevail as the theorists can be framed into either of two theoretical approaches, *the residual* or the *marginal*.

The concept of rent was first applied to land as means of agricultural production (in particular to the cultivation of a single crop)[42] and it was further extended to other economic agents.

---

[40] See Worcester, D.A. Jr., 'A Reconsideration of the Theory of Rent', *The American Economic Review*, (June 1946), for a full discussion on the developed theories of rent.

[41] See Kurz, H.D., 'Rent theory in a Multisectoral Model', *Oxford Economic Papers*, XXX, (1978). " A comparison with classical rent theory as formulated, in particular by Ricardo brings out a close affinity of the two approaches such that Sraffa's analysis may be called 'neo-Ricardian' ", op.cit., p.16.

[42] In the classical view land was considered a factor in fixed supply, having an inelastic supply curve thus not changing in time. As the factor became scarce due to the intensity of cultivation, the different qualities of land determined the use of the most fertile land in the first instance; lands of declining fertility were progressively used and there always remained a type of land (the less fertile) at the margin of use. A production cost was associated with every type of land; the difference in production costs between the most fertile land and the marginal land was the

Since the oil refinery embeds capital equipment -durable goods-, the works developed by Sraffa,[43] an followers, and Salter,[44] provide us with the basis to link the economic classical concepts applied to land to a different kind of production factors. Capital equipment, though a changeable factor when referred to the long run, it becomes fixed in supply in the short run because of the time-lag required by industries to absorb new technology. Thus in the short run the oil refinery processing unit capacities may become scarce and produce a rent partly determined by the difference in technical productivities associated with the different processing units, and on the other hand, dependent on the oil market conditions.

Appendices A and B provide a review of the concepts will be applied hereinafter.

This Chapter stands on theoretical grounds: first, several conceptual interpretations of rent as found in the literature are put forward; second, the concept as applied here and its use in analysing the level of oil refinery profitability are considered; third, the rent formation at a disaggregated level of refinery processing unit is analytically developed. The latter aims at finding a fomalization of the rent formation to show that under competitive conditions (which this thesis has assumed, Chapter 1), refinery processing units as any other kind of capital equipment in fixed supply can earn a surplus or rent in the same way as the natural resources such as land and mineral deposits do. Hence the analytical treatment often used for the determination of the rent of land applies to the oil refinery without great conceptual variations.

---

rent accruing to the refiner. Different capital and labour intensive methods were also employed on land of an homogenous quality; rent here was defined as the difference between unequal returns to capital when applied to land in different proportions.

[43] Sraffa, P., *Production of Commodities by Means of Commodities*, op.cit.

[44] Salter, W.E.G., and Reddaway, W. B., *Productivity and Technical Change*, (Cambridge: Cambridge University Press, 1966).

3.2 THE CONCEPT OF RENT

3.2.1 Theoretical Review

Generally speaking, rent refers to the payment to or the cost of any factor of production. Worcester[45] points out:

> While "land" may be defined at the outset, this procedure is quite impossible for "rent". Some of the things to which it might refer are
>
> 1. An entrepreneural payment to certain agents of production.
> 2. Part of the entrepreneural payments made to certain agents of production.
> 3. The income received by owners of certain productive resources.
> 4. Part of the incomes received by owners of certain productive resources.

Further, [45]

> Those who prefer numbers 1. and 3., apparently believe that the essential characteristic of "rent" is that is the full long run remuneration of a certain group of productive agents called, collectively, "land", while the other group believes that the essential characteristic of rent is that it is a surplus return.

The concepts of the residual, surplus rent are to be found in the classical and paretian theories whereas the concepts of long run equilibrium rent and quasi-rent are attached to the neo-classical concepts of marginal productivity.

For the paretians,[46]

> [...] rent is defined as being the return to any agent of production greater than that required to keep it in its present employment. It is a return over and above the normal return to an agent, and is clearly a "surplus" return.

---

[45] Worcester, D.A. Jr., 'A Reconsideration of the Theory of Rent', op.cit., p.260.

[46] Ibid., pp.261-262.

And as Robinson states,[47]

> The essence of the conception of rent is the conception of a surplus earned by a particular part of a factor of production over and above the minimum earnings necessary to induce it to do its work.

This conception implies that there is an alternative use of the production factor and an opportunity cost which allows it to gain a surplus. This latter as the difference between the actual output price and the lower opportunity cost of the alternative output produced by means of the resource in question; if the lower opportunity cost does not exist, this alternative output will remain non competitive and the rent will disappear.

Two definitions given in Worcester's discussion on rent theory are considered now. They relate respectively to rent and surplus returns.

Definition 3.1. Rent is the opportunity cost of land, a remuneration the firm must pay to the agent in order to secure the firm's use.

Definition 3.2. "Factor Profits" is the net income from sales after all costs have been paid including allowances for depreciation, interests, opportunity costs of management and payments to bondholders and stockholders.

In definition 3.2 "Factor Profits" is the surplus of value of outputs minus variable and fixed costs. "Factor Profits" keeps relation to what the paretians call surplus returns in so far as none has to be paid by the firm in order to retain the factors in present employment. In the refinery context "Factor Profits" is indeed related to rent, and the latter, in turn differs from definition 3.1 where rent is rather a fixed cost for the producer.

Under conditions of perfect competition and the single firm, there are no alternative uses for the production factors: in the short run the refiner has no alternative but to satisfy a generally fixed demand with the available fixed resources. Rent, if existing, will not be a surplus over an opportunity cost, rather, a payment to the owner of productive resources resulting from the difference between the value of outputs and the value of (variable) inputs to a production process:

[47] Robinson, J., *The Economics of Imperfect Competition*, (London: The Macmillan Press Ltd., 1969, reprinted 1976), p.102.

it is a residual payment. This leads to the third consideration of rent, which is related to the neo-classical thought.

Definition 3.3. *Rent* refers to the (return to a factor) price of a factor which in the long run is fixed in supply (such as land), and *quasi-rent* refers to the price of a factor which is fixed in the short run. The assumption of a competitive market implies that the quasi-rents become zero in the long run because the firm is in equilibrium and what it is left as earnings is just normal profits.

References to the term quasi-rent are found, among many, in Meriam,[48]

> Quasi-rent usually refers to the short run net income of machinery, equipment, and buildings. This income is peculiarly dependent upon the demand for the product.

And further,[49]

> In the theory of quasi-rent, on the other hand, the crux of the whole discussion is the demand for particular products and the derived demand for productive factors which cannot be used immediately for other purposes.

The concept of quasi-rent is graphically explained in Figure 9 on page 78. It depicts the short run cost-price curves for a single firm producing commodity q,[50] where the variables are defined as,

MC marginal cost curve of commodity-q,

AVC average variable cost per unit of output,

ATC average total cost per unit of output, and,

---

[48] Meriam, R. S., 'Quasi-Rents', in: *Explorations in Economics: Notes & Essays in Honor of Frank W. Taussig*, (New York: Kelley, reprinted 1936 edition), Chapter X, p.317.

[49] Ibid., p. 319.

[50] Certainly this is not the case in an oil refinery, a multi-product multi-process firm, however, because of the variability of proportions among oil products, the production of a particular oil product can be treated separately, its output level varied while keeping the other oil production levels fixed and the marginal costs expressed in terms of the remaining joint products values.

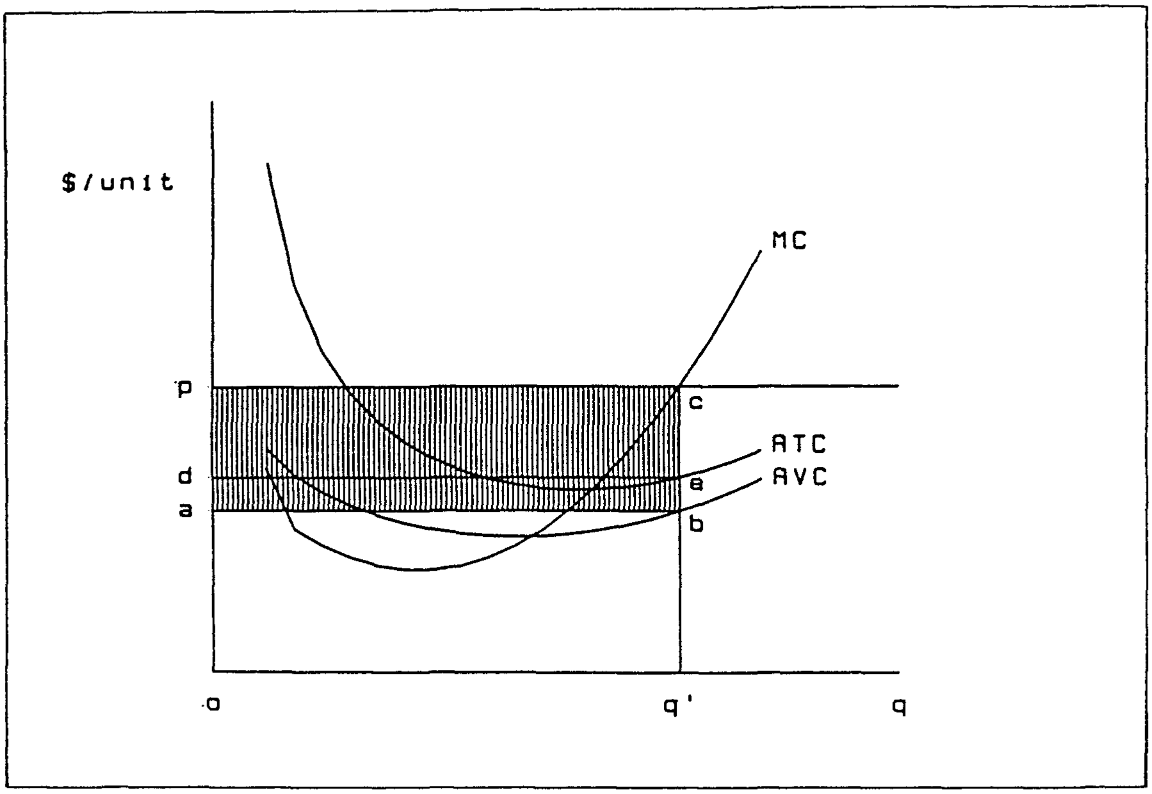


Figure 9. Cost-price curves and Quasi-Rent.

p market price per unit of output.

The firm maximizes its profit at output level q'; total returns, TR, total variable costs TVC, total fixed costs TFC and total costs (fixed plus variable) TC at q' are given by,

TR $= q'p$ which is the area oq'cp in the diagram,

TVC $= q'AVC$ which is the area oq'ba in the diagram,

TC $= q'ATC$ which is the area oq'ed in the diagram,

TFC $= q'(ATC - AVC)$ which is the area abed in the diagram,

and the difference between total returns and total variable costs is the quasi-rent, i.e.,

QR $=$ TR - TC which is the area abcp in the diagram.

From the quasi-rent the firm must pay the total fixed costs, TFC, and the residual, if any, is the excess profits accruing to the resource's owner. This latter is then,

Profit = QR - TFC,

in other words, for a firm being in a position of producing a profit, in the short run, the quasi-rent it receives from the use of the fixed productive factors, must at least cover the fixed costs of producing output q.

The quasi-rent is also called *economic rent*. Since the present study is confined to the short run, hereinafter either term will be called simply *rent*.

It is noted that all above definitions imply that rent in the production process and as attached to a fixed production factor (resource), is a difference, whether a difference between two opportunity costs, between a marginal cost and an average cost, or between total revenues and total costs, a surplus or residual factor always occur, hence the similarity in concepts.

Extensions or modifications to these concepts are necessary to define the oil refinery rent. This follows next.

3.2.2 The Rent in the Oil Refinery Context

Definitions 3.1 and 3.2 above do not suit the study of refinery rents: the refinery has been assumed a fully paid resource and the oil refiner (or industry) at the same time the owner of the land where the refinery is installed. Definition 3.2 as mentioned above is also related but does not give a definition of rent. Definition 3.3 relies on assumptions that apply to the refinery units, hence the reader may find a relation between that and definitions 3.4 and 3.5 next which pose the conception of rent from the point of view of the refiner.

At the refinery, conditions of existence and determination of rent are dependent on:

- the quantities of input factors required to produce the output demand (crude supply and capacity usage),

- the price of the input factors,

- the output demand, and the available refinery upgrading facilities (capacity and complexity) to satisfy it,

- the market price of oil products.

The conception of rent in the present study is now given:

Definition 3.4. Rent at the refinery level is a surplus above value of outputs minus value of inputs exclusive of fixed charges. A profit/loss for the refiner will arise on payment of the incurred fixed costs of production out of the refinery rent, if any.

Definition 3.5. The rent at the refinery arises as follows:

1. The marginal rent of a refinery processing unit-u is the reduction in the overall variable costs, or the increase in total revenues, for a small increase in the capacity of processing unit-u.

2. The total rent of the refinery processing unit-u is the marginal rent multiplied by its installed capacity.

3. The total rent of the oil refinery is the sum over all total refinery processing units rents as in 2., i.e., over all refinery processing units-u for which *marginal rents are positive*.

Definitions 3.4, 3.5-2. and 3.5-3. are equivalent. Indeed, there exist economic balances at the refinery processing unit-u or at the whole of the oil refinery which make the definitions equivalent. Moreover, the rents defined above have been already expressed as the economic summaries in algebraic terms, that is, in terms of the LP primal/dual formulation/solution (see section 2.4.2, Chapter 2): rent as in Definition 3.5-3. is accounted for in the economic equation (2.13), and rent as in Definition 3.5-2 is implied by the expression $b_u \rho_u$ there, and worked out for the particular single refinery case in Appendix B. The equivalence of concepts follows from expressions (2.11) to (2.13) in the same section.

It is noted that under surplus capacity the marginal rent becomes zero, and if that occurs for all refinery processing units, the total rent too. The existence of rent is linked to output expansion and capacity availability (capacity usage) since as production expands, the available capacity progressively becomes restrictive, the marginal cost of producing the output increases and with it the marginal rent. Such a process is pointed out by Adelman,[51]

> In general, incremental cost rising with higher output expresses the resistance of output to expansion, and gives the signal that production is pressing against the limits of capacity.

When the 'limits of capacity' are reached, it is said the refinery processing units become scarce, and to cite Adelman again, "Scarcity is the strain imposed by consumption on available supply, registered in competitive markets by price."[52]

That scarcity may be related to both capacity and product supply, in any case the resource in question will reach production limits and rent will arise. For the refiner output prices and the associated refinery rents should be competitive with market prices. Indeed, the prices generated by the LP solution of the Oil Refinery System are those to be compared to spot market prices.

Subsequent sections will treat the subject in more depth, here it suffices to have conceptualized the *oil refinery rent*. However, using the Single Refinery model an example is now following. It aims at showing the equivalence between the *residual* (difference between output and input values), and the *marginal* (change in the objective function when capacity is incrementally changed). The reader may skip the example without loss of generality.

---

[51] See Adelman, M.A., *The World Petroleum Market*, op.cit., p.176. Marginal costs of crudes and oil products (prices in a competitive market, as interpreted in the present work) are to Adelman the corresponding incremental costs. For a discussion of this point see Adelman on the same work, pp. 3-4 and ff.

[52] Ibid., p.1.

Example 3.1

Firstly, the constant returns to scale property of the LP production functions implies that an oil product output is paid the sum of its marginal products values. Secondly, on this latter basis the equivalence of the residual and marginal rent at the refinery processing unit level can be shown.

The multi-process nature of a refinery makes it possible for a single product to be obtained by a variety of processes; similarly, one single process may produce a variety of products in given proportions according to technologies available. It is possible to summarize the production of a single product in a single mass balance equation, as was already seen in Chapter 1. In this present example the one product is heavy fuel oil. Since in the models used in the thesis the factor labour is omitted, crude oil and the refinery processing units are the basic input factors to the refining process.

x_u and x_{pd} are as in section 2.2.1, components of activity vector-x, and p_{pd} and ρ_u components of price vector-p. These components, in turn, have sub-components associated, letting them be, for this example:

$x_u = (x_{eal}, x_{wal}, x_{gkt}, x_{wkt}, x_{ch1}, x_{ch7})$, $x_{pd} = (x_{hr}, x_{hd}, x_{hb})$,

$x_{pd} = (x_{fd}, x_{bit})$, $p_{pd} = (p_{wax}, p_{hr}, p_{gas}, p_{ccsh}, p_{ccsl}, p_h, p_{dfo})$:

x_{eal} amount of AL crude processed by atmospheric distillation process mode-E,

x_{wal} amount of AL crude processed by atmospheric distillation process mode-W,

x_{gkt} amount of KT crude processed by atmospheric distillation process mode-G,

x_{wkt} amount of KT crude processed by atmospheric distillation process mode-W,

x_{ch1} amount of waxy fuel oil as feed to the catalytic cracking process-ch1,

x_{ch7} amount of long-gas oil as feed to the catalytic cracking process-ch7,

x_{hr} amount of heavy fuel oil consumed as refinery fuel,

Table 3-1. Data of a Single Refinery solution case: processes' yields and product prices.

process	$dy/dx_i = a_{ij}$	$a_{ij}p_h$	process's activity level
x_{eal}	.483	72.45	17.67
x_{wal}	.211	31.65	6.16
x_{gkt}	.3032	45.48	8.67
x_{wkt}	.3032	45.48	0.39
x_{ch1}	.19	28.50	3.29
x_{ch7}	.145	21.75	1.83
x_{hr}	1.	150.00	1.01
x_{hd}	1.	150.00	8.416
x_{hb}	1.	150.00	4.05

SR LP solution prices ($ / metric tonne)

Catalytic Cracking Input Costs

p_{wax}	p_{hr}
155.00	150.00

Catalytic Cracking Output Values

p_{gas}	p_{ccsh}	p_{ccsl}	p_{hfo}
180.00	175.00	328.00	150.00

Quality specifications imputed costs = .53

Operating costs = .39

x_{hd} amount of heavy fuel oil transferred to final demand,

x_{hb} amount of heavy fuel oil transferred to bitumen demand,

y_h total amount of heavy fuel oil produced.

p_{pd} vector of marginal costs respectively of waxy fuel oil, refinery fuel, gas, heavy and light catalytic cracked spirits, heavy fuel oil and gas oil.

With the data of Table 3-1, it is implied that:[53]

1. The total amount of heavy fuel oil produced is given by,

$$y_h = \Sigma\, a_{hj}x_j \qquad \text{E.3.1}$$

where the a_{ij}'s are the technological coefficients and equal to the marginal product of y_h with respect to the change in the level of input process x_j, processes in distillation and cracking (for i=h; j=eal,wal,gkt,wkt,ch1,ch7).

The value of y_h is given by,

$$y_h p_h = \Sigma\ (a_{ij}p_h)\ x_j \qquad \text{E.3.2}$$

where the $a_{ij}p_h$'s are the values of the marginal products a_{ij}.

- The production function as in equation E.3.1 and its value as in equation E.3.2 are expressed in this particular case as in equations E.3.3 and E.3.4 below,

$$y_h = .483x_{eal} + .211x_{wal} + .3032(x_{gkt} + x_{wkt}) + .19x_{ch1} + .145x_{ch7} \qquad \text{E.3.3}$$

$$y_h p_h = .483x_{eal}p_h + .211x_{wal}p_h + .3032(x_{gkt} + x_{wkt})p_h + .19x_{ch1}p_h + .145x_{ch7}p_h \qquad \text{E.3.4}$$

[53] Data other than input and output prices, operating and quality specification costs, are taken from the SR technological matrix.

- The distribution function of y_h and its value in terms of its intermediate and final uses is given by equations E.3.5 and E.3.6,

$$y_h = x_{hr} + x_{hd} + x_{hb} \qquad \text{E.3.5}$$

$$y_h p_h = x_{hr} p_h + x_{hd} p_h + x_{hb} p_h \qquad \text{E.3.6}$$

- Replacing the input processes levels and marginal product values of Table 3-1, in equations E.3.3 to E.3.6, we have that:

y_h = 13.476 mmt, the total amount of heavy fuel oil produced (equations E.3.3 and E.3.5).

And the total value of heavy fuel oil is given by:

$y_h p_h$ = 17.67*72.45 + 6.16*31.65 + (8.67+.39)*45.48 + 3.29*28.50 + 1.83*21.75 = 2021.00 million $ (equation E.3.4)

$y_h p_h$ = 13.476*150.00 = 2021.40 million $ (equation E.3.6)

Inputs = Outputs (mass balance) and Value of Inputs = Value of Outputs (economic balance).

- In equation E.3.4, for instance,

$.483 p_h = p_{eal} =$
= .483 t-h/t-al dist.-E * 150. $/t-h =
= 72.45 $/t-al dist.-E

is the value one tonne of Arabian Light crude is worth when yielding heavy fuel oil by distillation process mode-E.

Similarly,

$.19 p_h = p_{ch1} =$
= .19 t-h/t-wax process-ch1 * 150. $/t-h =
= 28.50 $/t-wax process-ch1

is the value one tonne of waxy fuel oil is worth when yielding heavy fuel oil by using catalytic cracking process ch1.

- Although the refinery processing units do not enter explicitly as input factors in equation E.3.3 they are embeded in the technological coefficients or marginal products a_{ij}. In this way the value of the marginal product-j reflects the crude input value plus the value of the embodied technology: $y_h p_h$ is then *the sum of marginal productivities values.*

2. The economic balance equation at the catalytic cracking unit of using catalytic cracking process-chl is expressed as,[54]

$$(p_{wax} + .07p_{hr} + 1.\rho_{cc} + .92)x_{chl} = (.142p_{gas} + .21p_{dfo} + .17p_{ccsh} + .225p_{ccsl} + .19p_h)x_{chl} \qquad \text{E.3.7}$$

where the left parenthesis is the value of inputs plus the operating costs and rent contribution and the right parenthesis is the value of outputs.

- The value of heavy fuel oil out of the catalytic cracking unit via catalytic cracking process-chl is $.19p_h x_{chl}$. This can be obtained from equation E.3.7,

input value - output value (minus heavy fuel oil value) + rent + variable costs = $.19p_h x_{chl}$

By replacing activity levels and price values of Table 3-1 in the order of equation E.3.7 above, and for simplicity dividing the whole equation by x_{chl}, it is obtained:

165.89 - 162.92 + 25. +.92 = 28.50 $/tonne.

It should be noted that this is the same value as obtained from equation E.3.4 and expressed above as p_{chl}.

Similarly for every term in equation E.3.4 there exists a refining process (not necessarily a catalytic cracking process) such that an expression of its value in terms of inputs and outputs as in equation E.3.7 exits.

On the other hand, p_{wax} and $.07p_{hr}$ are the marginal costs of inputs to the catalytic cracking unit.

From **1.** it follows that the value of heavy fuel oil attached to a cracking process is equal to the value of the marginal productivity of

[54] See economic balance, equation (2.11), Chapter 1, and Appendix B, note B.1, algebraic formulation of catlaytic cracking rent in the Single Refinery Model.

that process, and from **2.**, equal to a sum of marginal productivities values.

The unitary rent is in consequence a *marginal* value (algebraic sum of marginal values). This can also be seen from equation E.3.7: the catalytic cracking rent, ρ_{cc}, is the derivative of the expression given by E.3.7 with respect to the capacity utilization coefficient attached to ρ_{cc}, when using catalytic cracking process-ch1. More generally, if expression E.3.7 is called Z, then,

$$\partial Z/\partial a_{ccj} = \rho_{cc}$$

where a_{ccj} is the catalytic cracking capacity utilization coefficient of process-j.

And the rent is obviously the difference between input values including variable costs and output values to the catalytic cracking unit therefore it is a *residual* economic term.

As it is precisely expressed by Baumol, " [...] It is clear that what from one point of view is a surplus, from another is just a marginal product."[55]

[55] Baumol, W.J., *Economic Theory and Operations Analysis*, (London: Prentice-Hall International, Inc., 1977), footnote, page 593.

3.2.3 Profitability in the Oil Refinery

An Oil Refinery is profitable in the short run as long as the total rents accruing to the refiner cover the total variable plus fixed costs and leave a positive gain. Or, in terms of a tonne of crude,[56]

> Refining a ton of crude oil is profitable, and will be done if the receipts cover the *total* costs of the operation, including the necessary profit; while no *individual* product will be forthcoming if the market price is above its incremental cost.

In the short run investment decisions are assumed to have been made; rents do not represent a cost for the refiner; refinery processing units are already installed and fully paid: they do not have present cost. As Salter precisely points out, capital equipment installed and paid "...is a gift of the past".[57] Or rather, capital costs (refinery processing units costs) become rents, and have no influence in output levels; contrariwise, changes in output levels due to changes in the pattern of demand are likely to change the operational levels of the available production processes, therefore the refinery processing capacity usages therefore the level of rents.

With the discussions put forward in 3.2.1 and 3.2.2 as a background, the conditions for a profitable oil refinery to prevail are now analysed. For this purpose equation (2.13), section 2.4.2, Chapter 2, is drawn here and the fixed charges term (which as pointed out earlier, are ruled out of the SR and 7-area WEM models) are added. Accordingly, calling r the total refiner's rent,[58]

$$r = b_{pd}p_{pd} - p_m x_m - b_{cs}p_{cs} - c_u x_u + c_{lr}x_{lr} - c_{pd}x_{pd} - b_{ncs}p_{ncs}$$

and the following variables being,

R revenue, value of finished products, $b_{pd}p_{pd}$,

TVC variable costs of refining (excluding labour costs), i.e., input supply and operating costs, $r - b_{pd}p_{pd}$, (equal to the sum of the remaining terms in the equation above),

---

[56] See Adelman, M.A., *The World Petroleum Market*, op.cit., p.175.

[57] See Salter W.E.G., and Reddaway, W.B., *Productivity and Technical Change*, op.cit., p.54.

[58] The total rent is called r to distinguish it from the individual (marginal) refinery processing unit rent-ρ.

TFC fixed costs: salaries plus labour and maintenance costs,

AD allowances for depreciation (if any) on existing capital, and,

P refinery net profit, (in Definition **3.2**, "Factor Profits"),

the rent and the economic net profit become:

$$r = R - TVC \tag{3.1}$$

$$P = r - TFC - AD \tag{3.2}$$

i.e.,

$$P = R - TVC - TFC - AD \tag{3.3}$$

What equations (3.2) and (3.3) say is that to work out profits it is necessary to know the level of rents and fixed costs (including depreciation) or alternatively, the revenue and total costs (including depreciation); knowing the latter, the level of rents is, of course, redundant.

For the refiner to be in the position of having positive, zero or negative net profits the following conditions should hold:

(i) Rents are less than or equal to zero, $r \leq 0$: under this condition, equation (3.1) gives,[59]

$$R = TVC \quad \text{and} \quad R < TVC \tag{3.4}$$

The expected revenue from the value of refined products is either just enough to cover total variable costs, or lower than total variable costs. It is obvious from (3.3) that profit, $P = - TFC - AD$, is a net loss, becoming greater when r is negative.

If fixed costs have already accounted for a *normal profit*, then P could be just enough to cover fixed costs plus depreciation, however not allowing for excess profits.

(ii) Rents are greater than zero, $r > 0$: when rents are positive, the profit P can be either positive or negative depending on whether the rent r is greater or equal to fixed costs plus depreciation.

If r is positive and equal to fixed costs plus depreciation, we have,

---

[59] The case of R negative, will never happen if the LP technique is used for no activity in LP would be employed if $R < TVC$.

$$R - TVC = TFC + AD \quad \text{and} \quad P = 0 \quad \text{even though} \quad r > 0 \tag{3.5}$$

If r is positive and greater than fixed costs plus depreciation, we have,

$$R - TVC > TFC + AD \quad \text{and} \quad P > 0 \tag{3.6}$$

Here again if the refiner includes already a normal profit in his fixed costs, he will enjoy or not excess profits on the above situations. If on the contrary, fixed costs do not include a normal profit, only the refinery at the second position will directly account for a profit.

3.2.4 Oil Refinery Rent: extensions

A comprehensive analysis of refinery profitability as outlined above can only be carried out by knowing every piece of intermediate information to place in the foregoing equations. This goes beyond the scope of this particular work. So far, it suffices to have set out the basis for profit analysis and to have highlighted the importance for the refiner of his short run level of rents. As far as this study is concerned, rents will be analytically treated and functionally determined. Then, rather than a "post-profit-analysis", it is aimed at finding the *primary revenue-profit indicator* as a reflect of *internal refinery performance* and *external market conditions*.

In so doing relevant questions relating refinery rents to past and current market conditions which will be treated separately in coming chapters, are put forward below:

1. The total rent accruing to the refiner as it derives from Definition 3.5-3, is a sum of individual processing units rents. There, some refinery processing units (in surplus) may be generating zero rent and others (in deficit) positive rents. Hence for an oil refinery to be profitable as in position (3.6) the scarce refinery processing units must give level of rents high enough to cover their own variable costs, the *other* refinery processing units operating costs and the fixed charges.

Generally, and in particular in North West Europe, the crude distillation units have been/are almost always surplus; reforming units have been/are also nearly surplus. In fact both units have to be necessarily in surplus at some time in the year otherwise no flexibility would be available to meet the seasonal requirements. Especially the catalytic reforming units which work to satisfy motor gasoline demand qualities (motor gasoline as well as the middle distillate products have clear seasonal movements). Hence that hydroskimming refineries usually give zero rent thus accounting losses, according to equation (3.2) above.

The rent in the refinery arises then from more complex units, e.g. all kind of cracking (catalytic and hydro) and alkylation. Often those rents have to account not only for their units costs but also for the losses in the hydroskimming units, hence a more complex refinery is usually better off.

European refineries have historically reflected this fact; oil refineries have been integrated within a company which controlled not only downstream but upstream operations as well as most of the transport system, explorations and even petrochemicals, this made the company's performance profitable as a whole.

Hydroskimming has been surplus; integration of the oil companies made up for the losses in hydroskimming. Consumers buy products not crude, refining was and is a necessary crude selling cost.

To prove or even support the statement that hydroskimming units do not generate a rent, is difficult since access to the oil companies' data is difficult. However, it is eventually possible to know (from companies' published reports) the total level of profits. But it is impossible to make an analysis (as an outsider) along the lines in (i) and (ii), previous section, for the whole of the North West Europe market, one lacks data on operating costs, fixed charges and refinery units rents.

The thesis that catalytic reformers are usually surplus is supported empirically. Single Refinery has been used in this work to test it by setting up deficit reforming capacity and results are in line with this hypothesis. This matter is reported in Chapter 4.

Recently, closures of hydroskimming refineries occurred; on top of the apparently 'normal' hydroskimming loss, refineries are now facing the economic and technical consequences of the changing pattern of supply/demand: the now heavier crudes on stream and thus the stronger environmental protections imposed to refineries together with the decreasing trend in the demand for heavy fuel oil, are forcing them to give up much of their previous hydroskimming capacity and adopt more complex processes to treat the heavier feeds.

It follows then that hydroskimming capacity in a refinery does not produce a rent; and, on the other hand, since installation of additional cracking, either by improving old available one, or by building is expensive and the margin left to refiners on a tonne of crude is small, the rent generated by existing cracking units, is in some cases, simply not high enough to cover losses on hydroskimming.

As precisely pointed out by Lummus,[60]

> [...] there is now a diverging pattern between production and consumption and the cost of bridging this gap is growing continuously [...]. The refining industry is once more facing a dramatic challenge: that is heavier and dirtier crudes and a shrinking market for heavy fuel oil. The decisions to be made are difficult and expensive.

2. The short term trends change: in equilibrium the refinery, in fact the cracking unit, generates normal profits: the marginal rent is equal to the total built-up cost of an additional unit of capacity. Short run disequilibrium positions may put level of rents up as has been observed recently in the market: there has been a worldwide expansion of cracking during the years 1980-83 that could be explained by the more heavy fuel oil being cracked as a result of several factors, *inter alia*,

- the increasing use of substitutes, i.e., coal, hydro and nuclear,

- the price of crude, which determines the price of oil products in the spot market (Unidimensionality), was high, the price of heavy fuel oil could not compete with the total cost of (marginal) coal in the market.

The foregoing situation caused:

1. An unbalance in prices, the system became (and still remains) no longer *unidimensional*: since the price of crude is dictated, the refiner wants to sell heavy fuel oil at its highest possible price and this will be higher the higher the price of crude is, but on the other hand the price of heavy fuel oil can not go beyond the limit imposed by the cost of its substitutes, i.e., by the total cost of coal. Hence two prices (in fact a price and a cost) need to be known in order to determine the price of heavy fuel oil,

---

[60] Rhoe, A., et al., 'Meeting the refiner's upgrading needs', *The Lummus Company*, paper presented at the National Petroleum Refiners Association, 81st. Annual Meeting, (California: March 20-22, 1983), p.1.

namely, the price of crude and the total cost of coal, the energy pricing system is then *two-dimensional*.

2. A surplus of crude and an increase of the cracking marginal rent to levels much beyond the equilibrium rent thus producing expansion.

It is shown that to restore equilibrium the price of crude (Arabian Light, Saudi Arabia) has to go down to about 14. to 20. $/bl (i.e., 100. to 145. $/mt). Heavy fuel oil price will then become competitive with coal and be (marginally) burned, crackers will go down to their 'normal' operational level and a 'normal' cracking rent will again be generated. Unidimensionality will be restored. A problem will remain however for the refiner, that of underutilization of capacity.

3. In the short run, rents can only be an indication of the (short run) level of profits since,[61]

> There is no way of determining how much of this rent is profit because fixed costs and depreciation interest charges are unknown, but a rise in the refinery rent will increase profitability of the refiners -the oil companies. If it drops, profits will drop.

In the long run, that refinery generating a consistently high level of rents able to pay fixed costs and giving a surplus at least equal to the *average annual capital charge* on investment, will allow for decisions on new plant options.

Points **1.** to **3.** are progressively dealt with in following chapters. Chapter 4 covers point **1.**, and introduces the case for point **2.** which is covered in greater depth in Chapter 7 where all factors are drawn together into a composite whole. Chapter 6 will make the link between short and long run rents by estimating the level of rents through time.

Sections 3.3 and 3.4 of this Chapter are devoted to state theoretically the rent formation within the refinery under competitive market conditions.

---

[61] Deam, R.J., 'Understanding Energy: A Rational Basis for Planning Alternative Strategies', in Dunkerley, J., *International Energy Strategies*, op.cit., p.293.

3.3 THE DETERMINATION OF THE OIL REFINERY RENT

The problem of rent determination within the oil refinery is analogous to that of the rent of land.

The refinery configuration is made up of various refinery processing units usually adapted to the market they satisfy.

Each refinery processing unit enters directly or indirectly in the production of final demand; to each refinery processing unit there is attached a particular set of processes directed to the intermediate/final satisfaction of refined products. Each process fulfils a different function according to specific chemical conditions and in order to satisfy predetermined volumes.

The diversity of refinery processing units within the refinery makes it comparable to the existence of *lands of different quality* in a given extension of land for cultivation. The quality of land depending on its fertility. In the case of capital equipment, as Salter puts forward: 'The fertility of land corresponds to the level of technical knowledge embodied in capital equipment'.[62] This applies in particular to the oil refinery. The *quality* of a refinery processing unit is determined by the ability it shows to adapt itself to the market requirements and to generate a surplus or positive rent.

In a LP context no production process gives a positive profit. Because of the equilibrium nature of an LP model, any basic activity will be zero-profitable implying that at the margin, the revenue it yields is equal to the costs it rises. The LP refinery models used do not include the wage rate since fixed costs, including labour costs, are omitted. Rent is then the only economic item accounting for a profit margin for the refiner as discussed in the previous section. And the total rent accruing to the refiner is the sum of the individual refinery processing unit rents as in Definition 5.3-3. The (marginal/individual) rents attached to each refinery unit are readily generated by the LP solution of the models.

In studying their formation, the SR is looked at and a two-step analysis followed by analogy to the economic formulation of the rent of land, i.e., *the extensive margin on refinery operations*, and *the intensive margin on refinery operations*, or shortly, *the extensive* and *intensive* margins respectively:

---

[62] Salter, W.E.G., and Reddaway, W.B., *Productivity and Technical Change*, op.cit., p.61.

- The *extensive margin* refers to the severity of operation in the presence of various refinery processing units of a *non-homogeneous* nature. Each refinery processing unit represents a different refining technology but a number of different technologies may be used to produce the same range of products. Competition between refinery processing units will then arise as refining capacity becomes scarce due to increases in product demands. The least costly refinery processing unit satisfying chemistry of demand and quality specifications is used in the first place and generates the highest rent per unit of feed processed. That refinery processing unit producing no surpluses (return) is said to be at the margin of use.

- The *intensive margin* refers to the severity of operation in the presence of various production processes within a particular refinery processing unit, i.e., a *homogenous* technology. Competition between production processes arises; a uniform rent per unit of feed processed will be generated at the refinery processing unit.

Some complications in analysis arise if extrapolating to the more integrated refinery system (the 7-area WEM) where on on top of competition among production processes and refinery processing units there exist various oil refineries also competing for satisfaction of demands. Transportation comes into play and rents are additionally affected by the location of the refinery and its imports/exports. Here the comparison to the case of land is not that obvious for every refinery area is to satisfy its own product demand with own particular market specifications and at minimum cost. Hence a refinery in (say) area B (North West Europe) would eventually import crude (to satisfy crude requirements) from area T (South Europe, North Africa) in the first place since transportation costs would be relatively lower.

The competition at world level is to satisfy a world product demand (and not only an area demand) where there exists competition between crude producers and shipowners on top of the competition of production technologies. The attention is focused first on a single refinery area with its associated technologies and product demands. The missing economic transport component (freight rate) is accounted for in the CIF price of imported crude and products; these prices in turn affect the level of rents and therefore the effect of freight rates is indirectly accounted for.

3.4 THE ANALYTICAL FORMULATION OF REFINERY RENT

Having defined the conception of rent as in definitions 3.4 and 3.5, its analytical determination at the refinery level is now pursued. In doing so the steps indicated above are followed. The concepts of extensive and intensive margin of land cultivation are relevant in the determination of rent. Extensions to more than a single product have been considered by Sraffa,[63] by means of his 'Standard System' of joint production and price determination.[64] Sraffa's work gave rise to a series of studies on multi-process multi-product firms and the distribution of product value. As it can be found in Kurz,[65] Quadrio-Curzio[66] and others.[67] It goes beyond the scope of this thesis to go into the details of these works, the analytical treatment of the system of joint production with special application to the determination of the oil refinery rent, is however followed up to some extent.

---

[63] Sraffa, P., *Production of Commodities by Means of Commodities*, op.cit.

[64] Joint production refers to those products which are necessarily produced by a given process as is the case in the oil refinery. A system of joint production as algebraically described by Sraffa is constructed in such a way that the aggregate proportions in which the various commodities enter the production system equals the proportions of the aggregate net product. An invariable commodity is chosen as standard in value, all other commodity's values are related to it. A set of commodities satisfying this commodity-composition is named a standard composite commodity or, a standard commodity. Two kind of commodities are distinguished, *basics*, those commodities entering directly or indirectly as means of production of all others; *non-basic*, those commodities which do not enter the means of production of any other commodity, alternatively, those which enter only their own means of production. Basic commodities are the only ones entering in the standard system. The system relates inputs and outputs (technological coefficients), labour (wage), rate of profit (uniform for every production process) and commodity prices. They are determined by fixing exogenously either the wage rate or the profit rate. The formulation is primarily used to analyse the relationships between the wage and profit rates.

[65] Kurz, H.D., 'Rent Theory in a Multisectoral Model', op.cit.

[66] Quadrio-Curzio, A., 'Rent, Income Distribution and Orders of Efficiency and Rentability', in: Passinetti, L.L., *Essays on the Theory of Joint Production*, (London: The Macmillan Press Ltd., 1980).

[67] See Pasinetti, L.L., Ibid.

Whereas each product should be treated simultaneously, here to simplify algebraic manipulation the analysis is illustrated with only premium motor spirit, the most valuable product at the refinery and the market place.

The notation is changed slightly in what follows though the reader may certainly find all definitions, variables and matrix coefficients, in the SR LP problem definition put forward in section 2.2.1, Chapter 1.

Let us then define,

k is either the number of refinery processing units-u, or the number of production processes available within the refinery processing unit-u depending on the analysis (extensive or intensive margin);

$A = (a_{ij}), i=1,n, j=1,k$ quantity of by-product input-i required by production process-j;

$B = (b_{oj}), o=1,m, j=1,k$ quantity of product-o produced by production process-j;[68] and,

$p = (p_l), l=1,n+m$ equilibrium price vector.

The equations below describe the relationships between production inputs and outputs (technological coefficients), available resources and prices. They are in fact the set of equations forming an economic balance at the intermediate level derived from the primal/dual LP relationships under the assumption of surplus capacity. Variable costs are neglected for simplicity.

An observation should be now noted: prices p_1 to p_n, and p_1 to p_m (in following equations sub-systems), are two distinct set of prices though kept with the same notation. Those attached to coefficients-A are input prices and those attached to coefficients-B are output prices.

[68] The union of matrices A and B as defined here is the technological coefficient matrix-A of the SR LP problem as defined previously in Chapter 2, section 2.2.1.

Sub-System S.1:[69]

$$(a_{11}p_1 + a_{21}p_2 + \dots + a_{n1}p_n) = b_{11}p_1 + b_{21}p_2 + \dots + b_{m1}p_m$$
$$(a_{12}p_1 + a_{22}p_2 + \dots + a_{n2}p_n) = b_{12}p_1 + b_{22}p_2 + \dots + b_{m2}p_m$$
$$\dots\dots\dots = \dots\dots\dots$$
$$(a_{1k}p_1 + a_{2k}p_2 + \dots + a_{nk}p_n) = b_{1k}p_1 + b_{2k}p_2 + \dots + b_{mk}p_k$$

A further arrangement is made so to express every equation of sub-system S.1 in terms of only one product, say product-o,[70] then we obtain,

Sub-System S.2:

$$(a_{11}p_1 + a_{21}p_2 + \dots + a_{n1}p_n) - (b_{11}p_1 + b_{21}p_2 + \dots + b_{m1}p_m) = b_{o1}p_o$$
$$(a_{12}p_1 + a_{22}p_2 + \dots + a_{n2}p_n) - (b_{12}p_1 + b_{22}p_2 + \dots + b_{m2}p_m) = b_{o2}p_o$$
$$\dots\dots\dots = \dots\dots\dots$$
$$(a_{1k}p_1 + a_{2k}p_2 + \dots + a_{nk}p_n) - (b_{1k}p_1 + b_{2k}p_2 + \dots + b_{mk}p_m) = b_{ok}p_o$$

[69] All subsequent price equation systems will be call sub-systems to mean a sub-set of equations from the full SR dual price system. If for sub-system S.1 all a_{ij}'s and b_{ij}'s are non-zero, then all commodities are said to be basic in the Sraffa's standard system. Moreover, each equation of sub-system S.1 would include the wage rate, w, and the rate of profit, r, as follows: $(a_{1j}p_1 + a_{1j}p_2 + \dots + a_{mj}p_m)(1+r) + l_m w = b_{1j}p_1 + b_{2j}p_2 + \dots + b_{mj}p_m$. Unlike Sraffa's system, commodities here are non-basic. In an oil refinery oil commodities are produced in a vertical processing way, the by-product of a refining process does not enter again to its own production. The only exception could possible be the recycle fuel oils and fractions of gas, straight-run benzine and straight-run gasoline used as refinery fuel but not being directed again to their respective pools.

[70] Such formulation has been anticipated by a direct example from the SR in section 3.2.2 of this Chapter.

It is worth to point out here:

- A coefficient a_{ij} is interpreted indistinctively as the marginal product (technological coefficient) or as the total amount of input $a_{ij}x_{ij}$ to production process-j where x_{ij} is the activity level of refining process-j (as in the LP context); x_{ij} is implicitly assumed to be unity here and in following sub-systems for simplicity. Similarly with outputs for coefficients b_{ij}.

- Any refining process has a dual constraint associated which can be expressed as any of the equations in sub-system S.1; and for any intermediate or final oil product a sub-system like S.2 can be derived from the dual problem.

The sub-system including the rent term is but an extension of S.2 and is equally derived from the dual LP refinery problem. For both, the extensive and the intensive margin treatments, sub-system S.2 will be constructed in terms of premium motor spirit (being p_o in sub-system S.2) as was pointed out earlier.

3.4.1 The Extensive Margin Formulation on Refinery Operations

It is assumed the existence of k refinery processing units of different efficiency and by this latter is meant their average production costs in relation to a homogeneous output, premium motor spirit in this case.

Now refinery rents are introduced as a reflect of the refinery processing units scarcity as severity of operation increases and/or production expands. Production expansion can occur either by a direct increase in the demand for premium motor spirit, or by increases in the demand of any other products of final demand whose intermediate blending components are jointly produced with intermediate motor spirit components (namely with catalytic cracked spririt, motor spirit reformates, alkylates, hydrocrackates, etc.), for instance, the fuel oils and middle distillates (gas oils) produced by cracking processes blending to the final fuel oil and gas oil pools.

The level of rents depends on the order of efficiency: the least efficient refinery unit, always in terms of premium motor spirit production, is assumed to pay no rent. Two more variables, namely ρ_u and Λ_u, are added to each equation of sub-system S.2.

$\rho = (\rho_u)$, u=1,k vector of refinery rents, ρ_u, is the rent in \$/mt of processing feed to refinery processing unit-u;

$\Lambda = (\Lambda_u)$, u=1,k vector of available refinery processing unit capacities;

p_p price of premium motor spirit;

b_{pu} amount of premium motor spirit (intermediate component) produced at the refinery processing unit-u;

$A = (a_{iu})$, i=1,n, u=1,k,

is the matrix of technological coeffients as defined above with j=u; and,

$$B = \left[\frac{b_{ou}}{b_{pu}} \right], \quad o=1,m, \quad u=1,k,$$

is the output matrix as defined before, where b_{ou} and b_{pu} are the amounts of intermediate product-o jointly produced with premium motor spirit at the refinery processing unit-u and the total amount of premium motor spirit produced at the refinery processing unit-u.

Sub-System S.3:

$$(a_{11}p_1 + a_{21}p_2 + \ldots + a_{n1}p_n) - (b_{11}p_1 + b_{21}p_2 + \ldots + b_{mp}p_m) + \Lambda_1\rho_1 = b_{p1}p_p$$

$$(a_{12}p_1 + a_{22}p_2 + \ldots + a_{n2}p_n) - (b_{12}p_1 + b_{22}p_2 + \ldots + b_{m2}p_m) + \Lambda_2\rho_2 = b_{p2}p_p$$

$$\ldots\ldots\ldots = \ldots\ldots\ldots$$

$$(a_{1k}p_1 + a_{2k}p_2 + \ldots + a_{nk}p_n) - (b_{1k}p_1 + b_{2k}p_2 + \ldots + b_{mk}p_m) + \Lambda_k\rho_k = b_{pk}p_p$$ [71]

[71] Appendix B describes in detail such a formulation for the catalytic cracking unit case, i.e., with u=cc, and $\Lambda_u\rho_u$ being the variable R in there.

There exists a *marginal* refinery processing unit-t that yields no rent:

$$\rho_t = 0, \quad \text{and hence,} \qquad \prod_{u=1}^{k} \rho_u = 0.$$

Refinery processing unit-t is the refinery processing unit which being in surplus capacity generates no rent. In the oil refinery these are usually the atmospheric and vacuum distillation units, and catalytic reforming units depending on season.

Unlike in Sraffa's and similar equilibrium systems, the existence and uniqueness of solution of sub-system S.3 is guaranteed as this sub-system together with the whole set of linear equations are solved by the LP simplex algorithm. The price vector-p and the rent vector-ρ are (as known from the SR problem formulation, Chapter 1), the dual marginal values attached to the intermediate product balance equations and the refinery processing unit constraints.

It is clear that vector price-p will always exist for it corresponds to equality constraints in an LP problem. Once these prices are known the rents, if any, are readily obtained by substituting them in equations S.3; rents are then price determined and the price of premium motor spirit resolves itself into three parts, namely, the value of inputs used by the refinery processing unit-u, the value of outputs jointly produced by it and the refinery processing unit rent.[72]

For the refinery processing unit at the margin of operation, let it be as above, refinery processing unit-t, there will be an equation in S.3 expressed as follows:

$$(a_{1t}p_1 + a_{2t}p_2 + \ldots + a_{nt}p_n) - (b_{1t}p_1 + b_{2t}p_2 + \ldots + b_{mt}p_m) = b_{pt}p_p \tag{3.7}$$

Following the argument, the variables Q_u and λ_u are defined respectively as in (3.8) and (3.9) below. They are introduced to obtain simplified expressions of equations in sub-system S.3 (and S.4 in section 3.4.3) from which the marginal refinery processing unit rent can be clearly identified in terms of average values.

[72] Any other joint product (intermediate or final) can be expressed in terms of two/three economic components. Premium motor spirit is used as a matter of illustration.

$$Q_u = \left(\sum_{i}^{n} a_{iu} p_i - \sum_{o}^{m} b_{ou} p_o \right) / b_{pu}, \quad u=1,k \tag{3.8}$$

Q_u is in this discussion *the average production cost relative to premium motor spirit production at the refinery processing unit-u*, or simply, *the relative average production cost*.

Q_u is clearly an average value. It is not the average production cost of premium motor spirit alone because of the joint product nature of the petroleum products. As Adelman writes,[73]

> [...] individual petroleum products are joint products, and it is altogether useless to seek or to pretend to have found the costs of the individual products -costs that do not exist.

And,

$$\lambda_u = \Lambda_u / b_{pu}, \quad u=1,k \tag{3.9}$$

the proportion of capacity of refinery processing unit-u used to produce a tonne of premium motor spirit, or *severity of operation of the refinery processing unit-u*.

Sub-system S.3 is now written in short as,

Sub-System S.3':

$$Q_1 + \lambda_1 \rho_1 = p_p$$

$$Q_2 + \lambda_2 \rho_2 = p_p$$

....

$$Q_k + \lambda_k \rho_k = p_p$$

It is clear from (3.8) and sub-system S.3' that costs here refer to value of inputs to the refinery processing unit-u and value of outputs

---

[73] See Adelman, M.A., *The World Petroleum Market*, op.cit., p.175.

from the refinery processing unit-u so that costs in fact convey an average surplus factor in terms of which the price (marginal cost) of premium motor spirit, p_p, equivalently, the price of any other joint product, can be expressed; to cite Adelman again,[74]

> [...] when products are joint in variable proportions, the incremental cost of a single joint product does exist within certain limits even if its average cost does not.

Q_u could then be positive or negative as value of inputs exceeds value of outputs or contrariwise. The terms $\lambda_u \rho_u$, u=1,k, represent the value in \$ per tonne of premium motor spirit due to the use of the refinery processing unit-u. It is also interpreted here as the rent paid to the refinery processing unit-u on the proportion of capacity used to produce the premium motor spirit output, or simply the rent on output.

3.4.2 Expression of Marginal Rent at the Refinery Processing Unit Level

Expressing the rent ρ_u in terms of production costs and severity of operation at the refinery processing unit-u, we have from S.3',

$$\rho_u = (p_p - Q_u) \frac{1}{\lambda_u} \tag{3.10}$$

Expression (3.10) implies that for a positive rent ρ_u to exist it is necessary the marginal cost of premium motor spirit production, p_p, is greater than the average production costs relative to premium motor spirit Q_u at the refinery processing unit-u. If for a refinery processing unit-u, ρ_u is zero, meaning p_p is equal to Q_u, the refinery processing unit earns no surplus.

On the other hand, at the whole of the refinery, other refinery processing units could eventually earn a surplus. Supposing there are two refinery procesing units v and w for which the output price (marginal cost) of premium motor spirit is greater than the relative average production costs, then,

[74] Ibid., p.176.

$p_p - Q_v > 0$ and $p_p - Q_w > 0$.

Suppose further two different possibilities:

First, $p_p - Q_v > p_p - Q_w$: this implies that $Q_v < Q_w$, i.e.,average production costs in refinery processing unit-v are lower than those in refinery processing unit-w.

Thus, from (3.10), $\lambda_v \rho_v > \lambda_w \rho_w$, implying,

$$(\lambda_v \rho_v) / (\lambda_w \rho_w) > 1 \tag{3.11}$$

the refinery processing unit-v, that with lower relative average production costs, generates a greater value on the premium motor spirit price due to the use of its capacity than the value on premium motor spirit due to the use of the capacity of refinery processing unit-w.

Also, $(\rho_v/\rho_w) > (\lambda_w/\lambda_v)$, (3.12)

the marginal rent ratio is greater than the inverse of the capacity usage ratio between the two refinery processing units with respect to the output production -premium motor spirit in this particular case.

Second, $p_p - Q_v = p_p - Q_w$: this implies that average production costs are equal in the two refinery processing units, and thus the premium motor spirit value embeded in the use of the refinery unit capacities:

$Q_v = Q_w$, $\rho_v \lambda_v = \rho_w \lambda_w$, and, $\rho_v/\rho_w = \lambda_w/\lambda_v$.

Equality in relative average production costs does not necessarily bring about zero-rent for the refinery processing units unless $Q_v = Q_w = p_p$, i.e., unless average production costs are equal to the marginal cost of producing premium motor spirit for both refinery processing units.

From the first and second alternatives it follows, $\lambda_v\rho_v \geq \lambda_w\rho_w$. This relation does not necessarily imply the marginal rent ρ_v is greater or equal than the marginal rent ρ_w, i.e., $\rho_v \geq \rho_w$, for a set of different combinations of λ's and ρ's may satisfy that condition. The relation between the marginal rents of two different refinery processing units depends on their capacity usage ratio, λ_w/λ_v.

If relative average production costs are equal, then the marginal rent on the refinery processing unit-v will be greater than that of the refinery processing unit-w only if λ_v is less than λ_w. In words, only if the intensity in capacity use of refinery processing unit-v is less than that of refinery processing unit-w. The same does not apply to the case of unequal average production costs for expression (3.12) only says the marginal rent ratio is greater than the capacity usage ratio, and if $\rho_v > \rho_w$, the expression holds inrrespectively of λ_w being less than or equal to λ_v. What one can conclude from here is the ratio of marginal rent of two different refinery processing units with unequal relative average production costs must be greater than the inverse of the ratio of their capacity usages.

3.4.3 The Intensive Margin Formulation on Refinery Operations

It is now considered the existence of only one homogeneous resource, a single refinery processing unit, in particular, and because the analysis is referred to premium motor spirit, the catalytic cracking unit is chosen. There exist k production processes of different productivities and working at different operational severity levels. They yield a range of intermediate oil products, inter alia, catalytic cracked spirit.[75] The production processes are the dual equations associated with the catalytic cracking activity vector.[76]

A uniform rent per unit of processing feed to the catalytic cracking unit will be generated on deficit of catalytic cracking capacity and as a result of the kind of production process(es) in use. It is assumed that on surplus capacity conditions, the most efficient process will be used first. The most efficient process is that production process with the least average production cost and satisfying specified product qualities. Processes of decreasing efficiency are progressively being used as the chemistry of demand requires it.[77]

There is a distinction between the most efficient process at the intensive margin of operation and the most efficient unit at the extensive margin of operation; the most efficient process -the least costly- need not to be the most productive in terms of output per unit of catalytic cracking capacity used. Moreover, to a given pattern of production processes in use the *order of efficiency* need not to coin-

---

[75] It should in fact be catalytic cracked spirit instead of premium motor spirit for the former is the direct output from the catalytic cracking unit which contributes to the premium motor spirit pool. However, given the linear relationships of the production functions it is valid to express the premium motor spirit formation in terms of the catalytic cracked spirit with the appropriate coefficient, and vice versa.

[76] The reader is asked to refer to Appendix B, note B.1, for the detailed formulation in terms of the LP oil refinery terminology.

[77] Whereas observing this progressive use of refining processes from an optimal LP solution is not possible, the simplex algorithm itself works in this way when a minimizing criterion is set up. The search for every feasible point in the convex polhyedron (primal feasible space) represents the consecutive introduction of the always cheapest production method available which satisfies restrictions. As long as the LP model is representing well a refinery (and thus the optimal solution is reached by the Simplex algorithm) its operational way is certainly compared to the intensive and extensive margins of rent of land.

cide with the *order of rentability*[78] generated by the level of activated processes. Although the unitary (marginal) rent is uniform, the individual rent values differ as they depend on the actual levels of processes' activities. Orders of efficiency and rentability change as levels of processes' activities change.

For the intensive margin formulation sub-system S.3 differs in the term of rent. Unlike the latter, here the unitary rent is unique per unit of catalytic cracking capacity used. Therefore, $\rho_u = \rho$ for all refinery process-j, j=1,k.

The sub-system for the intensive margin is introduced next,

Sub-System S.4:

$$(a_{11}p_1 + a_{21}p_2 + \ldots + a_{n1}p_n) - (b_{11}p_1 + b_{21}p_2 + \ldots + b_{mp}p_m) + \Lambda_1\rho = b_{p1}p_p$$

$$(a_{12}p_1 + a_{22}p_2 + \ldots + a_{n2}p_n) - (b_{12}{}^{1}p_1 + b_{22}p_2 + \ldots + b_{m2}p_m) + \Lambda_2\rho = b_{p2}p_p$$

$$\ldots\ldots\ldots = \ldots\ldots\ldots$$

$$(a_{1k}p_1 + a_{2k}p_2 + \ldots + a_{nk}p_n) - (b_{1k}p_1 + b_{2k}p_2 + \ldots + b_{mk}p_m) + \Lambda_k\rho = b_{pk}p_p$$

The capacity utilization constraint, $\sum_{j=1}^{k} \Lambda_j \leq \Lambda_{cc}$, where Λ_j is the catalytic cracking capacity used to produce the premium motor spirit output when using production process-j; and Λ_{cc} is the maximum catalytic cracking capacity available.

Similarly to the case of extensive margin, an equation such as (3.7) will be attached to that production process-s not in use,

$$(a_{1s}p_1 + a_{2s}p_2 + \ldots + a_{ns}p_n) - (b_{1s}p_1 + b_{2s}p_2 + \ldots + b_{ms}p_m) = b_{ps}p_p \qquad (3.13)$$

---

78 Orders of *efficiency* and *rentability* will be dealt with in section 3.4.5 of this Chapter.

To study the marginal rent-ρ at the intensive margin of operation, we let q_j and λ_j again be, respectively, *the average production cost relative to premium motor spirit when using production process-j*,

$$q_j = \left(\sum_{i}^{n} a_{ij} p_i - \sum_{o}^{m} b_{oj} p_o \right) / b_{pj}, \qquad j=1,k \tag{3.14}$$

and *the proportion of refinery unit capacity used by production process-j to produce a tonne of premium motor spirit* at the refinery processing unit in consideration, or *the severity of operation of production process-j*.

$$\lambda_j = \Lambda_j / b_{pj}, \qquad j=1,k \tag{3.15}$$

Sub-system S.4 is further written in terms of (3.14) and (3.15) as:

Sub-System S.4':

$$\begin{aligned} q_1 + \lambda_1 \rho &= p_p \\ q_2 + \lambda_2 \rho &= p_p \\ &\cdots\cdots\cdots\cdots \\ q_k + \lambda_k \rho &= p_p \end{aligned}$$

3.4.4 Expression of Marginal Rent at the Refining Process Level

The various production processes within the refinery processing unit will enter the production system in order of efficiency. Were the demand for premium motor spirit such that only one process sufficed to satisfy it, the least costly satisfying quality specifications, would be employed, let us say, process-s.[79]

---

[79] Since the refiner is seeking to satisfy demand at the overall least attainable cost, (profit-miximization/ cost-minimization criterion) there it is implicitly assumed the least costly method of production will be always used in the first place provided it satisfies required specifications.

The equation of production process-s in sub-system S.4' yields an expression for rent-ρ equal to:

$$\rho = (p_p - q_s) \frac{1}{\lambda_s}, \tag{3.16}$$

ρ will be positive as long as the price of premium motor spirit exceeds its relative average production costs. From (3.16) any unemployed production process will pay no rent since b_{ps} will be zero in the expression $\lambda_s = \Lambda_s/b_{ps}$.

As production expands due to an increase of the demand for premium motor spirit (or for any of the other products jointly produced), a second process, let us say process-t, will enter the production system. Considering the two equations for premium motor spirit as in sub-system S.4':

$$\begin{aligned} q_s + \lambda_s\rho &= p_p \\ q_t + \lambda_t\rho &= p_p \end{aligned} \tag{3.17}$$

and from them the expression for rent is given by,

$$\rho = \frac{q_t - q_s}{\lambda_s - \lambda_t} \tag{3.18}$$

From (3.18) three different situations can arise:

First, the rent-ρ is zero: if relative average production costs of using process-t remain unchanged, i.e., average production costs q_s and q_t are equal, no rent will arise. The chemistry of demand is such that both processes are needed to satisfy volumen and quality specifications (otherwise only one process, in principle, would suffice). The fact that both processes are employed without rising relative average production costs reflects the still surplus catalytic cracking capacity available.[80] Were more production processes being in

[80] The mathematics of LP guarantee a zero-rent being associated with conditions of surplus capacity. The interested reader may refer to Appendix B, note B.2, for an algebraic demonstration of the point.

use, and the rent-ρ being zero, this condition should hold between every pair of distinct production processes in use.

Second, the rent-ρ is positive: It is only when current processes fail to satisfy demand's volumes and qualities that a costlier process (also a more productive and less capital intensive process) is employed; this latter, in turn, reflects the fact that capacity is becoming scarce since premium motor spirit production can only be expanded by rising relative average production costs. Under these circumstances, it is obvious from (3.17) that a positive rent is paid on the catalytic cracking unit by using production process-t (the second having entered the refining process). The refiner expects a uniform unitary rent on the whole of the available cracking capacity so that production process-s will also generate a positive rent on the part of the cracking capacity it is actually using. As from expression (3.18), the rent-ρ will be positive provided the process with the higher relative average production cost has associated a lower severity in capacity use. Processes with higher severity have associated lower productivity and production costs.

As more production processes are being used, the continuously increasing unit costs bring about increases in the rents paid to the proportional catalytic cracking capacities used by the already operating production processes; similarly, a rent will be paid on the proportion of catalytic cracking capacity used by the process entering last in operation.

The proportions in which the operating processes use the total available capacity is determined by the demand from premium motor spirit relative to the demands for the joint-products, namely, gas oil and fuel oils.[81]

Third, the rent-ρ is negative: to higher processes costs correspond also higher capacity severities (catalytic cracking capacity use per unit of output). The refinery is no longer working efficiently, a reallocation of resources is called for.[82]

The problem of rent determination becomes more complex as the number of produced commodities increases. A sub-system like S.3 or S.4 is

---

[81] How rents vary when chemistry of demand is changed is an experimental matter to which Chapters 4 and 5 refer.

[82] Sraffa, op.cit., p.75, paragraph 87: "While any two methods (methods of cultivation) ..., they must satisfy the economic condition of not giving rise to a negative rent ...". As rents are here dual variables constrained to be always greater than or equal to zero, the LP solutions guarantee the named condition.

viable for it is always possible to choose one of the refined products and express its price in terms of the others' prices, input production costs and rents.

In a multi-product, multi-process firm like an oil refinery more than one production process could be simultaneously employed and there would still be surplus capacity.[83] Similarly, there could be only one production process in use and capacity in deficit. Again, this only depends on the actual product demand composition and the market requirements relative to upgrading facilities.

On these grounds there is not an 'absolute least cost' process, but it is always possible to find the least costly process or processes for the given supply/demand structure. That means, to optimize resources and use of technologies in the most economic way.

3.4.5 Orders of Efficiency and Rentability of Refinery Operations

The concepts of efficiency, rentability, severity of operation and productivity in the refinery (as understood here) have been already introduced.

When related to a homogeneous output (premium motor spirit, for instance) and to a given chemistry of supply/demand:

A refinery processing unit/production process is *efficient* if its relative average production costs are lower than any other refinery processing unit's/production process's and satisfies chemistry.

A refinery processing unit/production process is *rentable* when under conditions of a uniform marginal rent per tonne of processing feed, it has associated a relative higher rent on the proportion of capacity it uses.

A refinery processing unit/production process is more *productive* if its yield per tonne of processing feed is higher than any other refinery processing unit's/production process's.

---

[83] Unlike the conventional case of the rent of land for corn production where the existence of two or more methods of cultivation necessarily rises a positive rent.

A refinery processing unit/production process is more *capital intensive* (higher severity) if it uses relatively more capacity per unit of output produced.

The severity of operation-λ - in (3.9) and (3.15) - is the ratio of capacity used to output produced. The productivity ratio-e is the inverse of the severity ratio, that is, $e_u = 1/\lambda_u$ for refinery processing unit-u, and $e_j = 1/\lambda_j$ for production process-j.

Then orders of efficiency, rentability, severity of operation and productivity for the extensive and intensive margins and relative to a particular product output and chemistry of supply/demand follow.

- For the extensive margin of operation it follows from sub-system S.3' the order of efficiency is an increasing order in the relative average production costs of the refinery processing units contributing to the production of the homogeneous output:

  $$Q_1 < Q_2 < \dots < Q_k \tag{3.19}$$

  where refinery processing units-1 to k are the refinery processing units used.

  The order of rentability is opposite related, i.e., with $\rho'_u = \lambda_u \rho_u$, u=1,k,

  $$\rho'_1 > \rho'_2 > \dots > \rho'_k \tag{3.20}$$

  From existing orders of efficiency and rentability does not directly follow the direction of the orders of severity of operation and productivity, for if $Q_1 > Q_2$ implies $\rho'_1 < \rho'_2$ (section 3.4.2), therefore $\lambda_1\rho_1 < \lambda_2\rho_2$ and one can not conclude from this latter the relation of order between λ_1 and λ_2. Experiments with the 7-area WEM show that to orders of efficiency (3.19) and rentability (3.20), the following orders of severity of operation and productivity result:

  $$\lambda_1 > \lambda_2 > \dots > \lambda_k \tag{3.21}$$

  $$e_1 < e_2 < \dots < e_k \tag{3.22}$$

  The less costly a processing unit is, the more rentable, the more capital intensive and the less productive is.

  However, the orders of marginal rents do not necessarily coincide with any of the above relations' order.

- For the intensive margin of operation, it follows from the equilibrium equations of sub-system S.4' that the order of efficiency is an increasing order in the relative average costs of the production processes employed in such a way that,

  $$q_1 < q_2 < \ldots < q_k \tag{3.23}$$

  where process-1 and process-k are respectively the first and the last processes employed.

  The order of rentability attached to it with $\rho'_j = \lambda_j \rho$, j=1,k, is,

  $$\rho'_1 > \rho'_2 > \ldots > \rho'_k \tag{3.24}$$

  Unlike the case of extensive margin, the orders of severity of operation and productivity can be derived analytically. From (3.17) if $q_s < q_t$, then $\lambda_s \rho > \lambda_j \rho$, implying the severity of operation of process-s is greater than process-j's, i.e., $\Lambda_s/b_{ps} > \Lambda_t/b_{pt}$. Thus the less costly process is also the more capital intensive and the less productive since $b_{ps}/\Lambda_s < b_{pt}/\Lambda_t$.

  Then, the orders of severity of operation and productivity are,

  $$\lambda_1 > \lambda_2 > \ldots > \lambda_k \tag{3.25}$$

  $$e_1 < e_2 < \ldots < e_k \tag{3.26}$$

  where, $e_j = b_{pj}/\Lambda_{pj}$, j=1,k.

Relations (3.19) to (3.26) show that in an oil refinery, under exogenous conditions of market competitiveness (marginal cost=price), whether referred to a refinery processing unit or production process, the refinery operational performance in satisfying the product demand is the same. In other words, the extensive and intensive margins of refinery operations produce similar performances: with increasing relative average production costs are associated increasing productivities and decreasing proportional rents and capital intensities.

Although sections 3.4 and 3.5 have been developed in fundamentally theoretical basis, great deal of computational work was carried out with the SR and the 7-area WEM in order to study the analytical rent determination which concludes with section 3.5 here. No effort is placed in reporting numerical results since it would not provide additional information to the present discussion.

3.5 CONCLUDING REMARKS

The foregoing discussions suggest that at a particular refinery unit, for a given supply/demand chemistry and relative to a particular product, the level of marginal rent is associated with average production costs and product prices (marginal costs). This confirms Definition 3.5-1.

The analysis on this Chapter shows that an analogy indeed exists between the theoretical treatment given to the rent of land and that given (here) to the oil refinery rent. Unlike the case of rent of land, it has been shown that not great discrepancy arises in using either the *extensive* margin (where the whole refinery is the fixed supply and the refinery processing units the production methods) or the *intensive* margin (where a particular refinery processing unit is the fixed supply and its production processes the production methods) approach to explain analytically the rent formation, in either case the marginal rent is expressed as the difference between a marginal cost and an average production cost. Rent in this context is a surplus over and above average production cost arising only on scarcity of the resource.

It should be noted that no contradiction exists between the latter which is the analytical expression of (marginal) rent and Definition 3.4 of refinery rent. For the refiner the rent is a total amount, i.e., the sum of (individual) marginal rents times the individual capacities; the latter should ideally be high enough to cover fixed charges leaving a short run surplus profit, but producing a long run equilibrium rent.

CHAPTER 4: EXPERIMENTATION AT THE LOCAL LEVEL: SINGLE REFINERY

The conceptual platform set out in Chapters 2 and 3 provides the basis to carry out experimentation. A set of experimental variables is selected to test some experimental hypotheses: the different values assigned to them are assumed to represent particular market and refinery operating conditions under which particular refinery rent responses are expected to arise. Parametric Programming is performed on the identified set of experimental variables.

4.1 INTRODUCTION

The exercises proposed and carried out in this Chapter aim at showing the *empirical* effects of given external patterns on inner refinery operations and economic performance. Because the SR is the model used in this experimental phase, the effects of trading and worldwide oil competition disappear, instead a deeper understanding at the micro level is achieved.

Throughout the different experiments the focus is centred in the refinery rents behaviour. The refinery rent formation has been dealt with in the previous Chapter: knowing that expansion of output, increase in input/output prices, and/or reduction in capacity determine in one way or another the rent, experiments are set up on these grounds.

No attempt is made here to estimate a model of rent in terms of its determinants. This is done in Chapter 5 on the basis of a more comprehensive set of results, outcome of the 7-area WEM experimentation. Three main experimental points are tackled here:

1. Chemistry of demand: changes in the premium motor spirit and middle distillates demands under surplus and deficit catalytic reforming capacity (sections 4.2.3 to 4.2.8).

2. Price structure: the price exchange ratio Arabian Light/Heavy Fuel Oil (hereinafter AL/HFO) is exogenously fixed and progressively changed. This generates a two-dimensional system in oil pricing. Thus though the demand for heavy fuel oil remains unchanged, there is room for heavy fuel oil to enter the system at

a particular price; the amount of hydroskimmed/cracked heavy fuel oil produced at the refinery will be eventually lowered, and the heavy fuel oil imported to the system is suggested to represent marginal coal (coal penetration), i.e., the effect of coal/heavy fuel oil substitution is looked at (sections 4.2.9. and 4.2.10).

3. Capacity availability: given a priori an *equilibrium* rent, it is investigated the catalytic cracking and reforming capacities to be built in line with the desired equilibrium rents (sections 4.2.11 and 4.2.12).

4.2 SINGLE REFINERY EXPERIMENTATION

Before presenting the experiments' setting up the economic concept of Comparative Statics Analysis (CSA) is considered briefly.

According to Baumol,[84]

> Comparative Statics is the comparison of the equilibrium values of the endogenous variables of an economic model corresponding to alternative values of the parameters selected for study.

The analysis of two different equilibrium positions by Comparative Statics (CS) is called CSA (also Comparative Statics Equilibrium Analysis). It is then evident that under the SR and 7-area WEM modelling hypotheses, CSA can be used to examine different oil market equilibrium positions as regards crudes and oil product prices and marginal rents (which are the endogenous variables mentioned above).

Parametric Programming (discussed in section 2.5.4, Chapter 2) is applied throughout; it provides the tool for generating the series of equilibrium positions when the underlying parameters are changed.

---

[84] Baumol, W.J., *Economic Theory and Operations Analysis*, op.cit., p.320.

4.2.1 The Single Refinery Original Version

The Single Refinery original version is an aggregate representation of the UK (BP) oil system. It reproduces 1976 conditions in chemistry of supply/demand, and marker pricing. However, due to the fact that market structure during 1976-1979 was rather stable (just before the oil price increase at the end of 1979), the SR may adequately represent the market in any year of the period by changing the Arabian Light spot price accordingly: the aligned equilibrium price structure should then emerge (Unidimensionality).

The following data make up the original version (variables and technological coefficients from the SR Technology Matrix, Appendix E):

Chemistry of Supply	Resoruce vector-b_{cs} -	(mmt/y)
Hassai Messaoud (Algeria)	1.62	
Tia Juana (Venezuela)	.82	
Nigerian Light	3.50	
Kuwait Export	9.11	
Forties (UK)	32.94	

Marker crude supply/price	Activity/Coefficient-$p_m x_m$	
Arabian Light supply (endogenous)	48.26	(mmt/y)
Arabian Light price (exogenous)	89.10	($/tonne)

Chemistry of Demand	Resource vector-b_{pd} -	(mmt/y)
Premium Motor Gasoline	12.33	
Regular Motor Gasoline	4.18	
Light Distillate Feedstock	5.36	
LPG, Refinery Gas	1.66	
Kerosine	7.70	
Gas Oil	25.58	
Light Sulphur Fuel Oil	15.00	
High Sulphur Fuel Oil	28.05	
Bitumen	5.42	

Refinery Units Capacities	Resource vector-b_u -	(mmt/y)
Crude Atmospheric Distillation	147.24	
Vacuum Distillation	80.00	
Catalytic Cracking	13.29	
Catalytic Reforming	18.43	
Hydrofining	24.50	

4.2.2 Experimental Hypotheses and the Single Refinery Base Case

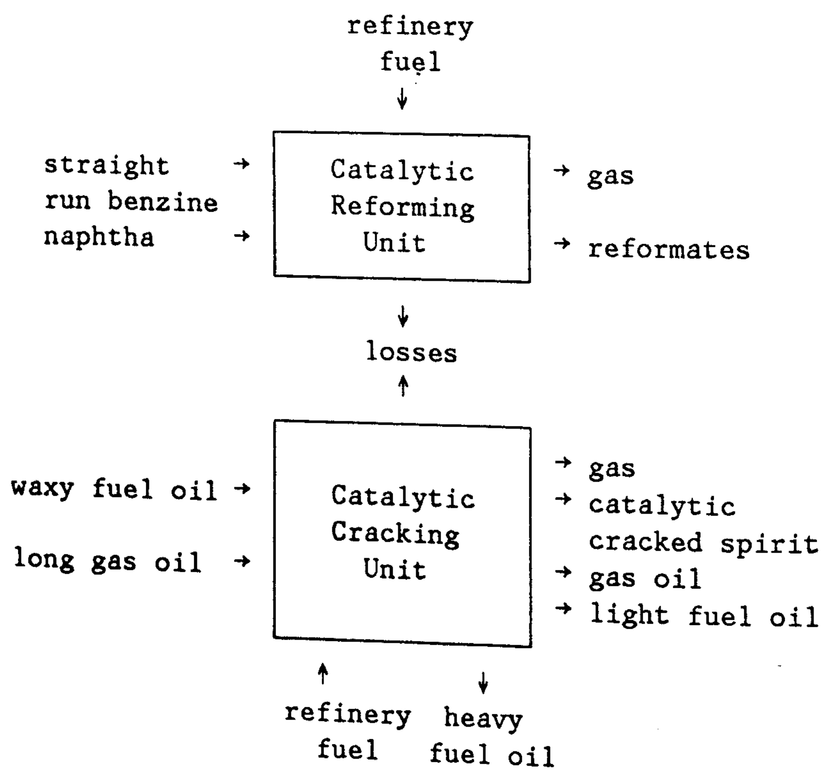
Two experimental hypotheses are to be tested, the first one refers to hydroskimming refineries, namely, those refineries possessing only simple distillation processes and reformers:

Experimental Hypothesis EH.1

Hydroskimming refineries produce low or no rent, and the complex refinery produces rent arising from the use of cracking processes.

This hypothesis (as discussed in section 3.2.4, Chapter 3) appears to be valid in North West Europe. The (simple) SR model will help in confirming/rejecting it. Although the SR does not actually represent the whole North West Europe region but a european sub-region, the SR system is not far from other european refineries.

The complex unit at the SR is the catalytic cracking unit; the following simplified diagrams show the SR's underlying reforming and cracking logistics:



From the reforming unit output some 80% is reformates which goes to the motor spirit pools. The cracking unit processes being more flexible, have a wider output range, produce about 40 to 45% of catalytic cracked spirit (to blend to motor spirit) and 20 to 30% of gas oil.

The operational limit of reforming makes it difficult for the unit to generate always a rent whereas the flexibility of cracking, and the fact that it is a costlier technology, make it better adapted to changes in the chemistry of demand and enable it to produce often high rents.

The oil company profitability is to a large extent the result of product pricing, reforming rent depends on the motor spirit and naphtha prices, cracking on waxy fuel oil, motor spirit and gas oil prices. In turn, reforming and cracking economics will dominate respectively the price relationships between naphtha and gasoline, fuel oil and middle distillates, in a refinery with the above configuration. Even in refineries having additional complex units installed such as thermal cracking, visbreaking and hydrocracking, it is shown the catalytic cracking is economically the dominant one.[85]

On this basis, it is intended to study the pricing and the catalytic cracking rents on a two-dimensional system. A mechanism which is outlined next:

The waxy fuel oil (cracking input) and the heavy fuel oil (cracking output) blend to the heavy fuel oil pool, and from here the latter is distributed to its end uses (see diagram on page 148). If there is an exogenous (fuel oil equivalent) input to the heavy fuel oil final demand, this is, *heavy fuel oil imports*, then the proportion of heavy fuel oil produced in the refinery for final use will eventually diminish. This exogenous input may be *coal* or *nuclear*, since both substitute for heavy fuel oil. How much of the marginal coal enters the market (in other words, how much of heavy fuel oil is sold in the market) depends basically on the price of coal relative to the price of heavy fuel oil.

If coal enters the heavy fuel oil final demand, i.e., if part of the heavy fuel oil is replaced by coal (marginal), there is an excess of waxy/heavy fuel oil in the refinery, this is to be cracked/burned, and thus less crude (which will be eventually in surplus) and more crackers will be required: the relation heavy fuel oil cracked/heavy fuel oil sold determines greatly the level of rents.

---

85 See Longley, R.C.M., 'Price Cost Assignment and Back-up Notes', (London: Chem Systems International Ltd., 1983), unpublished reference.

The heavy fuel oil price depends on the price of crude but it is limited by the total cost of coal; the latter will eventually determine the amount of heavy fuel oil replaced/cracked. The second experimental hypothesis is now formulated:

Experimental Hypothesis EH.2

There is an equilibrium price ratio AL/HFO which determines an equilibrium cracking rent for the refiner, and a minimum marginal coal substituting for heavy fuel oil. Above that equilibrium there will be higher rents and the incentive to expand, below there will be losses, and in both an instability in the system seeking for a new equilibrium.

A third consideration which is implied by the experimental hypothesis EH.2 is the possibility for the system to become two-dimensional thereby generating disequilibrium prices and rents. A two-dimensional, two-tier price structure, relaxes the First Law or Principle of Energy Substitution temporarily. As far as experimental hypothesis EH.2 is concerned, fixing exogenously the prices of Arabian Light and heavy fuel oil (or marginal coal) may produce a two-tier price structure in so far as the price of heavy fuel oil may aligne with coal price and not with the Arabian Light's, and on the other hand, prices of other products may aligne with the Arabian Light price, thus a price combination AL/coal must exist such that a one-tier price structure prevails. To observe this price mechanism and the effect of coal/heavy fuel oil substitution on rents, both prices are fixed in the SR to carry out experimentation.

The **SR base case** is then set up for current experimentation as follows:

- chemistry of supply/demand, and refinery unit capacities as in section 4.2.1;

- the price of heavy fuel oil is assumed to be 1.5 times the total cost of coal;[86]

  the crude/heavy fuel oil prices adopted are:

Marker crude price	216.00 $/tonne	(29. $/bbl)
Heavy fuel oil (import) price	150.00 $/tonne.	

From the relation,

Price-hfo = 1.5 Total cost of coal

we have,

Total cost of coal = (1/1.5) 150. \$/tonne-hfo

The amount of heavy fuel oil imported (marginal coal) is the amount of heavy fuel oil substituting for coal at the total coal cost of 100. \$/mt, or 1.5 times the total amount of coal imported is the amount of heavy fuel oil replaced.

The SR variant cases are set up on the basis of the SR base case as above. They include as mentioned in the Introduction, changes in chemistry of demand, in the price exchange ratio AL/HFO, and in the refinery units' capacities.

The SR variant cases are presented next, experimental results are put forward and preliminary conclusions on the hypotheses are drawn.

Although hypotheses EH.1 and EH.2 are to be tested throughout, sections 4.2.3 to 4.2.8 are particularly devoted to EH.1 and sections 4.2.9 to 4.2.12 to EH.2.

---

[86] The conversion factor in any *BP statistical review of world energy* is 1.5 tonnes of coal (fuel oil equivalent) to 1 tonne of fuel oil.

4.2.3 Changes in Motor Spirit Demand under Surplus Platforming

The aim of the first experiment is to see the catalytic reforming/catalytic cracking rents patterns when allowing for heavy fuel oil (coal) imports and an *increasing demand of premium motor spirit*. The refinery's limits in making motor spirit are also tested, a limit imposed by the refinery's own (fixed) physical infrastructure and the chemistry of supply/demand.

The solution of the **SR base case** (which is not reported here) shows that there is underutilization of platforming capacity of about 47%, the actual capacity is 18.43 mmt/y, hence it is expected the rents obtained to be attached to catalytic cracking since platforming is in surplus. It will be seen whether greater amounts of premium motor spirit demand increase the reforming capacity utilization, eventually generating a reforming rent.

All those cases where platforming remains at its original capacity of 18.43 mmt/y will be referred hereinafter to *surplus platforming case(s)*.

The first set of experiments is then carried out by varying the premium motor spirit final demand above and below the base case value of 12.33 mmt/y. Accordingly, premium motor spirit demand is varied progressively from 0. up to the point where the refinery physical limits are reached, all other factors remaining fixed.

Most relevant experimental results are summarized in Table 4-1, Figure 10 on page 124 and Figure 11 on page 125. From them it can be assessed:

(i) Under a two-tier price structure, i.e., Arabian Light and coal (heavy fuel oil import) prices determining crude and product prices, the refinery's limits (in satisfying premium motor spirit demand) happen to be at the latter's level of 20. mmt/y: under the given refinery configuration (only two units able to produce motor spirit) premium motor spirit levels greater than or equal to 21. mmt/y can not be satisfied. The system becomes infeasible (in LP terms) and uneconomic in refinery profitability terms. In those cases there appears either, a surplus in heavy fuel oil locally produced (non imported) or a surplus in premium motor spirit since there is excess of heavy fuel oil, which is additionally cracked. The premium motor spirit is given away at a *nominal* price of $ 2.411/mt.

Table 4-1. Single Refinery: experimental results, cases of changes in motor spirit demand under surplus platforming.[87]

Case PMS-d mmt/y	AL Lift mmt/y	HFO Imports mmt/y	PMS Price $/ton	HFO Price $/ton	Cat Refor $/ton	Cat Crack $/ton	Ratio AL/HFO
0.-5.	36.9	18.3	0.	154.0	30.4%	76.0%	1.40
6.	36.6	19.4	170.5	178.0	32.2%	83.0%	1.21
7.	36.5	20.8	180.6	207.2	37.0%	86.5%	1.04
8.	36.5	21.9	186.0	213.7	37.0%	86.5%	1.01
9.	37.4	22.0	277.7	256.3	37.7%	19.4	0.84
10.	39.2	21.3	282.5	245.7	39.2%	23.2	0.88
11.	40.9	20.8	305.5	253.8	45.4%	29.1	0.85
12.	43.3	19.7	359.7	187.7	51.0%	35.3	1.15
12.33	**44.0**	**19.3**	**359.7**	**187.7**	**52.7%**	**35.3**	**1.15**
13.	45.8	18.4	359.7	187.7	56.4%	35.3	1.15
14.	48.5	17.0	376.4	150.0	62.0%	48.5	1.44
15.	51.4	15.3	379.0	150.0	67.7%	49.5	1.44
16.	54.3	13.6	379.0	150.0	73.4%	49.5	1.44
17.	57.2	11.9	379.0	150.0	79.2%	49.5	1.44
18.	60.8	9.6	471.9	150.0	84.4%	94.4	1.44
19.	65.3	6.3	536.1	150.0	89.2%	128.3	1.44
20.	70.7	2.2	569.6	150.0	95.0%	146.5	1.44

Arabian Light price = 216.00 $/tonne - 29.00 $/bl
Heavy Fuel Oil (import) price = 150.00 $/tonne
Catalytic reforming capacity = 18.43 mmt/y processing feed
Catalytic cracking capacity = 13.29 mmt/y " "

Each experimental case is identified by a different premium motor spirit demand in column 1, thus each row represents results from a single experiment.

[87] In Table 4-1 and followings: prices are FOB refinery; whenever '%' appears on any entry of columns 6 or 7 , it refers to the percentage of capacity utilization (hence zero rent), otherwise rent; and, the highlighted figures refer to the SR base case solution.

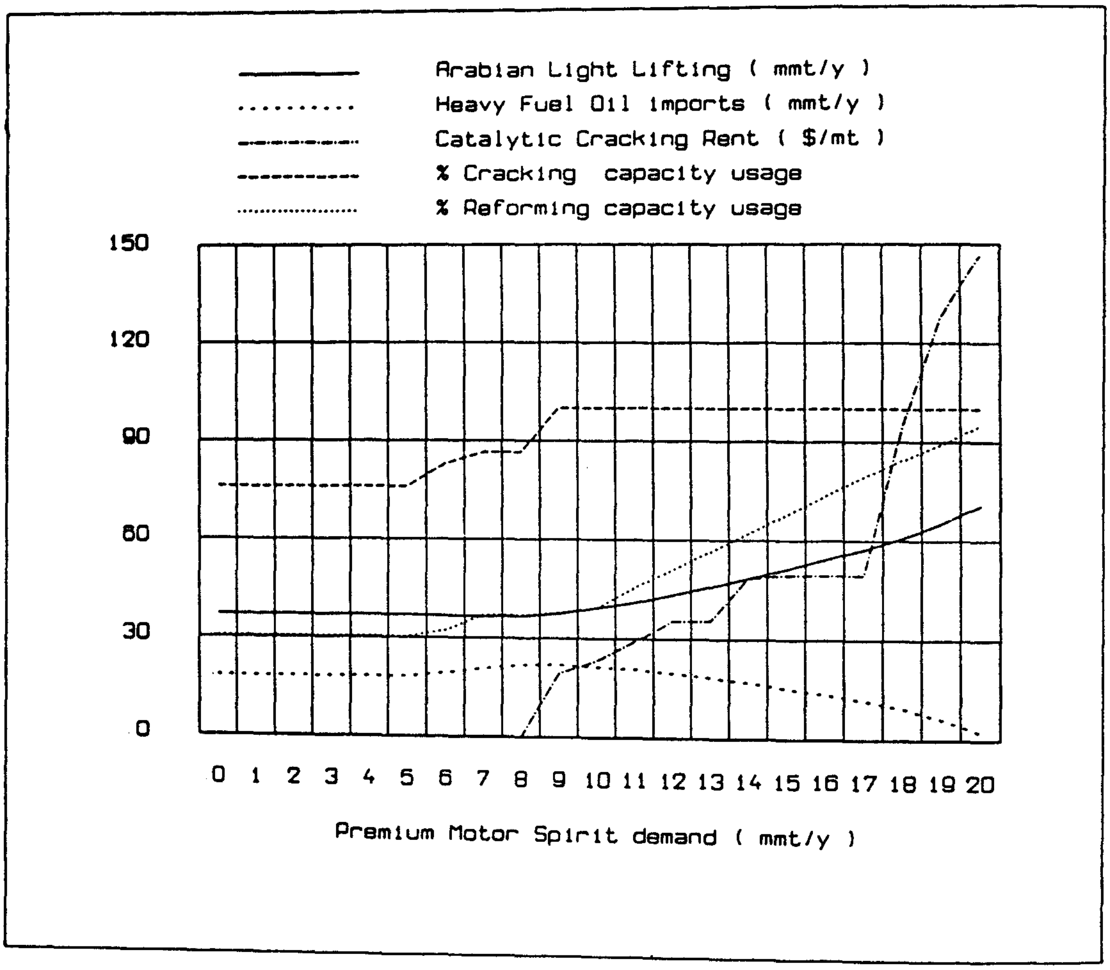


Figure 10. Single Refinery: rents/hfo imports under premium motor spirit demand changes and surplus platforming.

(ii) Rents: premium motor spirit demands below 9. mmt/y do not generate a catalytic cracking rent for the refiner. As crackers are being loaded (hence the percentage of capacity utilization in both catalytic cracking and reforming units increase), an increasing cracking marginal rent with the increasing demand is produced. The catalytic reforming unit though reaching high percentage utilization levels, *does not generate* in any case a marginal rent. This suggests there should be room for the catalytic reforming unit to increase output with increasing premium motor spirit: however, keeping other product demands fixed, there does not seem to be flexibility to reach limits of platforming output.

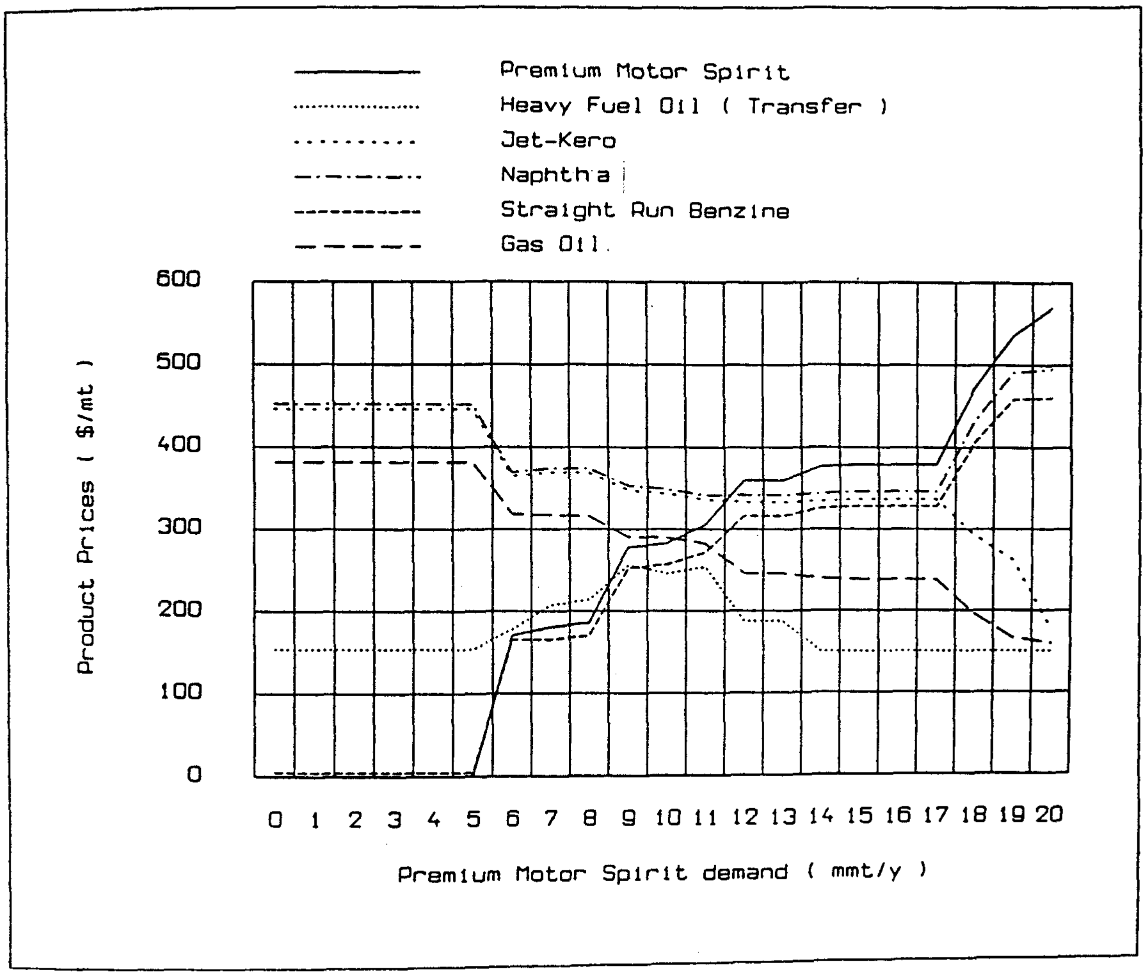


Figure 11. Single Refinery: product price profile under premium motor spirit demand changes and surplus platforming.

In other words, a fixed AL/HFO price exchange ratio and increasing premium motor spirit demand, bring about an increase in catalytic cracking capacity utilization and generation of progressively increasing marginal rents; the latter, in turn, results in increasing platforming utilization rates, which however never cause the unit capacity to become restrictive. Most probably a change in chemistry of demand, for instance simultaneous increases of gas oil and motor spirit demands, or a change in platforming capacity would produce a platforming rent.

On the other hand, increases of heavy fuel oil imports (lower priced than that produced at the refinery) also occur with increasing demand up to a point of 100% catalytic cracking capacity utilization, positive marginal rent, this is at 9. mmt/y. This is also the point of maximum heavy fuel oil imports (Figure 10 on page 124). However, as catalytic crackers are continuously fully loaded, less and less heavy fuel oil is imported and more Arabian Light crude required. More hydroskimmed heavy fuel oil is produced, increasing waxy fuel oil amounts feed crackers; domestic fuel oil price decreases to level off the 150. $/tonne import price, point at which imports continue to decrease more rapidly.

(iii) Product price profile: for demands between 0. and 9. mmt/y the refiner faces and extremely unactractive product price profile (Figure 11 on page 125); indeed, except for the straight-run benzine price, all other product prices lie above the premium motor spirit one. The extreme values corresponding to premium motor spirit demand between 0. and 5. mmt/y. There is still an interval (9. to 11. mmt/y) where naphtha and kerosine prices are higher than the premium motor spirit price.

From 9. mmt/y premium motor spirit demand on the system recovers stability and price structure is in equilibrium though aligned with a two-tier price structure.

At Rotterdam kerosine and gas oil prices higher than premium motor spirit price, and naphtha price higher than kerosine price have occurred at different points in time, from Table D-7, Appendix D, we have:

- Gas oil higher than premium motor spirit price: Oct-Nov 1973.
- Kerosine higher than premium motor spirit price: Oct to Dec 1973; Jan 1974; March,Jul-Aug,Sept to Dec 1979; Jan to March 1982.
- Naphtha higher than kerosine price: Feb to July, Sept 1974; April to June 1975; Jan to Oct 1976; July to Nov 1978; Jan to March, Jun-Jul 1981; April-May 1983.

Almost all dates above represent low motor spirit demand seasons, thus the observed low demand and prices for motor spirit. It is not intended to enquire about the past situation at Rotterdam as regard prices, but it is worth noting that depressed motor spirit prices are accompanied by low to zero cracking rents.

(iv) The AL/HFO price exchange ratio (for heavy fuel oil prices FOB refinery) is increasing with the premium motor spirit demand. The increasing ratio produces, in turn, an increasing rent. Though this effect does not come clearly from these experiments (since demands of 0. to 8. mmt/y produce unstable situations) there is the tendency for the rent to increase with the ratio: since the Arabian Light price is fixed, increases in the ratio are due to decreases in the local heavy fuel oil price, in turn, producing increases in motor spirit price, hence generating higher rents. Further studies on the AL/HFO price exchange ratio are carried out, reports in section 4.2.10.

4.2.4 Changes in Motor Spirit Demand under Deficit Platforming

The previous experiments show a maximum of 95% reforming capacity utilization which of course does not produce a platforming rent. Since greater increases of premium motor spirit (other demands being fixed) make the system infeasible, the catalytic reforming capacity can not be fully used in this way, but instead, *reducing* reforming capacity will, in principle, force reformers to reach physical limits when expanding output.

This is done here, every reduced platforming case is referred to as a *deficit platforming case*, set up as follows:

- catalytic reforming capacity was restricted to the capacity range 6. to 16. mmt/y (from the surplus case of 18.43 mmt/y capacity). Exploratory tests have shown that for a catalytic reforming capacity less than or equal to 5. mmt/y the system becomes infeasible, since a minimum of reforming capacity is required for satisfying motor spirit demand qualities and part of the final demand (the minimum being, in this paticular refinery, 6. mmt/y);

- for every deficit platforming setting up, premium motor spirit demand was also varied from 0. mmt/y to physical output limits in order to test the amount of premium motor spirit the refinery was able to produce by cutting down reforming capacity.

Accordingly, the *maximum* premium motor spirit output satisfied in every deficit platforming case is presented in Table 4-2. It is worth pointing out that for every premium motor spirit demand case, irrespectively of the deficit platforming capacity fixed, results are reproduced exactly as in the surplus case, *except* when maximum pre-

Table 4-2. Single Refinery: experimental results, cases of changes in motor spirit demand under deficit platforming.

Refor Cap mmt/y	Case PMS-D mmt/y	AL Lift mmt/y	HFO Imports mmt/y	PMS Price $/ton	HFO Price $/ton	Cat Refor $/ton	Cat Crack $/ton
6.	7.	36.4	20.8	189.4	219.0	11.5	86.5%
6.	8.	37.5	20.7	305.7	280.4	176.8	29.6
7.	10.	40.7	19.9	1635.8	187.2	1312.5	617.4
8.	11.	42.9	18.8	1701.8	174.6	1363.6	643.2
9.	12.	45.6	17.3	1536.8	150.0	1144.6	507.8
10.	13.	48.9	15.3	1500.3	150.0	1100.4	493.8
11.	14.	52.2	13.2	2310.8	150.0	1889.9	813.8
12.	15.	55.7	10.9	2367.5	150.0	1944.5	835.0
13.	16.	59.3	8.5	2367.5	150.0	1944.5	835.0
14.	17.	63.0	6.0	2543.1	150.0	2087.2	886.4
15.	18.	67.3	3.1	4642.6	150.0	3945.3	1731.0
16.	18.	67.3	3.1	4642.6	150.0	3945.3	1731.0
17.	19.	65.3	6.3	536.1	150.0	96.7%	128.2
18.	20.	70.7	2.2	569.6	150.0	97.3%	146.5

mium motor spirit output limits are reached: Table 4-2 presents *only* the limit cases; for instance, referring to the reforming capacity of 10. mmt/y, results from 0. to 12. mmt/y premium motor spirit demand, for the surplus and this particular deficit case (10. mmt/y) are those of Table 4-1, and for 13. mmt/y premium motor spirit demand (the maximum produced at 10. mmt/y), results differ (see tables 4-1 and 4-2 at that level).

The following is noticed from the results:

(i) The maximum level of premium motor spirit production appears to be the same as in the surplus case, namely, 20. mmt/y, at the reforming capacity of 18. mmt/y (just some .40 mmt/y of capacity less than the surplus's). For every deficit platforming case the maximum output is about 2. mmt/y greater than the actual platforming capacity. Heavy fuel oil imports do not show any significant difference to the surplus case: they increase with increasing demand until a particular combination of the price exchange ratio AL/HFO and the premium motor spirit demand causes the refinery to produce a catalytic cracking rent: this combination point is precisely the one generating rent in the surplus case, i.e., the 9. mmt/y premium motor spirit demand case. From here on imports and liftings of Arabian Light decrease and increase respectively.

(ii) For each restricted reforming capacity, catalytic cracking rent increases considerably at the refinery's maximum premium motor spirit demand level satisfied, as compared to the surplus case. Except for the catalytic reforming capcity of 6. mmt/y, the catalytic cracking rent is (invariably) first generated at 9. mmt/y of premium motor spirit demand, like in the surplus cases. Thereon, it increases, and eventually takes the high marginal value when the refinery produces the maximum premium motor spirit amount possible.

Alongside, the catalytic reforming capacity utilization increases, and only when at maximum premium motor spirit, there arises a catalytic reforming rent, a *high* reforming rent. Figure 12 on page 130 depicts the catalytic cracking and reforming rents only at the premium motor spirit output limits (since the pattern for the other levels is that of the surplus case) and for every platforming deficit case.

(iii) At maximum premium motor spirit levels, product prices increase greatly, price structure, however, remains unchanged (as compared to the surplus case) for lower levels of premium motor spirit demand and for every deficit platforming case. This is illustrated by the following example: taking the catalytic reforming capacity of 11. mmt/y and comparing prices at 10. and 14. mmt/y (the maximum at 11. mmt/y deficit case) premium motor spirit demand respectively with those of the surplus case we have,

Refor Cap mmt/y	PMS Demand mmt/y	PMS $/ton	SRB $/ton	NAPHTA $/ton	KERO $/ton	DFO $/ton
11.	10.	282.5	256.7	348.5	343.0	290.0
18.43	10.	282.5	256.7	348.5	343.0	290.0
11.	14.	2310.0	169.0	454.0	437.4	267.8
18.43	14.	376.4	326.0	343.0	331.0	240.0

Except for the straight-run benzine price, the overall price tendency is to increase while catalytic reforming capacity is being reduced.

Because prices keep the surplus case pattern for premium motor spirit demand levels other than the maximum, product prices are not tabulated or depicted in a graph similar to that of Figure 11 on page 125. Only the premium motor spirit prices are depicted in Figure 12 on page 130.

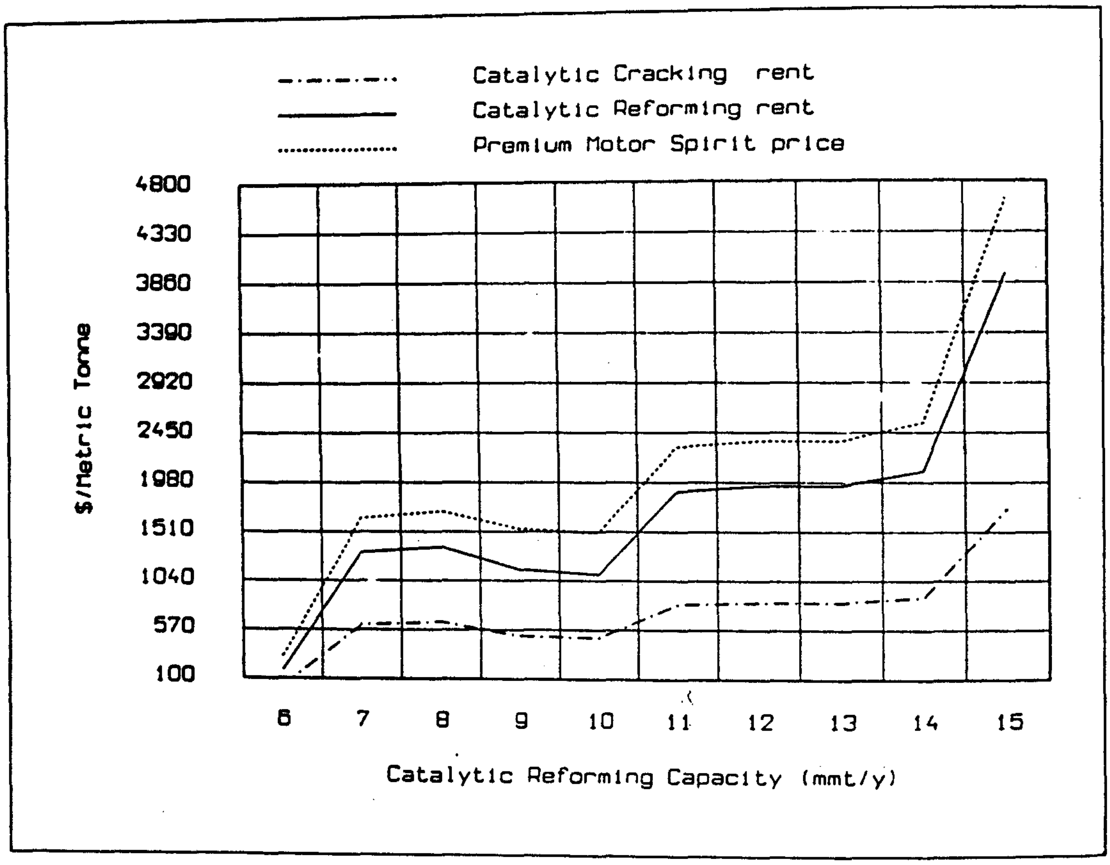


Figure 12. Single Refinery: catalytic cracking and catalytic reforming rents under pms demand changes and deficit platforming.

4.2.5 Concluding Remarks on sections 4.2.3 and 4.2.4

For a refinery with a configuration like the Single Refinery's, and with a chemistry of supply predominantly of medium (to light) crudes (the 88% of supply), and satisfying a chemistry of demand of about 24% of middle distillate and 46% of fuel oils (in contrast to a 16% of motor spirits), there is an excess of catalytic reforming capacity. If part of the fuel oil final demand is substituted by imports, there is room for more waxy fuel oil to be cracked and, eventually less of the reforming capacity to be used.

It has been shown that increasing premium motor spirit demand does not load platformers: capacity is too much in excess. The combined effect of changing premium motor spirit demand and reducing platforming capacity can generate a catalytic reforming rent. However, experimentally, cutting down catalytic reforming capacity (all other inputs and production factors remaining fixed) does not generate a rent unless the refinery is to satisfy a *particular* premium motor spirit demand. This latter is incidentally the ***maximum*** premium motor spirit level under deficit platforming, and the one making reforming capacity to be fully used. Decreasing by one tonne this maximum will not generate a reforming rent; a tonne above the maximum will not be feasible for the refinery to produce.

Given the seasonal movement of most oil product demands, inter alia, motor spirit demand, it seems to us, firstly, that platforming capacity *should* (mostly) be in surplus so to account for the seasonal swings in motor spirit demand, which at some point could make the unit to generate a rent for the refiner; secondly, platformers might be more efficiently used when cracking units are directed to produce middle distillates rather than cracked spirits, given the above chemistry of supply, a one prevailing with slight changes in North West Europe.

Due to the relevance of middle distillates in final demand, increases in their levels should force the use of platformers. Indeed, platformers are fully used (hence producing a rent) for fixed values of motor spirit demand and varying levels of gas oil demand, particular combinations of both produce rent.

4.2.6 Middle Distillate Demand under Surplus Platforming

The aim of this and the next experimental section is to see the effects on refinery rents of varying the demand for middle distillates, under both surplus and deficit platforming (section 4.2.6) capacities, and at various premium motor spirit demand levels.

The experimental runs under surplus platforming and changing middle distillate demand were carried out in this section on the following basis:

- platforming capacity remained as surplus, 18.43 mmt/y (base case);

- premium motor spirit demand was fixed within a predetermined interval, this is 9. - 20. mmt/y: from previous results these limits are respectively, the minimum premium motor spirit output producing rent, and the maximum feasible output for the refinery to produce;

- for every premium motor spirit demand fixed within the above interval, the gas oil demand was left free to move from 0. mmt/y to the maximum gas oil demand the system could produce.

Under this setting up, the *minimum* and *maximum* amounts of gas oil the refinery was able to satisfy at any premium motor spirit demand in the range 9. - 20. mmt/y were respectively 4. and 48. mmt/y.

Great deal of computational effort was placed on this exercise, results are presented in graphical form. Reports on the whole set of runs indicate that:

(i) At a physical level: the higher the premium motor spirit demand, the greater the Arabian Light liftings, and for a fixed premium motor spirit demand, the Arabian Light increases with the gas oil demand. The opposite occurs for heavy fuel oil imports, the higher the premium motor spirit demand, the lower the heavy fuel oil imports, and for a fixed premium motor spirit demand, heavy fuel oil imports tend to decrease (as both, Arabian Light and hevy fuel oil are complementary inputs), see Figure 13 on page 133.

The same applies for fixed gas oil demand and varying premium motor spirit, as can be seen from the same figure.

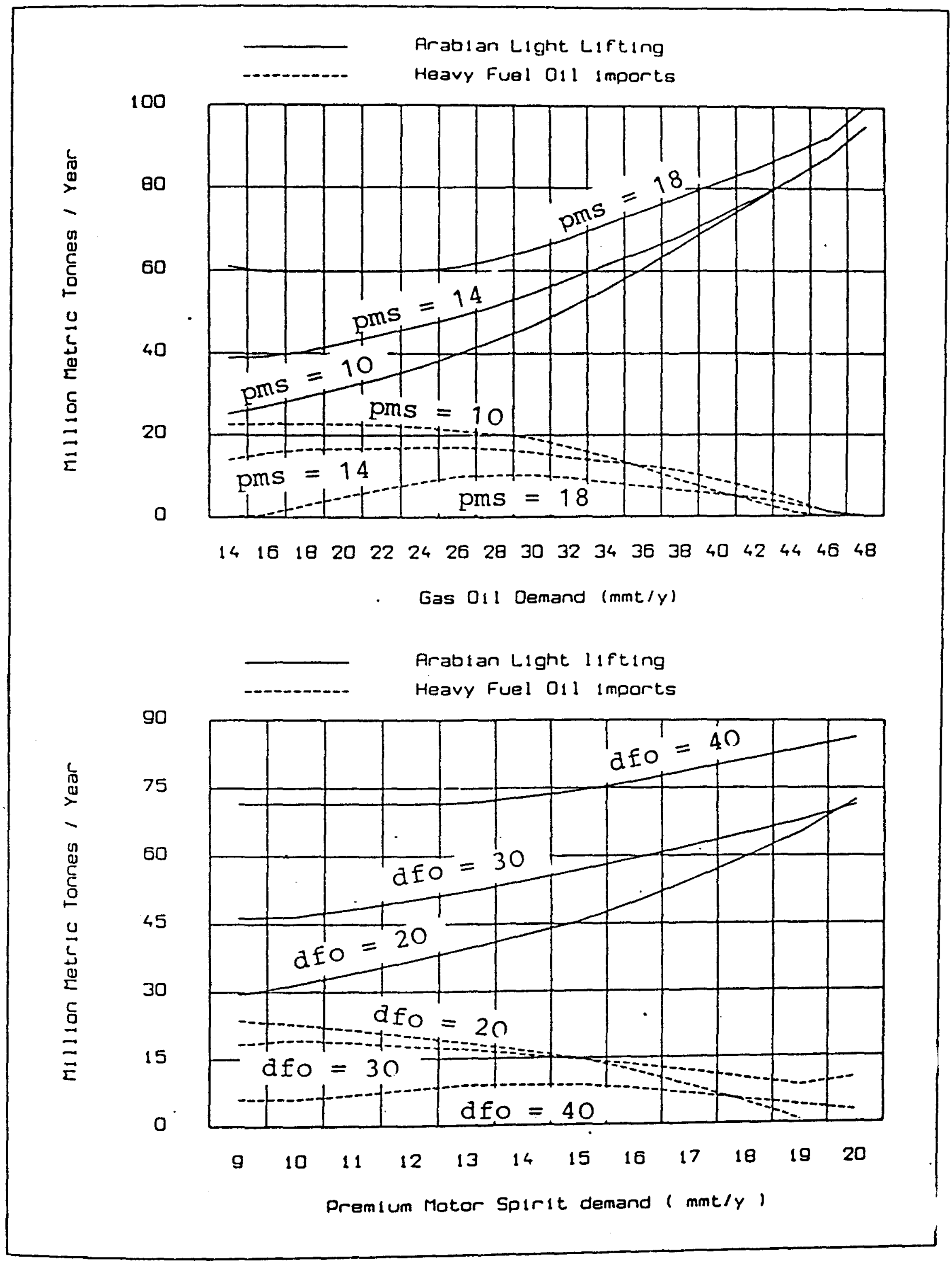


Figure 13. Single Refinery: Arabian Light lifting and heavy fuel oil imports, pms and dfo demand changes, surplus platforming.

The tendency is then for Arabian Light liftings to increase and heavy fuel oil imports to decrease with the increase of gas oil or motor spirit demands. This causes great volumes of gas oil and heavy fuel oil to be produced via crude distillation. The excess of heavy fuel oil produced goes then to satisfy final demand since crackers are already fully loaded; no substitution heavy fue oil/coal occurs in the system for high gas oil demand, and at the fixed price exchange ratio AL/HFO.

Although the waxy fuel oil going to crackers remains almost the same in every case (cracking at full capacity), the logistics of gas oil shows that crackers are *mostly* utilized to produce gas oil whereas the fraction of catalytic cracked spirit diminishes, and the use of platformers increases due to the (now) excess of straight-run benzine in the refinery. Hence, the major motor spirit component is invariably and increasingly, reformates.

The fractions of naphtha and kerosine (also from distillation) blending to gas oil, increase steadily.

(ii) Rents: catalytic reforming capacity remains mostly iddle, though the percentage of capacity utilization increases with the gas oil demand for a fixed premium motor spirit demand. At the maximum output combination of 20. mmt/y - 44. mmt/y premium motor spirit/gas oil demands, a reforming rent arises. Further increases of both together make the system infeasible, so that there is *only one* option for the reforming unit to generate a rent.

Catalytic cracking rents, as could be expected, are generated. They have been depicted in two ways: at different fixed levels of gas oil demand and varying motor spirit (Figure 14 on page 135), and at different fixed levels of premium motor spirit demand and varying gas oil demand (Figure 15 on page 136).

Figure 14 on page 135 shows that at any fixed level of gas oil demand, the greater the demand for premium motor spirit, the higher the catalytic cracking rent is. And within the family of catalytic cracking rent curves, the lower the gas oil demand, the greater the rent is.

From Figure 15 on page 136 it can be seen that catalytic cracking rent attains high values at both low and high gas oil demand levels when keeping fixed the premium motor spirit demand. The overall tendency is, however, to decrease with increasing gas oil demand.

There are particular premium motor spirit/gas oil combinations infeasible to make, e.g., very low gas oil levels and high levels of premium motor spirit since once gas oil demand is satisfied, the excess of hydroskimmed gas oil/heavy fuel oil does not have a way out.

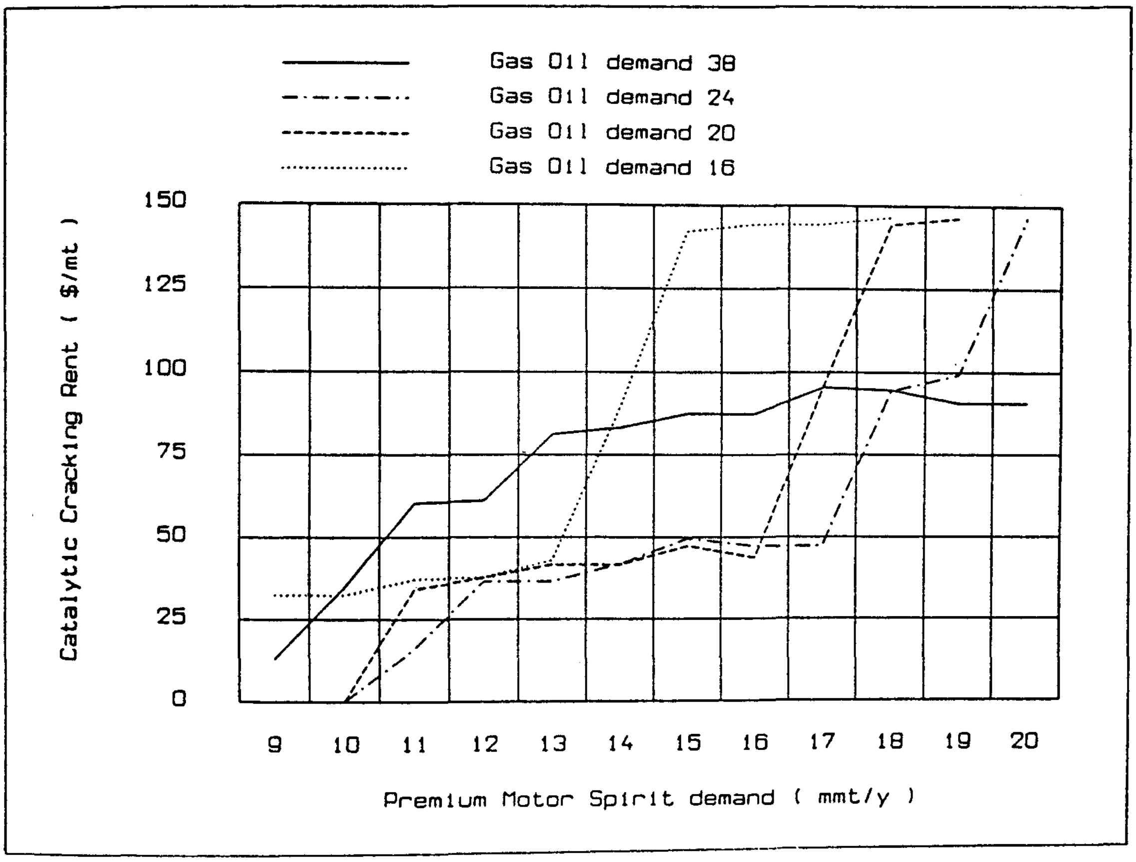


Figure 14. Single Refinery: catalytic cracking rent lines for varying pms demand and fixed dfo demand, surplus platforming.

There are also *limit* combinations, namely, those involving the maximum gas oil output of 48. mmt/y. The corresponding catalytic cracking rent line at any premium motor spirit level, is out of the range of Figure 15 on page 136, hence not plotted there:

pms demand (mmt/y)	9. - 15.	16. - 17.	18.
at 48. mmt/dfo demand			
cracking rent ($/tonne)	2550.	2530.	2480.

But the behavioural pattern of rents with gas oil demand changes suggests the mentioned decreasing pattern, particularly for gas oil levels from 0. to 34. mmt/y.

The fact that catalytic cracking *rent* decreases and platforming use increases with increasing gas oil demand, lies on the refinery units physical *performance*; as explained in (i), the catalytic cracking unit

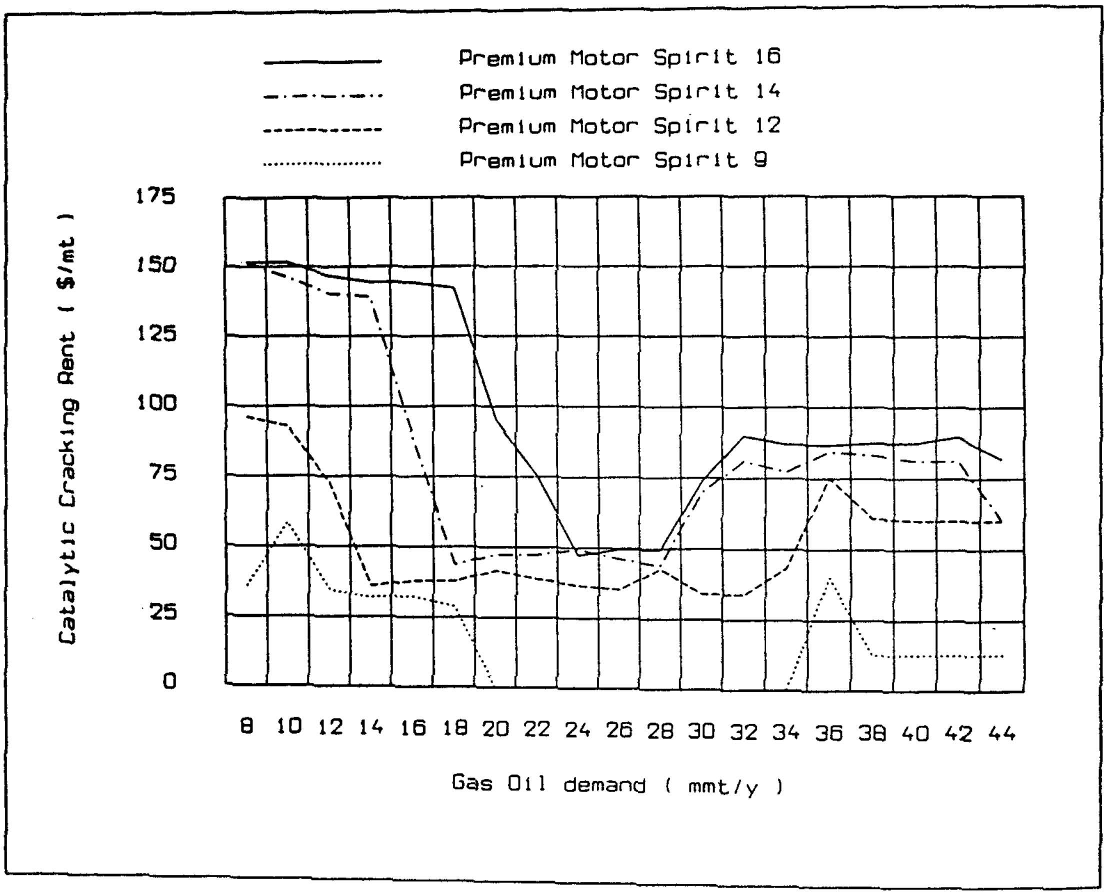


Figure 15. Single Refinery: catalytic cracking rent lines for varying dfo demand and fixed pms demand, surplus platforming.

works chiefly to satisfy gas oil demand; that is not the best position for the refiner with the given refinery configuration, he would certainly be better off with a high premium motor spirit demand comparatively to the *gas oil demand*, and with a cracking unit satisfying greater motor spirit fractions than gas oil ones, as the former is higher priced and rents are price determined *in the short run*.

(iii) Prices: the product price structure presents, in general, a stable behaviour, a particular case with premium motor spirit demand fixed *in* 15. mmt/y is depicted in Figure 16 on page 137. As gas oil demand increases, gas oil prices also increase while straight-run benzine and motor *spirit* prices fall.

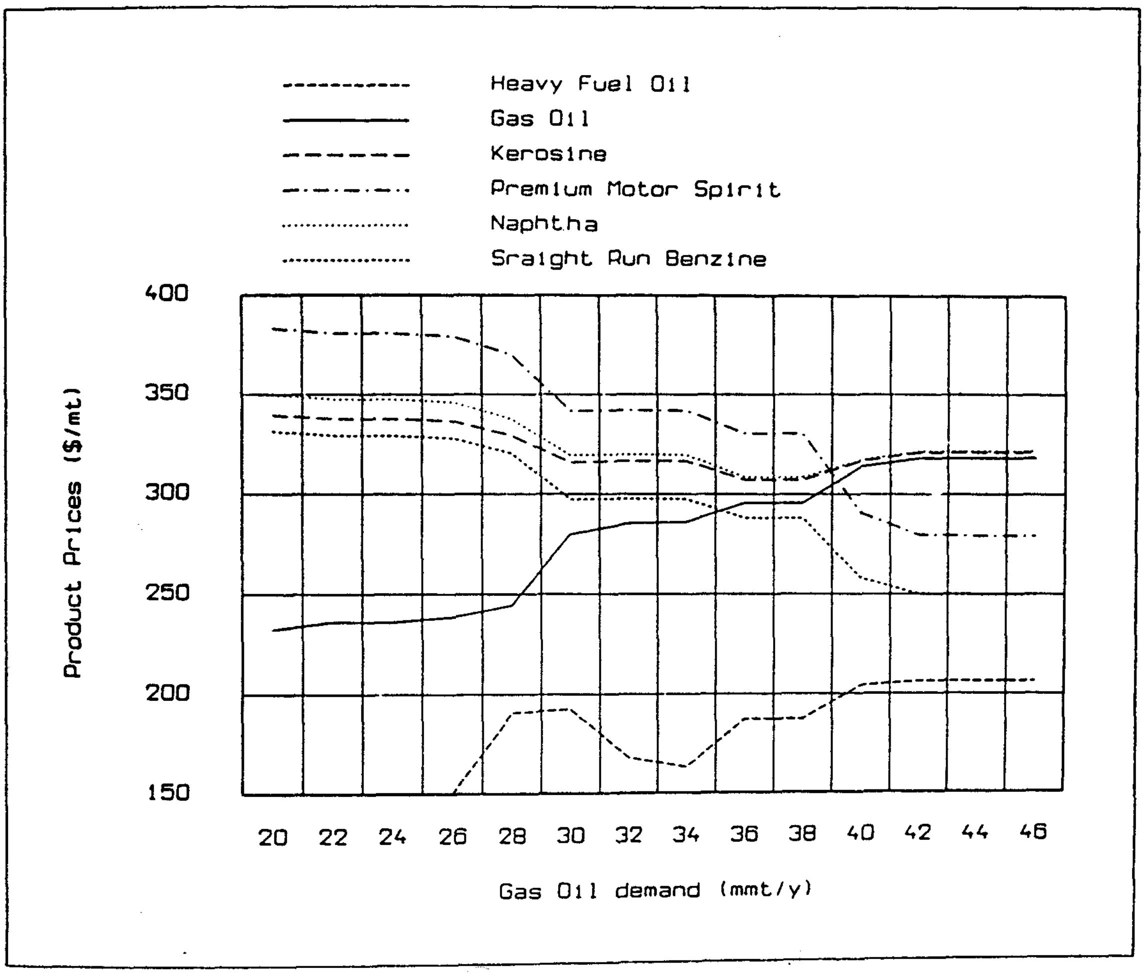


Figure 16. Single Refinery: product price profile under gas oil demand changes and surplus platforming.

Naphtha shows the opposite gas oil price pattern: though its price is ever higher than gas oil's, it decreases slowly, this combined with the increasing gas oil price tend to aproximate both price curves, a fact which occurs at high levels of gas oil (where naphtha price seems to recover slightly).

As regarding fuel oils, their intermediate, transfer prices (Figure 16 depicts the heavy fuel oil transfer price curve) are closely related to the Arabian Light price since the major heavy fuel oil fraction in the pool comes from pure crude distillation (about 70% is hydroskimmed component), the pattern is increasing. However, at the market level, the price of marginal heavy fuel oil (coal) for imports imposes a bound on the heavy fuel oil market price, hence that the 150. $/mt for heavy fuel oil of final demand remains fixed along with the gas oil demand variation. The following concluding remarks of this section are drawn:

- the lower the gas oil demand and the greater the premium motor spirit demand (within limits), the higher the catalytic cracking rent; platforming usage increases, however never becoming restrictive;

- the greater the gas oil demand for fixed premium motor spirit demand, the lower the catalytic cracking rent; platforming usage also increases;

- great levels of both gas oil and premium motor spirit demands generate a *platformer rent*. Crackers and crude distillation components act as gas oil *satisfiers*, since platformers are increasingly fed with straight-run benzine to meet demands.

4.2.7 Middle Distillate Demand under Deficit Platforming

A similar set of runs to the previous section's was carried out here but cutting down platforming, setting up conditions are as follows:

- platforming capacity progressively reduced in the range 6. to 18. mmt/y;

- premium motor spirit varying in the interval 9. to 20. mmt/y (for reasons already noted in section 4.2.6); and,

- gas oil demand varied between 20. mmt/y to maximum attainable output for every premium motor spirit case within the corresponding deficit platforming setting up.

Due to the great amount of computational information obtained from experimentation, no attempt was made in tabulating it here. Instead, relevant points and graphs are presented:

(i) As regard physical refinery performance, no difference arises between this and the previous surplus case: Arabian Light liftings and heavy fuel oil imports increase and decrease respectively, crude distillation and cracking meet gas oil requirements.

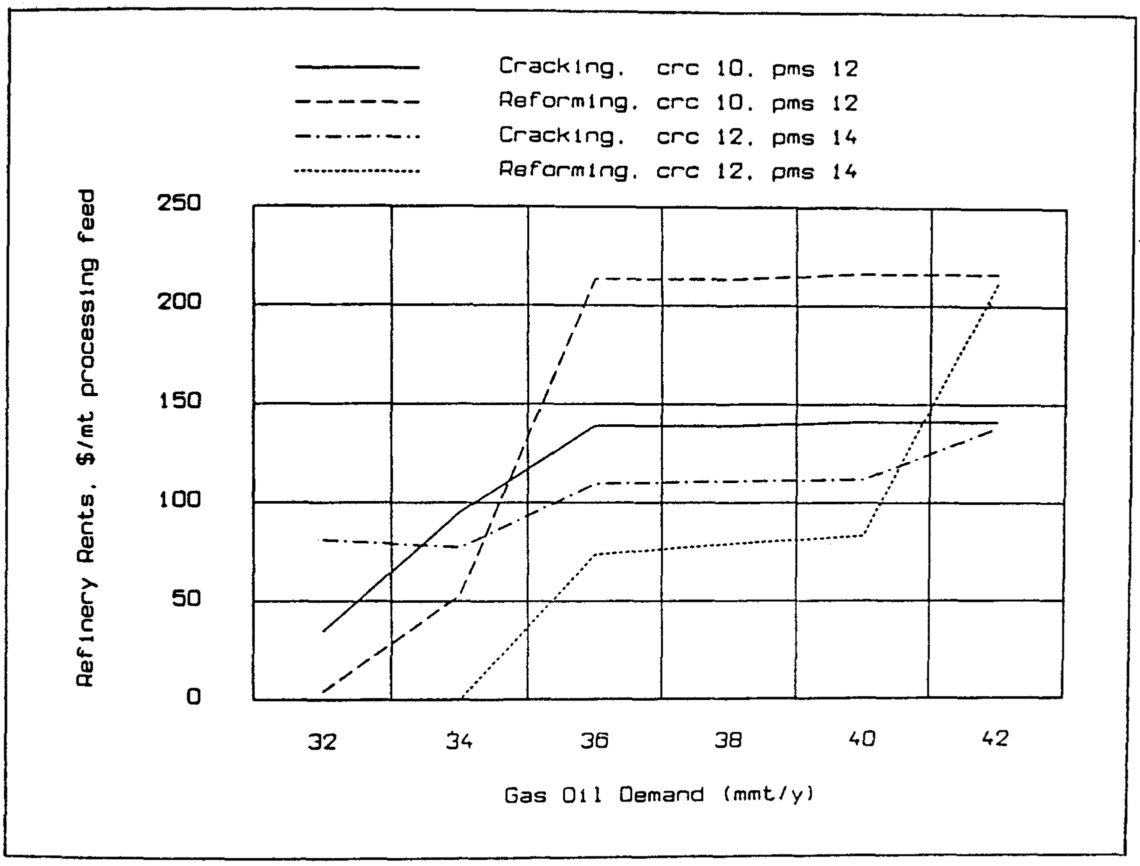


Figure 17. Single Refinery: refinery rents under gas oil demand changes and deficit platforming.

It is noted, however, that for a given deficit platforming capacity, there are limits on the refinery's production of premium motor spirit and gas oil, (also seen effect in 4.2.4), for instance if the platforming capacity is fixed at 6. mmt/y, the maximum combination is 7. mmt/y premium motor spirit and 32. mmt/y gas oil demands. The limits on premium motor spirit are the same of Table 4-2 for the changes in gas oil demand have no effect whatsoever on the ability of the refinery to expand premium motor spirit output beyond those limits.

(ii) Rents: under deficit platforming and increasing gas oil demand, and for every premium motor spirit demand case, there is an output combination generating a catalytic reforming rent. Unlike the case described in section 4.2.4, (deficit platforming, varying premium motor spirit demand, fixed gas oil), from that *rent point* on, catalytic reforming rent increases along with the catalytic cracking rent. As the gas oil demand increases, and less catalytic cracked spirit is

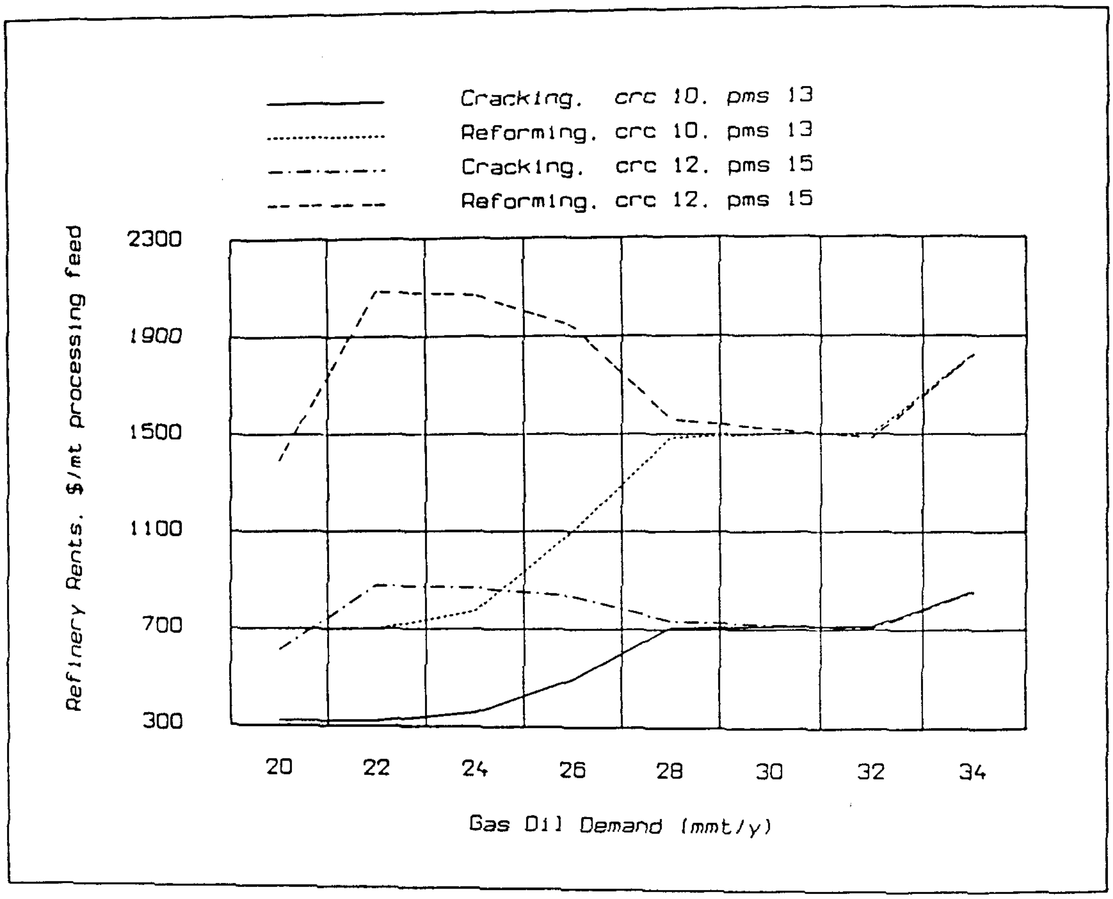


Figure 18. Single Refinery: refinery rents at maximum outputs under gas oil demand changes and deficit platforming.

blended to premium motor spirit, more straight-run benzine is reformed: a *critical* point arises where catalytic reforming rent becomes greater than catalytic cracking rent. See for instance, Figure 17 on page 139: catalytic reforming and cracking rent lines are shown for the platforming capacity of 10. mmt/y and premium motor spirit demand of 12. mmt/y, and for the platforming capcity of 12. mmt/y and premium motor spirit demand of demands of 14. mmt/y.

The rent points are noticed to be 32. mmt/y gas oil demand for the 10. mmt/y platforming case, and 34. mmt/y gas oil demand for the 12. mmt/y platforming case. The critical points being around 34.5 mmt/y and 40. mmt/y for the same cases.

And, moreover, at maximum premium motor spirit outputs, and for every deficit platforming case, the platforming rent becomes higher than the catalytic cracking rent for *every gas oil demand change*, see Figure 18. There are depicted the rent lines for platforming capaci-

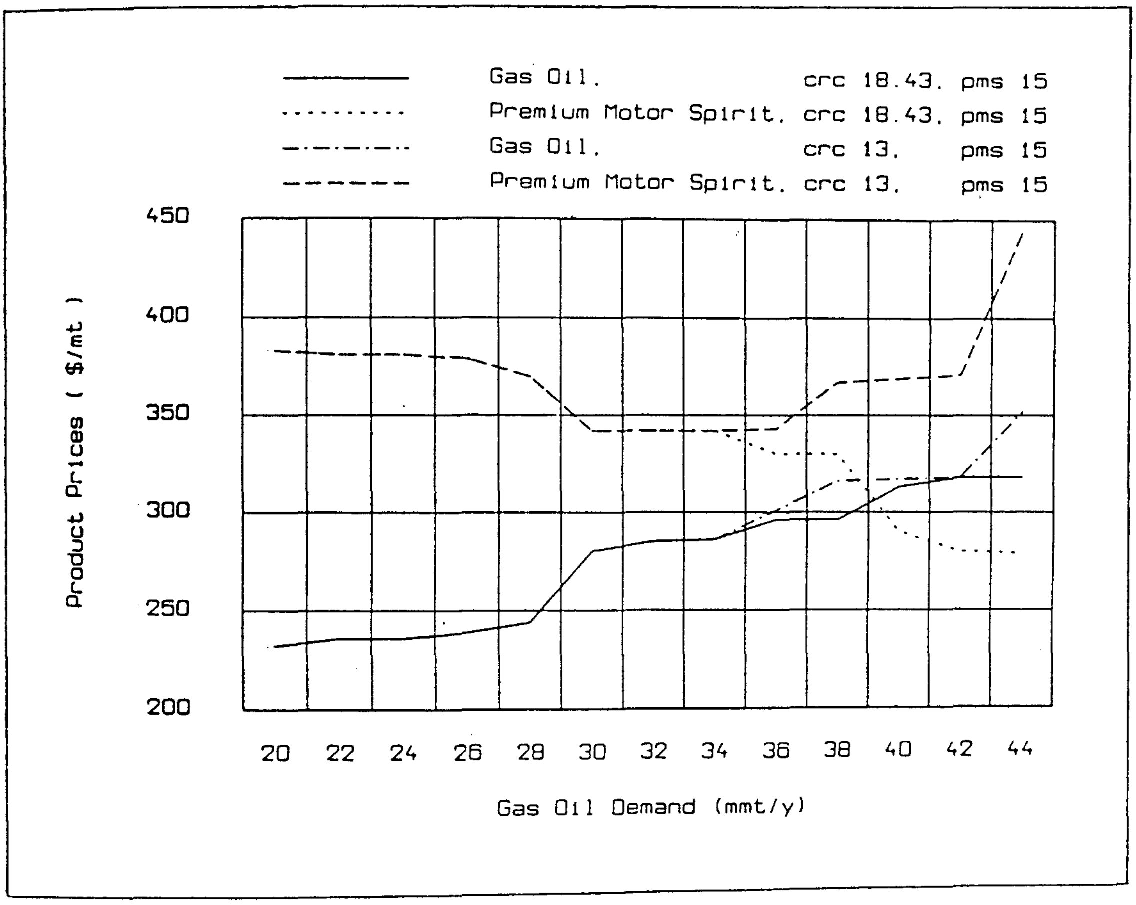


Figure 19. Single Refinery: product prices under gas oil demand changes and surplus/deficit platforming.

ties of 10. mmt/y and 12. mmt/y and for the respective maximum premium motor spirit outputs of 13. and 15. mmt/y.

(iii) Prices: surplus and deficit platforming poduct price structures are identical up to the point of platforming generating rents: then the prices of motor spirits rise appreciably, and other prices tend to remain constant. In Figure 19, the prices of gas oil and premium motor spirit for the platforming capacities of 13. mmt/y and 18.43 mmt/y (surplus), and for the premium motor spirit demand of 15. mmt/y are depicted. The critical point being 38. mmt/y of gas oil demand.

4.2.8 Concluding Remarks on *sections* 4.2.6 and 4.2.7

A premium motor spirit/gas oil output combination generating both catalytic reforming and cracking rents, is the best economic one for the refiner. Since this fact is out of the refiner's control, what is left to him is to predict his short run (marginal) rents on the basis of known/estimated price supply/demand curves.

Experimentally, increasing gas oil demands push gas oil and fuel oil prices up, and lower catalytic cracking rent. Increasing premium motor spirit demands push light product prices up and increase catalytic cracking rent. In other words, catalytic cracking rent is an increasing function of motor spirit demand an prices; similarly, it is a decreasing function of gas oil demand and prices and of heavy fuel oil price.

Catalytic reforming rent is suggested tó be an increasing function of catalytic cracking rent, therefore an increasing function of premium motor spirit price and a decreasing function of heavy fuel oil price; similarly it is a decreasing function of straight-run benzine and naphtha prices. It could also be said that it is an increasing function of gas oil demand and prices as increasing gas oil demand forces platformers to increase severity. But on the main, its short run rent, as attached to a fixed resource, depends on the available capacity and particularly on the capacity relation cracking/platforming and the chemistry of demand.

Under surplus platforming conditions, for this unit to produce eventually a marginal rent, it is necessary that the cracking unit is also generating a rent, and that there exists an output combination premium motor spirit/gas oil making the plaforming resource restrictive, but this combination is, as in the case of deficit platforming and premium motor spirit demand changes, not any one but a *particular* one, suggesting a *non-rent* tendency of the catalytic reforming unit in this refinery under fairly fixed supply/demand conditions.

These exercises reflect the variety of feasible correlations between refinery unit rents and input/output prices. A sensible choice should be to correlate the marginal rent to the crude price, or rather to its straight components, i.e., the straight-run benzine, naphtha and kerosine prices for the sake of reforming rent estimation; and to heavy fuel oil (waxy fuel oil), premium motor spirit and gas oil prices for the cracking rent estimation. This is done and reported in chapters 5 and 6.

4.2.9 Price Exchange Ratio Arabian Light/Heavy Fuel Oil

The second experimental hypothesis is tested here. The attention is turned to the *price markers*, both, Arabian Light and coal (in $/mt fuel oil equivalent) are progressively changed and effects on rents and the process of heavy fuel oil/coal substitution measured.

Although this aspect is dealt with in a more realistic context in Chapter 7 of this work, preliminar consideration is given here to what is thought to be the rationale behind refinery rents behaviour in a now changing *chemistry of energy*, this is to say, an energy market in which any, or both, of the chemistry of supply/demand experience structural changes (because of the process of energy substitution), due basically to changes in the supply price structure.

Accordingly, supply prices were changed in a predetermined range, giving rise to a set of price exchange ratios. For every fixed ratio, the refinery rents were observed, and a graph of contour lines produced.

Referring to the coal/heavy fuel oil price relation,

Price-hfo = 1.5 Total cost of coal

or,

Total cost of coal = (1/1.5) Price-hfo,

there is no loss of generality if talking about marginal coal and imported heavy fuel oil indistinctly since the factor 1.5 makes the tonnage equivalent, and regarding prices, when knowing one them, it is only needed to apply the above relation and the price of the other follows. Hereafter then, the marginal coal will be termed also the total cost of marginal coal. Similarly, a price exchange ratio AL/HFO will mean also a price exchange ratio Arabian Light/Total cost of marginal coal.

The experimental basis was set up as follows:

- platforming capacity as in surplus, 18.43 mmt/y;
- chemistry of demand SR original version (section 4.2.1) unchanged;
- the price exchange ratio AL/HFO was set by fixing prices according to averages found at the Rotterdam spot market in recent years: from end 1973 onwards the Arabian Light price has been always

greater than 10. \$/bl (73. \$/mt), hence the minimum considered here, at the same time prices of heavy fuel oil have been higher than those of Arabian Light, this is also reproduced here.

Table 4-3 presents the set of price combinations AL/HFO and the respective price exchange ratios used for experimentation.[88]

Table 4-3. Single Refinery: experimental Arabian Light/Heavy Fuel Oil price exchange ratios.

AL \$/mt	Price Exchange Ratios (tonnes of hfo per tonne of Arabian Light)										
250.	17.	7.	4.5	3.3	2.6	2.	2.	1.6	1.4	1.3	1.2
235.	16.	7.	4.	3.	2.5	2.	1.7	1.5	1.3	1.2	1.
212.	14.	6.	4.	3.	2.2	2.	1.6	1.4	1.2	1.	1.
205.	14.	6.	4.	3.	2.	1.8	1.5	1.3	1.2	1.	1.
191.	13.	5.	3.5	2.5	2.	1.7	1.4	1.2	1.	1.	1.
176.	12.	5.	3.	2.	2.	1.5	1.3	1.	1.	.9	.8
161.	11.	4.6	3.	2.	1.7	1.4	1.2	1.	.9	.8	.8
132.	9.	4.	2.4	1.8	1.4	1.1	1.	.9	.8	.7	.6
102.	7.	3.	2.	1.4	1.	.9	.8	.7	.6	.5	.4
73.	5.	2.	1.3	1.	.8	.6	.5	.5	.4	.4	.3
FOE \$/mt	15.	35.	55.	75.	95.	115.	135.	155.	175.	195.	215.

FOE = fuel oil equivalent in \$/tm.

88 A more comprehensive experimental frame is developed in Chapter 7 with the 7-area WEM. There, prices move within wider ranges: a wide spectrum of options for price structure and ratios, for refinery rents arises.

By analysing the refinery units physical/economic performance after experimentation the following can be stated:

(i) Product prices (Figure 20 on page 146) do not show any significant feature, they increase with the ratio, and are particularly high at high Arabian Light prices and high coal costs.

(ii) Rents: In the Single Refinery model, the reforming unit being in surplus, does not generate a rent at any price combination Arabian Light/Heavy Fuel Oil.

For the catalytic cracking unit there exist price combinations which produce rents, those giving ratios greater than 1; for ratios less than or equal to 1, no rent is produced. Figure 21 on page 147 shows the points (ratio,cracking rent), for ratios > 1: the general tendency is clear, catalytic cracking rent increases with the ratio. This is particularly noticed for ratios greater than 2 (second cut in the x-axes as the scale is logarithmic).

It is also noticed that there exist several price combinations giving the same price exchange ratio and for which the rent is the same. Each of these rent values represents a contour line of Figure 22 on page 149, where rather than the price ratio, both prices and the cracking rent (in $/mt processing feed) isolines are depicted.

In the same figure, those ratios giving no rent belong to the Zero-Rent zone. The maximum rent value (120. $/mt) occurring at the lowest heavy fuel oil price, 15. $/mt f.o.e. (i.e., 10. $/mt coal cost, down left corner) and the highest Arabian Light price, 250. $/mt (upper left corner), the higher the Arabian Light price and the lower the heavy fuel oil price, the higher the rent. Respectively in Figure 21 on page 147, the more to the right and to the top of the figure, the lower the heavy fuel oil price, the higher the Arabian Light price and the rent.

In other words, *the higher the price exchange ratio Arabian Light/Heavy Fuel Oil, the higher the catalytic cracking rent is.*

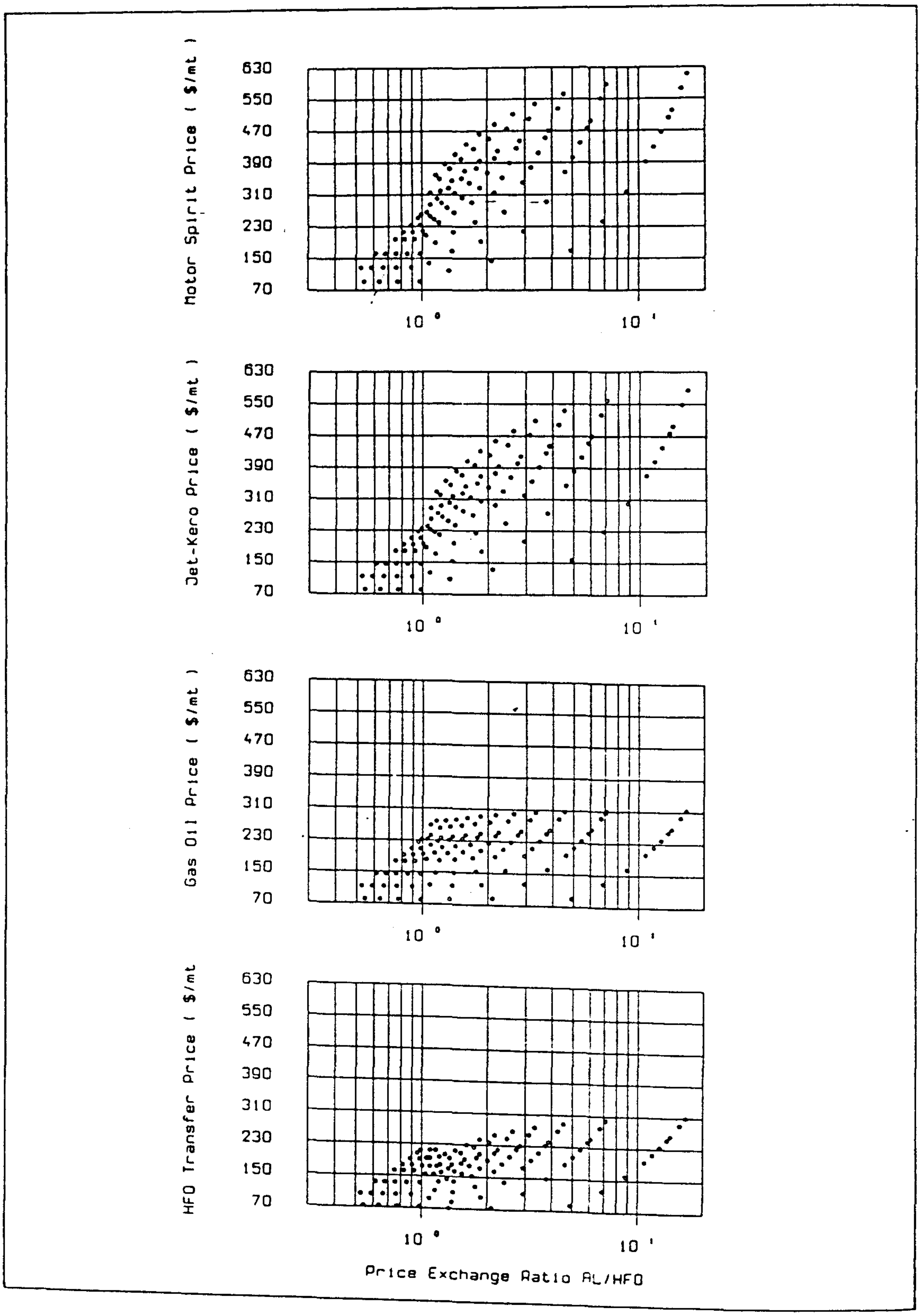


Figure 20. Single Refinery: product prices varying with the price exchange ratio AL/HFO.

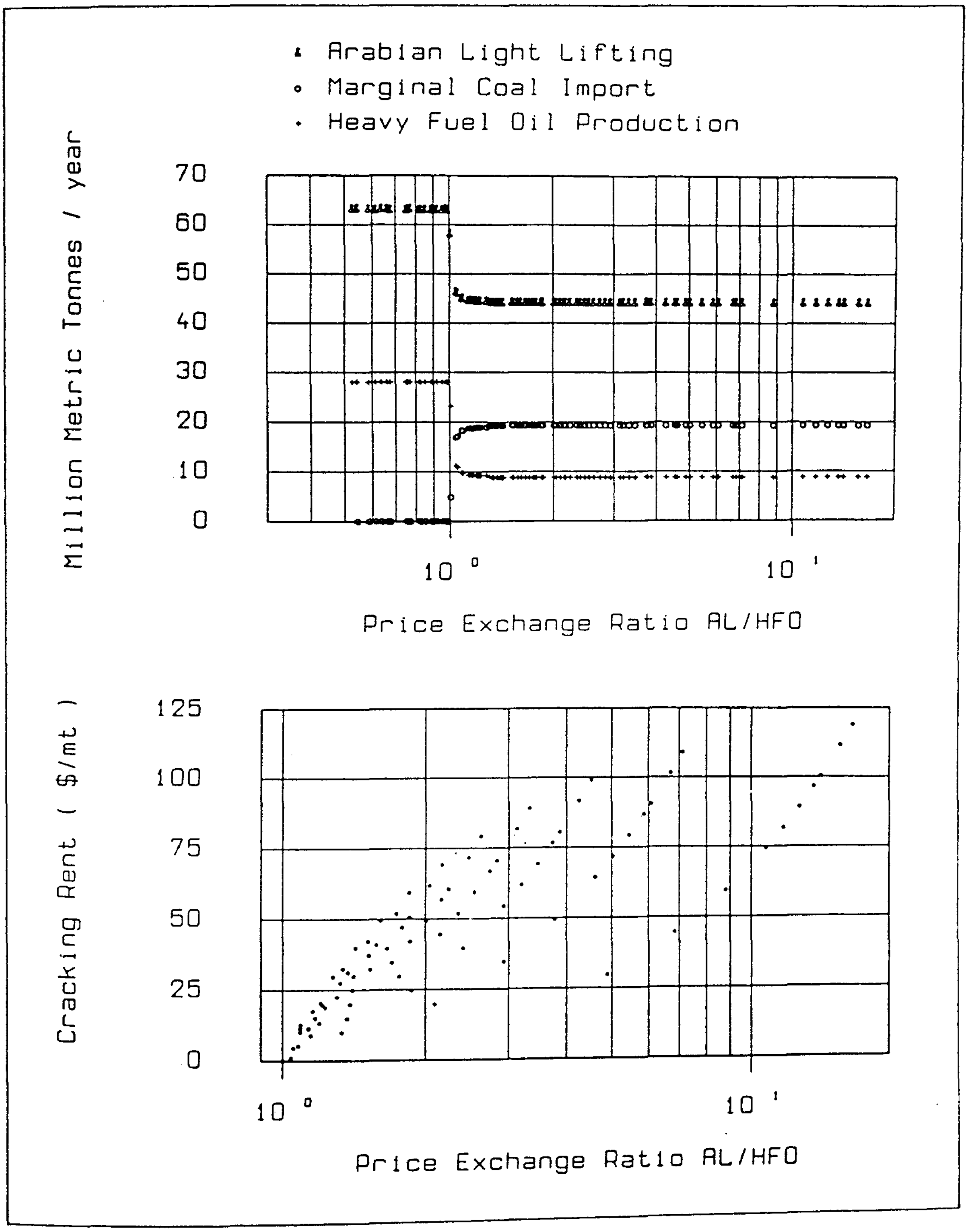
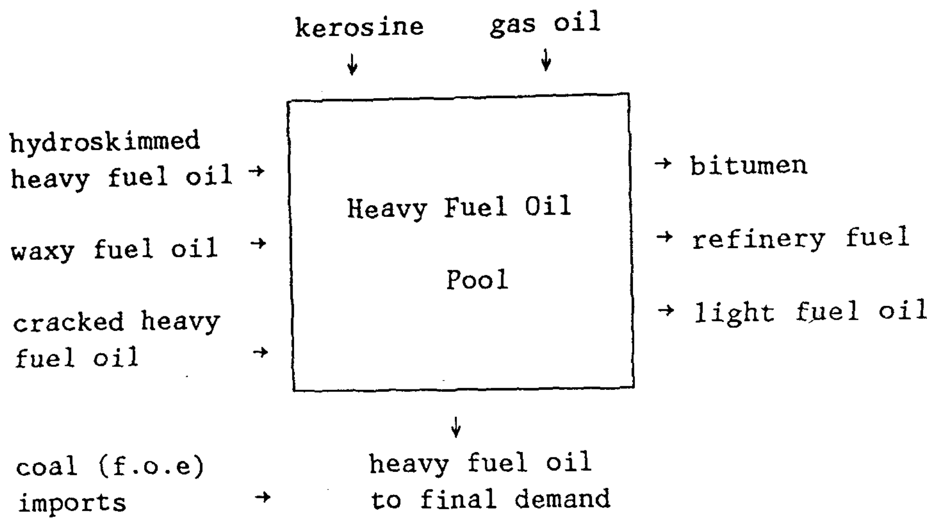


Figure 21. Single Refinery: exogenous input and catalytic cracking rent with varying price exchange ratio AL/HFO.

There are important physical changes associated with this behaviour. They can be explained on the grounds of the heavy fuel oil logistics and its links to economic factors (prices): a diagram of the heavy fuel oil pool input-output will help:



There are two components, namely, waxy fuel oil and hydroskimmed heavy fuel oil which compete for blending to the heavy fuel oil pool depending on the supply/demand chemistry, waxy fuel oil can be in turn directed to the cracking unit. When demands are fixed (our present case) and supply prices are varying, then it is the input side the one subject to changes, Arabian Light and heavy fuel oil compete as input feeds and this in turn determines the fractions of waxy fuel oil to be sent to the heavy fuel oil pool or cracking unit.

The heavy fuel oil imports is the amount of the total heavy fuel oil demand substituting for the non-refinery make of heavy fuel oil, i.e., it is the fraction of coal replacing heavy fuel oil in the market.

Two situations are distinguished as effects of changing supply prices:

(1) High price exchange ratio: this implies a high Arabian Light price, low heavy fuel oil price.

As was assumed, heavy fuel oil price is bounded by the total cost of coal, when this is low (excess supply, etc.) there is room for coal to enter the final demand, explaining the noticed higher level of imports.

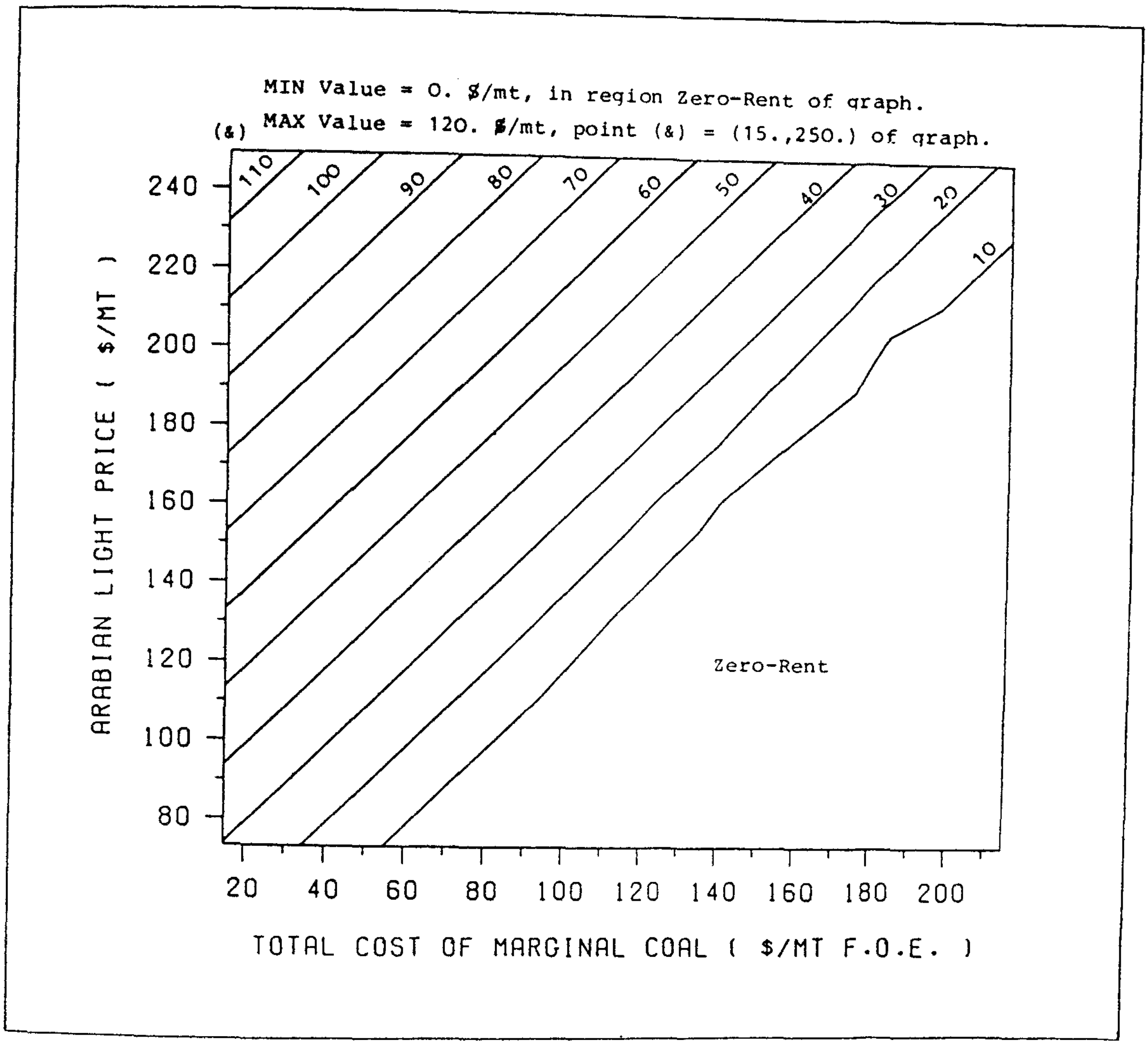


Figure 22. Single Refinery: catalytic cracking rent isolines under surplus platforming and varying price exchange ratio AL/HFO.

While a high ratio prevails, marginal coal is entering the market, making about 70% of the final demand for heavy fuel oil, that is replacing 70% of the heavy fuel oil of final demand, otherwise produced at the refinery mainly via distillation: the higher the imports, the lower the Arabian Light (high priced) liftings, hence the lower the hydroskimmed and waxy fuel oil amounts blending to heavy fuel oil.

The heavy fuel oil being replaced by coal, i.e., that which can not be sold in the market, has its way out in the refinery: more of it is burned and cracked, increasing the level of cracking rents. If they keep high (higher than the built-up cost) there will be an incentive for the refiner to invest.

(2) Low price exchange ratio: it implies a low Arabian Light price relative to the heavy fuel oil price.

Since the refiner is selling his heavy fuel oil as higher as he can, with a high coal cost prevailing, the best option is to sell it in the market and not to crack/burn it at the refinery; indeed less and less marginal coal is imported (reaching zero levels) and more crude comes in due to its lower price. This increases the hydroskimmed and waxy fuel oil amounts blending to the heavy fuel oil pool. The heavy fuel oil produced at the refinery satisfies entirely the final demand. Alongside, less heavy fuel oil is burned/cracked, rents decrease and eventually disappear.

Both situations can be seen in Figure 21 on page 147. Ratios lower than 1 (i.e., point 10^0 in x-axes) do not bring about either rents or coal/heavy fuel oil substitution. For cracking rent, ratios lower than 1 are not depicted for they belong to the Zero-Rent zone. After point 10^0, inputs of Arabian Light and coal decrease and increase respectively to remain then fairly constant; rents, however increase continuosly.

4.2.10 Concluding Remarks on section 4.2.9

Unidimensionality has been relaxed by fixing the Arabian Light/coal (fuel oil equivalent) price exchange ratio. A set of Arabian Light prices and costs of marginal coal were chosen for experimentation from which two main situations are distinguished from the results: the high and low price exchange ratio AL/coal. The first one brings about high cracking rents thus the incentive for the refiner to expand, and coal penetration in the market; the second one, minimum coal imports, low to zero cracking rents, and more heavy fuel oil via refining available for sale in the market.

It is now suggested, none situation is profitable for the producer and the refiner in the long run. In the first case, very high ratios (very high Arabian Light price) may cause a cut in the demand for crude in the market. The refiner having already expanded capacity, will find his refinery with excess, and therefore with falling rents, if not with more disastrous consequences. The second case, may not even bring about a rent high enough to cover variable plus built-up costs because the hydroskimming units do not produce rents.

A point exists where an equilibrium rent produces just enough to pay variable plus built-up costs, and neither overexpansion of capacity due to high rents nor losses due to low rents occur.

A graph like that of Figure 22 on page 149 combines supply prices and cracking rent behaviour. It is a useful and readily way of visualizing the cracking rent position under a two-tier price structure. The total cost of coal and/or the Arabian Light price could be replaced by any other supply prices if markers were others, and similarly, their relation to rents would arise.

During the mid seventies (1975-1977) the average cracking invesment cost in North West Europe was about 60. $/mt installed capacity, i.e., about 15. $/mt average annual capital charge or built-up cracking cost[89]. During that period too the Arabian Light Rotterdam spot price was around 12. $/bbl (i.e., 90. $/mt). In the isoline graph the 90. $/mt Arabian Light price corresponds to 30., 20., 10. $/mt or zero cracking rent, intersecting with a total cost of coal greater than or equal to 30. $/mt. Whitout precise figures of coal cost for those years, a cost of about 40. $/mt is taken as an approximation, under these conditions the refiner in North West Europe was at the time just breaking even, hence no expansion seems to have occurred; reality evidences this fact (see for instance, Figure 32 on page 296, Chapter 6).

The oil price increase of 1979 produced greater increases in crude and product prices in the early eighties. Arabian Light averaged 33. $/bbl (about 250. $/mt) at Rotterdam during 1979-1983. That means, Arabian Light price was high and cracking rents greater than 120. $/mt (highest cracking rent in graph), which is far beyond 28. $/mt, the average built-up cost of the time, cracking expanded.

It is in the interest of the markers' producers to keep supply prices in such a way as to maintain equilibrium. Fixing Arabian Light price is a political issue, the policy maker should ideally fix his price so as to keep the system in a one-tier price structure (restoring Unidimensionality). If this were not to happen, a change in marker source would occur and the next marker could well be coal or natural gas, or any other backstop technology competitive with present oil product prices.

The analysis is kept up to this point now, better results are reported and further analysis is done with the 7-area WEM, Chapter 7.

---

89 See Table 6-2, page 292, and Table 6-3, page 293, Chapter 6, and the explanation associated with the estimation of investment costs time series and average annual capital charges.

4.2.11 Catalytic Cracking and Catalytic Reforming Built-up

For fixed chemistry of demand and refinery capacity it has just been seen that the price ratio AL/HFO can determine the amount of marginal coal penetrating the system. Furthermore, the higher the ratio, the higher the imports and the cracking rent.

In this section the problem is seen from another perspective: fixing the rent on a fixed demand will determine the corresponding capacity combination cracking/reforming to satisfy that demand. If marginal inputs are allowed, that *right combination* may be less than otherwise required to satisfy the same demand.

Arabian Light and marginal coal inputs will limit the cracking/reforming capacities, obviously depending on their prices. If imports are high, less capacity will be required. Coal penetration will be at maximum when so rents are, i.e., when the heavy fuel oil demand has less hydroskimmed and cracked fuel oil in the blend, and instead more imported fuel oil (equivalent).

The interest is to see when maximum coal penetration, i.e., the maximum marginal input the refinery can tolerate, occurs:

- at a fixed AL/HFO price exchange ratio, varying expected catalytic cracking and reforming rents, and at a fixed chemistry of demand;
- at a fixed AL/HFO price exchange ratio, fixed catalytic cracking and reforming rents, and variable premium motor spirit demand.

In order to study these two different positions and arrive to maximum coal penetration, the Single Refinery is set up as:

- catalytic reforming and cracking capacities are set up to zero, rents are varied in both units from 5. to 110. $/mt, the expanding vectors will set the capacities in line with the expected rent. The zero-rent case for reforming is not proposed since this will mean surplus platforming and greater cracking capacity required.
- there are assumed rents for reforming and cracking, respectively of 28. and 60. $/mt feed, and of 43. and 90. $/mt feed. These are approximate investment costs of both technologies for the mid seventies and 1984 respectively (see Table 6-2, page 292, Chapter

6), i.e., rents greater than average annual capital charges or equilibrium rents to assure the refiner is in the position to expand. Maximum coal penetration at the fixed rents will then arise with associated capacities.

The outcome of the two cases is now showed:

In the first case, Figure 23 on page 154 shows the development of the reforming and cracking capacity expansion with an increasing pattern of equilibrium rents.

Catalytic reforming capacity is always greater than cracking, as it happens in North West Europe. For low expected rent, the cracking capacity is about 50% of the reforming capacity, and there is a rent, 55. \$/mt in the graph, where both capacities approach while reforming capacity decreases and cracking capacity increases. For an equilibrium rent of say, 10. or 25. \$/mt, cracking capacity is 50% of the reforming capacity.

Arabian Light liftings and heavy fuel oil imports remain fairly constant, however it is noticed they follow respectively the patterns of expansion of cracking and reforming capacities (same figure, bottom graph): increasing cracking rent is then associated with increasing cracking capacity and crude liftings, and with decreasing reforming capacity and heavy fuel oil imports.

In the second case, demand determines the amount of heavy fuel oil to be cracked, capacity adapts to demand and rent. Maximum coal penetration occurs for low levels of motor spirit demand. Greater demand implies more Arabian Light liftings, more heavy fuel oil for cracking, and more capacity. Heavy fuel oil import does not represent a strong competitive input to crude, probably due to the variability of demand, the (fixed) price combination AL/HFO (216. and 150. \$/mt, respectively), and the fact that reforming is forced to produce a rent, hence that platforming and crude liftings show similar growing patterns (naphtha feeding platformers, see Figure 24 on page 155). Figure 24 on page 155 shows the development of the reforming and cracking capacities, and of the refinery inputs when an equilibrium rent (i.e., that covering investment costs) and variable demand are assumed; and Figure 25 on page 156 singles out the capacities' expansion and shows

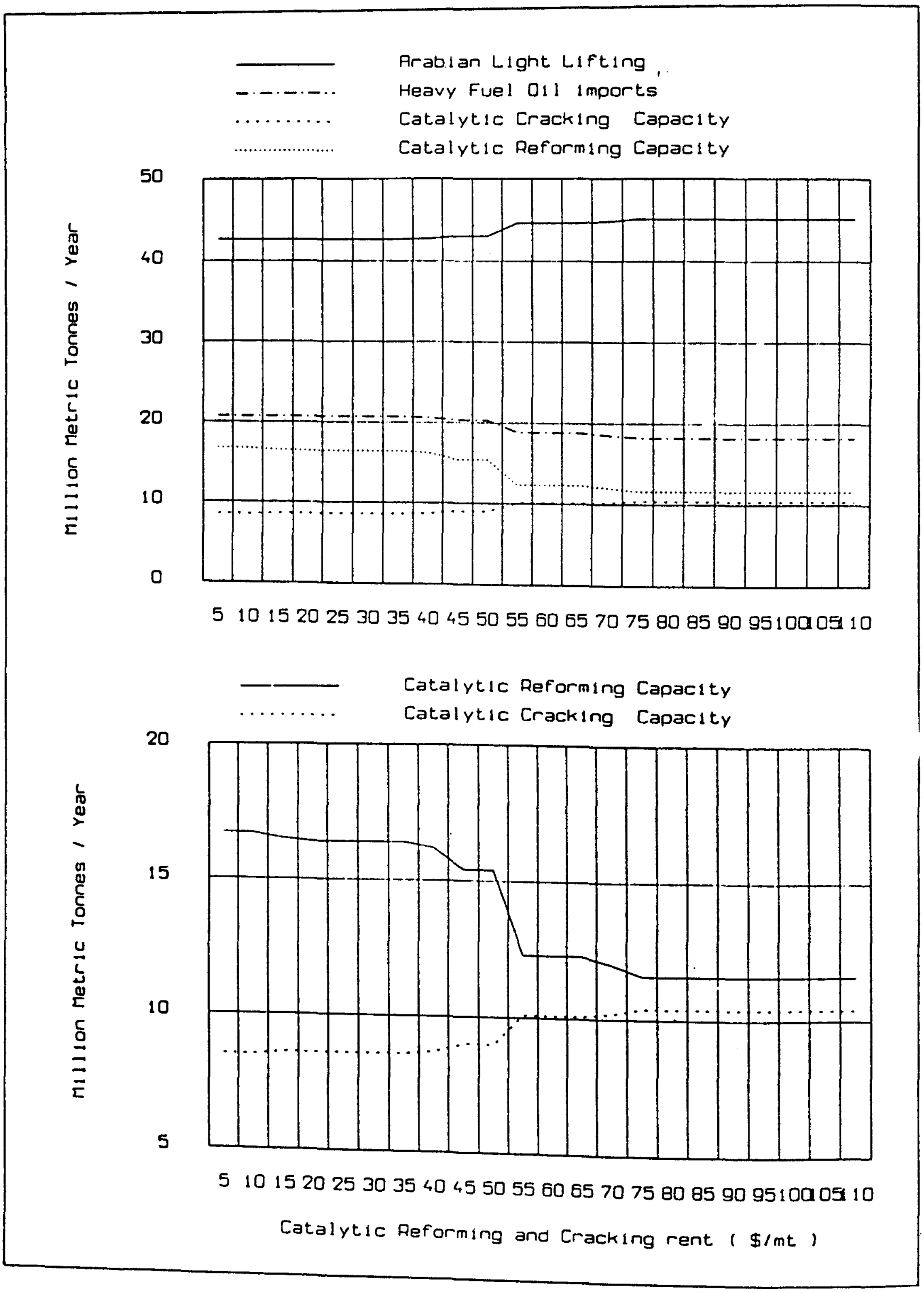


Figure 23. Single Refinery: supply pattern and capacity built-up under fixed demand and variable equilibrium rents.

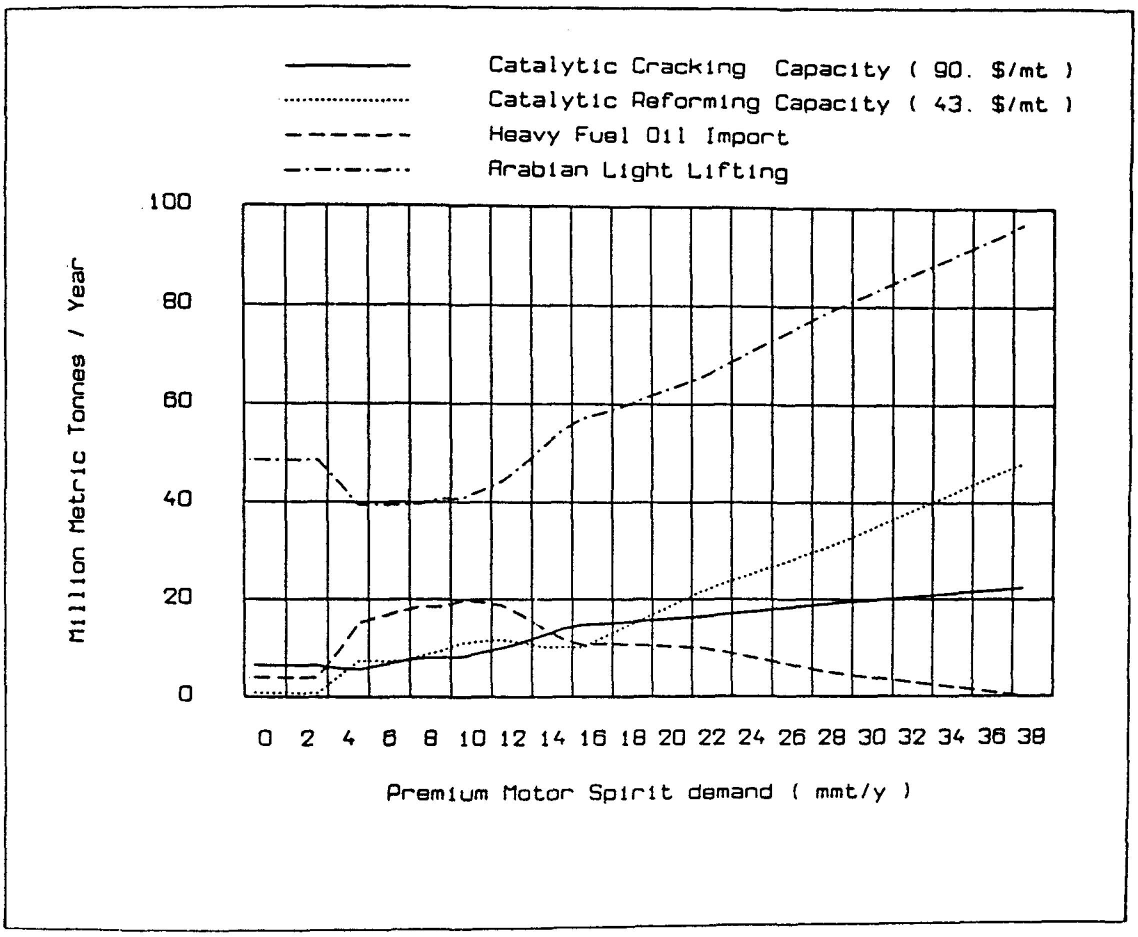


Figure 24.. Single Refinery: supply pattern and capacity built-up under variable premium motor spirit demand and fixed rents.

additionally the cracking capacity as percentage of reforming for variable motor spirit demand.

From both figures it can be seen that as premium motor spirit demand increases ever more capacity of both processes is required while the cracking capacity as percentage of reforming tends to decrease. Since both capacities increase, the former implies that greater changes in metric tonnes of processing feed per metric tonne increase in demand occurred in reforming than in cracking. A metric tonne increase in demand is, in turn, reflected in the increase of Arabian Light liftings and the decrease of heavy fuel oil imports.

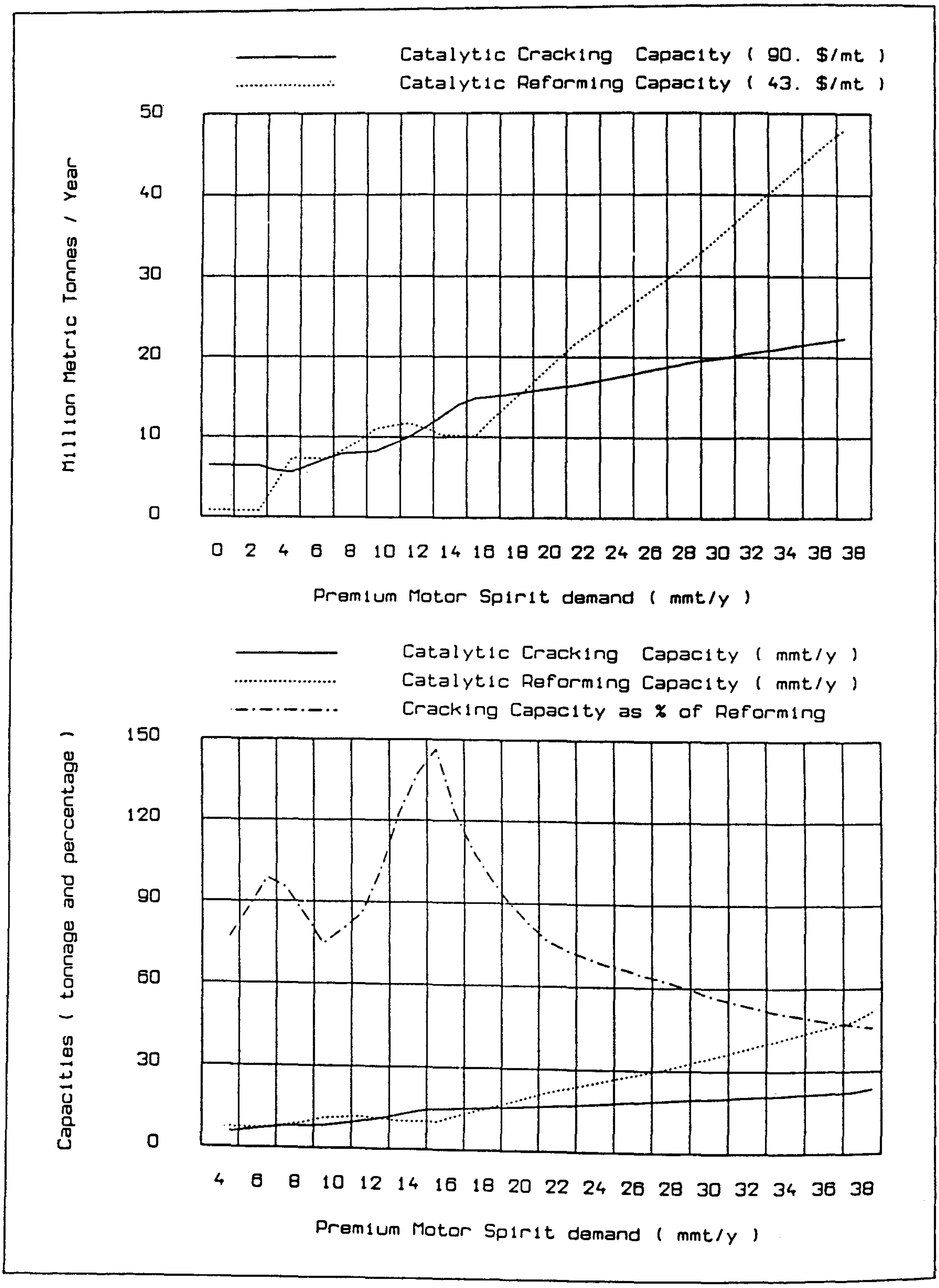


Figure 25. Single Refinery: catalytic reforming and cracking percentage capacity relation for fixed rents.

4.2.12 Concluding Remarks on section 4.2.11

The two case studies of section 4.2.11 tried to adapt equilibrium rents to capacities, first on variable rents, and fixed demand and AL/HFO ratio; and second, on variable demand, and fixed rents and AL/HFO ratio.

Although in the short run the refiner can not decide on capacity since his resources are fixed, the expectations of rents are necessary for long run investment decisions on the basis of the refiner's view of the market, i.e., demand projections, technology costs and technical progress associated, refining costs, etc.

The refiner planning for investment would take into account the estimated demand, and of course, the determinant economic factors, and fix the rent covering built-up (average annual capital charges) and variable costs. In so doing, maximum demand should be considered otherwise there may develop shortages in product supply. In the short run capacity may be fully used and the expected or higher rents arise, but seasonality drives demand, thus refinery output. Capacity may be enventually in excess, rents will then decline. Cracking being the costlier process is to assure a rent, the penalty due to low demand is ultimately paid by the cracking unit whose rent must account for the losses in platforming when seasonality, and/or structural changes in demand force capacity to be in surplus.

An important finding is that reforming capacity results greater than cracking in almost every variant case within both case studies.

Let us make an illustration. The Single Refinery base case and other variant cases of section 4.2.11 show diverse capacities and percentages with also diverse rents (see table below).

For the motor spirit demand of 12. mmt/y, three different capacities are associated depending on the rent. The base case solution shows a surplus in platforming and a rent of 35. \$/mt in cracking. Forcing platforming and cracking to generate rents of 35. \$/mt each, results in a considerable reduction of cracking capacity (becoming 52% of reforming, in contrast to 72% in the original base case). For the same demand and fixing higher rents (43. and 90. \$/mt processing feed in reforming and cracking respectively) results in lower platforming and higher cracking, 85% of platforming. To maintain capacity percentages of about 72, and the fixed rents of 43. and 90. \$/mt, motor spirit demand should be around 23. - 24. mmt/y, and not the 12.33 mmt/y of the base case.

Case	Motor Spirit mmt/y	Capacities Crack mmt/y	Refor feed	% Crack on Ref	Refinery Rents Crack $/mt	Refor feed
Base	12.33	13.29	18.43	72	35.3	0.0
Variant	12.33	8.60	16.40	52	35.0	35.0
Variant	12.00	10.00	11.70	85	90.0	43.0
Variant	24.00	17.30	24.50	71	90.0	43.0
Variant	23.00	16.90	23.10	73	90.0	43.0

The cases may complicate very much indeed, there is not an absolute capacity relation. The former depends on the expected rents which in turn depend on a variety of factors already known: with uncertainties in demand trends, price structure, supplies, etc., the refiner wanting to satisfy demand, will have more often than no a surplus in platforming capacity.

The latter is in line with former findings, as reported in sections 4.2.4 and 4.2.7:

Rents arise for cracking *and* platforming *simultanesouly*, if a *particular* demand is to be satisfied at a *paticular* price ratio AL/HFO. Cracking will almost ever produce a rent, and platforming seldom.

As regard coal penetration, the maximum does not depend greatly on the demand for motor spirit, rather on the AL/HFO price ratio, hence that the effect of expanding capacity and varying motor spirit demand and rents on imports is not appreciated.

4.3 CONCLUSIONS

Concluding remarks sections that report on main individual findings have been placed at the end of each section of this Chapter.

The conclusion to be posed here relates to the experimental hypotheses put forward in section 4.2.2, beginning of the Chapter, namely,

Experimental Hypothesis EH.1

Hydroskimming refineries produce low or no rent, and the complex refinery produces rent arising from the use of cracking processes.

Experimental Hypothesis EH.2

There is an equilibrium price ratio AL/HFO which determines an equilibrium cracking rent for the refiner, and a minimum marginal coal substituting for heavy fuel oil. Above that equilibrium there will be higher rents and the incentive to expand, below there will be losses, and in both an instability in the system seeking for a new equilibrium.

The exercises developed in this Chapter have brought about preliminar confirmation to EH.1 and EH.2:

As regards EH.1, it has been widely shown the reforming unit in the Single Refinery is nearly always in surplus, not to mention the crude and vacuum distillation units. Although every refinery is a case in itself, the SR is a fair representation of a north western european refinery in infrastructure and pattern of supply/demand, and moreover, it belongs to a particular refining area with which the spot market of Rotterdam is associated, being prices in any refinery of the area related to that spot, particularly to the spot Arabian Light price.

Evidence that hypothesis EH.1 has been taking place in the market is given in these lines:[90]

> During the period 1973-1983, refining with atmospheric residue upgrading became progressively the industrial standard, at the

[90] 'Petroleum refining and products marketing in Western Europe', (Paris: Enerfinance; direct communication of OPEC News Agency; article referenced in *OPEC bulletin*, May 1985, p.59 in: 'Difficult future for European refiners'), p.11.

> expense of hydroskimming refineries. But this meant that refining companies were obliged to invest in costly conversion projects whilst at the same time they cut back on their now largely excessive distillation capacities.

And further,[91]

> In refining, 150 million tons of distillation capacity dismantled over the period 1975-1983, whilst in the same period some 80 million tons of equivalent catcracking capacity were built.

In relation to EH.2, it has been also shown that a two-tier price structure may produce rents above or below equilibrium ones, depending on the price link crude/coal(f.o.e.), and there exists a particular price combination which produces exactly the equilibrium rents. What this combination was in the recent past and is at present market conditions is analysed in Chapter 7. It relates to the process of interfuel substitution giving rise to the Third Law or Principle of Energy Substitution.

---

[91] Ibid., p.12.

CHAPTER 5: EXPERIMENTATION AT THE GLOBAL LEVEL: 7-AREA WORLD ENERGY MODEL

As in the experimentation with the SR, Chapter 4, the theoretical considerations of Chapters 1 and 2 will be basically used for the experimentation with the 7-area WEM. Factorial Experimental Designs, statistical experimental method, and Parametric Programming, general LP/simulation method are the two experimental techniques applied to the 7-area WEM. Two kinds of Refinery Rent models per area and per refinery processing unit within each area are estimated whenever sufficient experimental data are available.

5.1 THE 7-AREA WEM BASE CASE VERSION AND UPDATING

The 7-area WEM base case version is to be viewed from two standpoints: the exogenous, market information requirements; and the endogenous, refinery technology configuration to meet exogenous demands at (endogenously determined) prices of market clearance. The two are considered now separately:

Firstly, the 7-area WEM base case exogenous input represents the 1976 worldwide oil market structure in terms of the pattern of supply/demand, refining capacities, shipping availability and price leadership; the exogenous 1976 data base is presented in tables C-4 to C-6 of Appendix C. Changes in the oil system require parallel changes in the model; even though the 7-area WEM is a static one period model, for it to represent any other year's situation an updating of external information is necessary. Accordingly, a current representation of the 7-area WEM should take account of:

1. A considerable reduction in refining capacity in particular in western Europe. The western european total crude distillation capacity decreased at an average rate of 2.5% from 1972 to 1982; this latter having been 1055. mmt in 1976 and 983. mmt in 1982.

2. A considerable change in the supply of crude as a result of:

   a. OPEC rationing due to the supply cutbacks of Iran and Iraq (in war since 1980). Iranian crude production has been reduced from 295. mmt (5.92 mbl/d) in 1976 to 158. mmt (3.175 mbl/d) in 1979 and to only 65.7 mmt (1.32 mbl/d) in 1981 -actual levels are around 2.5 mbl/d, 1983 figure. Similarly Irak's production pattern which in 1976 produced about 119. mmt (2.4 mbl/d), 130. mmt (3.48 mbl/d) in 1979 and 44. mmt. (.895 mbl/d) in 1981 -with actual levels of about 1. mbl/d. On the total, OPEC production accounted for 1540.2 mmt (31.09 mbl/d) in 1976 and from 1980 onwards the production levels were progressively reduced to come to about 895. mmt (18.28 mbl/d) in 1983 -despite the production floor of 17.5 mbl/d agreed in the OPEC conference of March 1983;[92]

b. the increase in crude supply of new oil exporting countries as for instance Mexico: the mexican oil exports in 1976 accounted for 43.6 mmt and for 149.4 mmt in 1983. The model does take into account crude supply of local production and indigenous crude as an aggregate in the area but for some crudes disaggregation at crude level is not represented -as for the mexican crudes. This is so to reflect the self-sufficiency in oil of some countries which despite of being oil producers do not however exert great influence on the oil market (or at least this was not the case at the time the 7-area WEM was being set up).

3. Decreases in product demands as substitutes for some products of final demand are being developed (coal substitutes for heavy fuel oil, methanol substitutes for motor spirit, etc.) by the consuming side.[93]

4. The oficial price of the crude marker Arabian Light which despite continuous increases since 1973 to reach US$/bl 11.5 in 1976 and US$/bl 34. in 1980 it dropped to US$/bl 29. in March 1983 and it is still officially at this price. At the Rotterdam spot market, AL price changes ranged from (average year) US$/bl 11.65 in 1970, US$/bl 36.44 in 1980 and US$/bl 28.96 in 1983; as an indicator of recent price levels at the same market the AL price averaged US$/bl 28. in 1984-1985.

The changes implied by 1. to 4. above can be easily done in the model since they are part of the exogenous fixed elements. Introducing new crudes in the model entails the addition of new variables with the processes' logistics associated. A more disaggregated model, a

---

[92] The figures indicated here are found in *BP statistical review of world energy*, (The British Petroleum Company plc, June 1984).

[93] Modifications to the 7-area WEM have been done in order to study product substitutability and demand elasticities, as in Giesecke, C., 'World computer model of oil markets: OPEC pricing strategy model in the short and long term', Ph.D thesis, op.cit.

22-area WEM (in existence at OPEC Research Division) does consider a wider spectrum of crudes per area as well as of shipping routes and pipelines. Its construction lies on the same hypotheses of a competitive oil market and Unidimensionality in pricing.

Secondly, the coefficients in the technology matrix unlike the exogenous data, are traced back to 1973 when the conceptual modelling of the 7-area WEM was iniciated as the reader may refer to the June 1973 issue of *Energy Policy*.[94]

No actual updating of technological coefficients took place since then. In order to adapt refineries to the changing chemistry of supply/demand, continuous progress in refinery processing technology is being realized in the technology market.

Although changing some technological coefficients does not cause great changes in the models LP solutions and moreover, there exists a technological sub-matrix which if disturbed does not cause any change in the system whatsoever (refer to section 2.5.5, Chapter 2), the now refinery technological changing pattern calls for more actual technology modelling; in particular, those processes newly developed to treat the heavier crudes on stream should be included.

As far as the 7-area WEM is concerned, updating the existing 1973 technology matrix does not imply in itself great deal of work and changes but does imply efforts in gathering data.[95]

Perhaps the more difficult task is the updating of the operating, transportation and fixed costs (which have clearly risen meanwhile), and the capacities associated with the areas' ports of discharge. Nelson,[96] has devoted great attention to the problem of cost estimation at all levels of the refinery, cost indexes are published systematically in the *Oil & Gas Journal* every month, though they apply to the

---

[94] Deam, R. J., et al., 'The Development of Western European Oil Prices', *Energy Policy*, I, 1, (June 1973), op.cit.

[95] This requires the specific actual parameters used in deriving the technological coefficients. Particularly troublesome become the coefficients on product quality specification rows whose computation is not straightforward requiring intermediate data and operations. As discussed already in section 2.5.1, the SR and 7-area WEM raw data have been tabulated for use by the OMNI software to provide the input deck to the matrix generator.

[96] Nelson, W.L., *Guide to Refinery Operating Costs (Process Costimating)*, (Tulsa, USA: The Petroleum Publishing Company, 1976).

USA refining sector only. A group of experts would indeed be required to undertake such a task.

Summarizing, the current 7-area WEM representation needs an effort in the technological side which includes a revision of existing units and processes in line with actual technology needs. This means, inclusion of visbreakers, all kind of flexicoking and all kind of complex processes which in the meantime have been improved or newly developed.

Regardless updating, on the basis of the fixed 7-area WEM configuration, empirical results show that the underlying relationships between prices and rents, refinery unit capacities and rents have been kept if compared to the actual refinery world. It is only the magnitude of the price variables what changes, the general trends are certainly comparable. As far as this thesis is concerned, the former suffices for the aim is to provide a platform of work which irrespective of the physical detailed structure of the system is a valid tool of experimentation and supports hypotheses.

5.1.1 A refinery in area-a: Assumptions

For the sake of further analysis, a refinery in an area of the 7-area WEM is viewed as:

- an oil refinery in area-a is understood as an average refinery of the area concerned. There is aggregation of unit capacities, crude supply and product demands within the area; technological data are assumed to be average or of a typical area refinery; and,

- a rent of a refinery unit in area-a would therefore be the rent in \$/mt of feed which is accruing to the unit of an average refinery of the area. In particular, to an average refinery located at the spot market attached to the area. For instance, in area B a typical refinery at Rotterdam, in area U at USA East Coast and in area J at Singapore.

5.2 EXPERIMENTAL METHODOLOGY WITH THE 7-AREA WEM

Oil refinery rents at the global level of interaction can be estimated of a wide range of ways. Bearing in mind the five linked sub-systems (see Figure 1 on page 8) of the world oil system, namely, oil production, refining, product demands and transport sub-systems, it directly follows that correlations between their representative variables may be found. It is clear that it is always possible to correlate any variable to any other by proposing any sort of (linear and/or non-linear) model which supposedly represents the variable's behaviour and applying statistical techniques, Regression Analysis (RA) in particular, to estimate their coefficients and then checking further on its validity: the clue is then to choose the variables giving appropriate results.

For rents to arise at the refinery, or for a marginal rent to exist at the refinery processing unit the following set of variables are determinant (see previous Chapters 3 and 4):

- the severity of operation: capacity utilization,
- the product price and demand structures, particularly the premium motor spirit price which is the most valuable product at the market place, and the middle distillates/motor gasolines demands; and
- the crude price and supply patterns.

It is discussed later how other factors turn up to have influence in determining levels of rents, as for instance the worldwide shipping capacity which is considered an experimental variable in some exercises.

It is pursued here to derive a model from a more integrated and large one (the 7-area WEM) which aims at explaining the behaviour of a reduced set of variables of the large model. This model is commonly known as a *metamodel*, "...a model explaining the simulation model."[97]

---

[97] Kleijnen, J.P.C., 'EXPERIMENTATION WITH MODELS: Statistical design and analysis techniques' in Cellier, F.E., (ed.), *Progress in Modelling and Simulation*, (London: Academic Press Inc. Ltd., 1982), Chapter X, p.174.

The metamodels referred to later refer as the *first* and *second kind of Refinery Rent models* are found by carrying out two alternative methodological approaches:

1. A Factorial Experimental Design (FED), is formulated in terms of five factors: each an element of a distinct oil sub-system. Responses are then correlated by Linear Regression (LR) to the factors and/or combination of factors which have a major effect on responses. Anticipating results, the models found are highly interactive in the factors chosen, for instance, some third-order interaction terms appear, making the models difficult to handle; this reflects the complexity in operation of such a highly integrated system.

2. Parametric Programing (PP) on the 7-area WEM was performed on the basis of premium motor spirit and gas oil demand changes (in the specific area of North West Europe).[98] The refinery rents at the area level are registered. LR models in terms of two and/or three independent variables are estimated from the set of the FED's responses in 1. and from the PP's responses.

The models proposed by 1. and 2. differ in the number and kind of terms. Models in 1. include about 6 terms with second and third-order interactions while models in 2. include at most four, all single terms.

Models of the second kind are preferred (as will be seen later) for the indepedent variables values (prices at Rotterdam) are readily available from the market place, making calculations straightforward for the refiner.

---

[98] Area B in the 7-area WEM, the countries comprising area B appear in Table C-1, Appendix C.

5.3 FACTORIAL EXPERIMENTAL DESIGNS

The purpose of this section is to set out in a general way the fundamentals of Factorial Experimental Designs (FED) and the Statistical Analysis of Experimental Data (SAED) for model estimation. A more formal set of related definitions is progressively presented.

The theory on FED is very extensive and it still develops at a very detailed and specialized level in the search for optimum experimental settings up and appropriateness of the estimated models. Little attention is given to the statistics involved in the FED formulation, contrariwise, the reader may note many statistical details are left out. The interested reader may refer, among many others, to Daniel, Davies, Kleijnen, Mendenhall, Myers, Peng and Raktoe[99] for a full account of the theory of FED and SAED. Similarly *Biometrika* and *Technometrics* provide useful sources of reference for up to date developments and applications.

Most of the statistical and mathematical methods applied to the design and analysis of experiments arose to meet the need of integrating and analysing considerable bulks of data involved in biological, engineering and chemical experiments. As theory developed, applications were made possible to a wider range of subjects including the social sciencies. The following work shows that applications are also viable to a different kind of subject. As far as the author has reviewed, few attempts have been made to apply the FED methodology to a *mathematical device*, that is, a LP model, the 7-area WEM, to produce sets of observations (refinery rents) for model estimation.

---

[99] Daniel C., *Applications of Statistics to Industrial Experimentation*, (USA: John Wiley & Sons, Inc., 1976); Davies, O.L., *The Design and Analysis of Industrial Experiments*, (London: Oliver and Boyd, 1963); Kleijnen, J.P.C., *Statistical Techniques in Simulation*, Part I and Part II, (New York: Marcel Dekker, Inc., 1974); Mendenhall, W., *Introduction to Linear Models and the Design and Analysis of Experiments*, (California: Wadsworth Publishing Co., Inc., 1968); Myers, R.H., *Response Surface Methodology*, (Boston: Allyn & Bacon, Inc., 1971); Peng, K.C., *The Design and Analysis of Scientific Experiments*, (USA: Addison-Wisley Publising Co., Inc., 1967); Raktoe, B.L., et al., *Factorial Designs*, (USA: John Wiley & Sons, Inc., 1981).

Exceptions are reported by Kleijnen,[100] "Whenever a scientist performs an experiment, be it with an abstract model as in simulation or with a physical system [...]," and further,

> Regression analysis will be known to the great majority of simulation practitioners. Unfortunately, experimental design is familiar to only a certain group of statisticians. Nevertheless, in recent years, applications of experimental design in simulation have started to appear [...].

Sub-sections 5.3.1 and 5.3.2 describe the generalities and setting up of the Factorial Design (FD) to conclude with the summary section 5.3.3.

5.3.1 Generalities

An experiment involves in general a set of variables of which one or more are considered by the experimenter to be *dependent* on the remaining or *independent* variables.

An Experimental Design, ED, is a plan of assigning experimental values or *levels* to the independent variables; the levels belonging to a domain of values of realization of the independent variables.

A single application of values from the ED to the individual variables is called a *treatment combination*, *run*, or simply *treatment*. A treatment defines the conditions under which the experiment is carried out.

An entity to which the treatments are applied is called the *experimental unit*. The outcome of an experiment based on a particular treatment is called *experimental observation*, *observation*, *measurement* or *response*.

An independent variable is also called *factor* and its values the *levels of the factor*. A *quantitative* factor comprises numerical values; a *qualitative* factor though it may have numerical levels, it is denoting a non-measurable factor.

---

[100] Kleijnen, J.P.C., 'EXPERIMENTATION WITH MODELS: Statistical design and analysis techniques' in Cellier, F.E., (ed.). *Progress in Modelling and Simulation*, op.cit., p.173.

A Factorial Design, FD, is an ED planned to estimate the responses behaviour and to study the individual and/or combined *effects* of factors on the response.

Meyers writes,[101]

> An important advantage of the factorial approach is that it lends itself to assessing the "joint effect" of two or more variables, that is, the interactions among factors can be measured.

And as precisely pointed out by Davies,[102]

> Many experimental situations require the examination of the effects of varying two or more factors. It is shown that in a complete exploration of such a situation it is not sufficient to vary one factor at a time, but that all combinations of the different factor levels must be examined in order to elucidate the effect of each factor and the possible ways in which each factor may be modified by the variation of the others. In the analysis of the experimental results the effect of each factor can be determined with the same accuracy as if only one factor had been varied at a time, and the interaction effects between the factors can also be evaluated.

A FD is said to be *complete* if the replication of every treatment is greater than zero; and it is said to be *minimal* if the replication is equal to one for all treatment combinations. A FD is a *fractional* design if some but not all treatment combinations have multiplicity greater than zero.

The changes in responses due to the change in the level of a factor is called the *effect of a factor*. The average effect of a factor, i.e. the effect of a factor averaged over all levels of all other factors is called the *main effect of the factor*. An *interaction* is the dependence of the effect of a factor on response upon the level of any other factor. Two factors interact when the effect of one factor on response is different at different levels of another factor. A *s-way interaction*, *s-cross product* or *s-way cross classification* is the factorial representation of an interaction of *s* distinct factors.

---

[101] See Myers, R.H., *Response Surface Methodology*, op.cit., p.42.
[102] See Davies, O.L., *The Design and Analysis of Industrial Experiments*, op.cit., p.247.

5.3.2 Setting up a 2^5 Factorial Design and the Proposed Model

Formalizing and relating the outlined above to the present study follows next:

7-area WEM, the mathematical representation of the integrated oil system, is the experimental unit; it is an LP mathematical model hence deterministic in nature,

$x = (x_j^l)$, $j=1,n$, is the number of factors used in the FD, and l is the level at which the factor is set.

In the present study the components of factor-x are elements of the LP primal feasible space of the 7-area WEM defined by the technological matrix-A, the right hand side vector-b and the objective function vector-c,[103] and appropriately coded to simplify operations (coming discussion);

$l = l_j$ number of levels at which a factor x_j may be set, which is equal to 2 for all j in this particular case;

k is the total number of treatments or runs applied to the experimental unit; it is determined by the number of factors and levels; here, n=5, $l_j = 2$ for all j, then, $k = 2^5 = 32$ different runs applied to the 7-area WEM.

$X = [\ f_j(t)\ ]$, $j=0,k-1$, $t=1,k$, is the matrix of the levels of the factors chosen and/or the combination of levels of two or more factors when applying treatment-t, t=1,k.

Matrix X is called the *design matrix* associated with the linear model proposed, for it characterizes the design plan; each element $f_j(t)$ is a single level value or the value of joint combination of levels' values.

$$X = \begin{bmatrix} f_0(1) & f_1(1) & \cdots & f_{k-1}(1) \\ f_0(2) & f_1(2) & \cdots & f_{k-1}(2) \\ \cdot & \cdot & \cdots & \cdot \\ f_0(k) & f_1(k) & \cdots & f_{k-1}(k) \end{bmatrix}, \quad \text{k treatment combinations,}$$

[103] See the LP problem formulation of sections 2.2.1 and 2.3, Chapter 2.

$y = (y_h)$, h=1,m, are the dependent variables or kind of responses to approximate the estimated model. Since only one kind of response is analysed at a time, h=1; these responses being, in turn, the dual variables attached to the refinery processing units;[104]

and,

$Y = (y_t)$, t=1,k, the k responses, realizations of the variable-y when applying the k treatment combinations to the 7-area WEM; these realizations move in a bounded space, the duality LP space defined by the same set of parameters as the primal, i.e., technology matrix-A' (transpose of A), cost vector-b, and resource vector-c.

The whole set of responses-Y is commonly known as the *response surface*. And the *response surface methodology* the statistical method selected to fit the observed responses to the experimental factor values. The selection of treatment combinations in the FD is the *response surface design*. Both problems, the response surface estimation problem and the response surface design problem are linked and play determinant roles in the search for the appropriate model to fit the response surface.

After applying the k treatment combinations and obtaining the responses-Y, the purpose is to find a *linear model* as in (5.1) below in order to fit the named reponse surface,

$$y_t = \sum_{j=0}^{k-1} f_j(t)\, b_j = f'(t)B, \qquad t=1,k \tag{5.1}$$

where,

$B = (b_0, b_1, \ldots, b_{k-1})$, is the k-unknown parameter vector of the proposed model.

Vector B is usually known as the *set of effects*, for it "essentially represents the behaviour of Y with respect to changes in the levels of the factors."[105] Alternatively, it is the set of main effects as previously defined and referred to in Davies.[106]

---

[104] SR LP problem definition, section 2.2, Chapter 2.
[105] The quotation is found in Raktoe, B.L., et al., *Factorial Designs*, op.cit., p.13, with a different notation for the Y.
[106] See Davies, O.L., *The Design and Analysis of Industrial Experimens*, op.cit., p.250.

And, $f(t) = (f_0(t), f_1(t), \ldots, f_{k-1}(t))$ is the vector of linear combinations of levels as defined by the design matrix X.

The set of responses y_t, t=1,k, are equal to their expected value for they are outcomes of a deterministic model thus no replication has effect on responses; the variance of y_t, t=1,k, with respect to its expected value is consequently zero, $E[y_t] = \mu_t$, $V[y_t] = 0$, for t=1,k. In matrix notation,

$$Y = XB, \quad E[Y] = \mu, \quad V[Y] = 0 \tag{5.2}$$

The problem of fitting the response surface is then one of estimating the parameter vector-B in such a way as to minimize the difference between the actual responses Y and the estimated responses Y'' by an appropriate statistical method. In other words, to minimize,

$$Y - Y'' = \varepsilon \tag{5.3}$$

where, $Y'' = X\beta$ is the estimation model of Y, $\beta = (\beta_0, \beta_1, \ldots, \beta_{k-1})$ is the parameter estimation vector of B, and $\varepsilon = (\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_k)$ is the vector of *residuals* or model estimation erros, with $E[\varepsilon] = 0$, and $Cov[\varepsilon] = \sigma^2 I$.

The *Least Square* statistical method is used to estimate the parameters-B. This latter is discussed after setting out some definitions as statistical background. The reader may skip without lost of generality definitions **5.1** to **5.4** following.

Definition 5.1. A vector of parameters $\beta = (\beta_0, \beta_1, \ldots, \beta_{k-1})$ is said to be a *linear parametric combination* of $\theta = (\theta_0, \theta_1, \ldots, \theta_{k-1})$ if there is a vector of known coefficients $C = (c_0, c_1, \ldots, c_{k-1})$ such that, $\beta = C\theta = \Sigma c_i\theta_i$.

Definition 5.2. Two linear parametric combinations $\beta_1=C_1\theta_1$ and $\beta_2=C_2\theta_2$ are independent if β_1 can not be expressed as a scalar multiple of β_2.

Definition 5.3. Two linear parametric combinations $\beta_1=C_1\theta_1$ and $\beta_2=C_2\theta_2$ are said to be (algebraically or mutually) *orthogonal*[107] if, $\Sigma c_{1i}c_{2i} = 0$.

Definition 5.4. A linear parametric combination $\beta=C\theta$ is said to be a *contrast* of $\theta=(\theta_0, \theta_1, \ldots, \theta_{k-1})$ if, $\Sigma c_i = 0$.

Definition 5.5. A vector of parameters $\beta = (\beta_0, \beta_1, \ldots, \beta_{k-1})$, function of the observations Y of a factorial design FD is a vector of *least square estimators* (LSQE) of the parameters $B = (b_0, b_1, \ldots, b_{k-1})$ if the values $b_i = \beta_i$, for i=0,k-1, minimize

$$(Y - XB)' (Y - XB) \tag{5.4}$$

If β is a contrast of θ and θ is the vector of least square estimators of Θ in

$$(Y - V\Theta)' (Y - V\Theta) \tag{5.5}$$

then β is the *best linear unbiased estimator* of B in (5.4).[108]

---

[107] The term 'algebraically orthogonal' is used by Raktoe, B.L., et al., *Factorial Designs*, op.cit., p.24 and ff.; and the term 'mutually orthogonal' is found in Ogawa, J., *Statistical Theory of the Analysis of Experimental Designs*, (New York: Marcel Dekker, Inc., 1974), p.2 and ff.. In the present study 'orthogonal' is used to mean both algebraically and mutually orthogonal linear parametric combinations.

[108] What the last four lines of definition **5.5** express is found in the literature as a conclusion of an important statistical theorem; see for example, Raktoe, B.L., Ibid., p. 27 and ff.; Drapper, N.R., and Smith, H., *Applied Regression Analysis*, (USA: John Wiley & Sons, 1981), and Mendenhall, W., *Introduction to Linear*

A best linear unbiased estimator has the following properties: it is expressed as a linear function of the observations; its mean is equal to the mean of the parameter it is estimating, i.e., $E[\beta] = B$; and it is the parameter of minimum variance among the class of linear unbiased estimators.[109]

The vector of estimators β of B is then obtained by minimizing the square of residuals given by expression (5.4) with $B = \beta$,

$$\varepsilon^2 = (Y - X\beta)' (Y - X\beta)$$

$$\varepsilon^2 = Y'Y - Y'X\beta - X'\beta' Y + X'\beta'X\beta$$

$$\varepsilon^2 = Y'Y - 2Y'X\beta + X'\beta'X\beta \tag{5.6}$$

deriving ε^2 with respect to β and setting it to zero, we have,

$$\frac{\partial\varepsilon^2}{\partial\beta} = 2\beta X'X - 2Y'Y, \qquad \beta X'X = Y'X$$

and from the last expression, the estimators β of B are given by,

$$\beta = Y'X(X'X)^{-1} , \tag{5.7}$$

with $E[\beta] = B$ and $Cov[\beta] = \sigma^2(X'X)^{-1}$.

The matrix $X'X$ is known as the *information matrix* attached to the FD with respect to the vector parameter β of the given model.

A k-square design matrix X formed of k orthogonal contrasts is said to be an *orthogonal design matrix*. And if X is an orthogonal design matrix, the information matrix $X'X$ is a diagonal matrix. It may be seen from (5.7) that when $X'X$ is a diagonal matrix the estimators β

Models and the Design and Analysis of Experiments, op.cit., Chapter 9.

109 See Johnston, J., *Econometric Methods*, (Tokyo: McGraw-Hill, Kogakusha Ltd., 1972), pp.18-25, for a formal definition of linear unbiased estimator.

are readily obtained saving in this way a great deal of computational operations and errors. And more importantly, the models' coefficients are made uncorrelated therefore each of them may be obtained independently on the other thus $Cov[\beta] = V[\beta] = \sigma^2(X'X)^{-1}$.

A FD comprising an orthogonal design matrix is called an *orthogonal design* and it is usually the kind of FD one is interested in for it provides a simple way of estimating the parameters B.

If the original design matrix, let it be V, is not an orthogonal matrix, one will try to find an *orthogonal transformation* $F_j = (f_j(1), f_j(2), \ldots, f_j(k))$, $j=0,k-1$, so that matrix V is converted into an orthogonal matrix X. Then if in the stated (original) problem the proposed model attached to the factorial design FD is $Y = V\Theta$, where V is the original design matrix and $\Theta=(\Theta_0,\Theta_1, \ldots,\Theta_{k-1})$ is the vector of the models' coefficients to be estimated, then a transformation f(s) applied to every factor or combination of factors' levels (element of V) will produce elements of an orthogonal design matrix X with proposed model $Y=XB$. The vector of estimable parameters B will satisfy the properties implied by definitions **5.1** to **5.5** above.

It is noted that an orthogonal design whereby the coefficientes B can be easily estimated has been so far referred to. Those coefficients are meaningless in relation to the original FD coefficients of the problem. They are to be 'retranslated' to obtain the actual Θ coefficients of the FD with $Y=V\Theta$ as the proposed model.

The orthogonal transformations are also called *orthogonal polynomials*. It goes beyond the scope of this work to study in detail the variety of methods available to derive them.[110]

The method used here is that proposed and developed by Yates.[111] It is shown that the so-called *Yate's effects* properly multiplied by a constant K, are the estimators of the proposed model $Y=XB$ of the Yates's orthogonal FD. Further it is shown that following an Analysis of Variance on the responses to estimate the models' coefficients and the variance of residuals of (5.3) is equivalent to apply the Yates algorithm to the original FD with the consequent advantages of an orthogonal FD.

[110] Extensive treatment is given in many good texts, such as Myers, R.H., *Response Surface Methodology*, op.cit.

[111] See Yates, F., 'The design and analysis of factorial experiments', *Imp.Bur.Sci.Tech.Commun.*, 35, (1937), p.15.

5.3.3 Summary

The experimental study of a particular phenomenon or system comprises in general the following steps:

- the design of the experiment or factorial design;
- specification of the model in terms of the independent variables under study;
- the treatments application to the experimental unit;
- estimation of the proposed model's parameters from responses, fitting of the model to the observations and estimation of errors; and,
- validation of the proposed model.

The steps above are closely linked for the proposed model determines the FD to be used, and the opposite holds, once a design is planned, it directly follows the kind of model to be estimated. On the basis of previous discussions, the steps are developed in following sections by providing,

1. A transformation of the original factors, commonly called *coding* of the variables or factors.
2. The construction of the set of contrasts, elements of the orthogonal design matrix X by using the Yates algorithm.
3. The estimation of the models coefficients β.
4. The *retranslation* of the estimated models through the orthogonal design to the original models (of the non-orthogonal designs): the construction of a *decoding* matrix D.
5. A criterion for truncating the models for the full original model will consist of 32 terms (each term represents a factor or combination of factors in a 2^5 factorial design) of which only the more significant ones according to the criterion selected will remain. The rest are added up to make an estimate of the variance of residuals of the estimated response model with respect to the true response.

5.4 THE 2^5 7-AREA WEM FACTORIAL DESIGN

The 2^5 7-area WEM FD is a complete, minimal, non-randomized and *balanced* factorial design. By this latter it is meant a FD with the same number of responses per treatment combination. That is, there exist nine refinery marginal rents (one per each refinery processing unit, namely, alkylation, catalytic cracking, coking, atmospheric distillation, hydrofining, hydrocracking, catalytic reforming, residue desulphurization and vacuum distillation) in each of the seven areas, in total, 63 responses per treatment combination or run of the 7-area WEM model. However, many of the responses happen to be zero for not all the refinery processing units are fully utilized therefore able to generate a rent for the refiner in the concerned area. This makes the estimation of any model of refinery rents poor in some cases and infeasible in others.

One may say that the FD carried out on the 7-area WEM is the simplest of all designs comprising the methodology of FD. It is clear that both randomization in selecting the order of application of treatments and replication of runs will not affect the responses whatsoever.[112]

5.4.1 The 7-area WEM Factors

The 2^5 FD applied to the 7-area WEM has been designed in such a way that each factor selected represents a variable of the five major components of the oil system. All factors have been set at two levels, low and high, or '0' and '1' respectively. Factors and levels are presented in Table 5-1 below.

---

[112] This 2^5 7-area WEM FD has been designed by R.J. Deam and V. Poleo at Queen Mary College, University of London. It has been in principle planned to estimate and test another set of responses, namely, crude oil and oil product prices at the different market locations, part of V. Poleo doctoral thesis in preparation. In his work Poleo discusses at a sensible degree of detail the principles and aims behind the choice of the present 2^5 7-area WEM factorial design as well as the difficulties in designing, particularly with regard to the preliminary computational experimentation to find the levels of factors which ensure feasibility to the system. The relevance this experimental design has in the search for rents estimation models is discussed later in this Chapter.

Table 5-1. 2^5 7-area WEM FD factors and levels.

Sub-System	Variable Factor	Experimental Levels	0	1
Market pricing	1 Arabian Light price	$/mt	84.85	150.00
Crude Supply	2 Arabian Heavy	mmt/y	44.00	52.81
Product demand	3 Worldwide motor spirit demand	mmt/y	521.84	548.03
Refining	4 Worldwide CC cap	mmt/y	449.80	386.85
Transport	5 Worldwide shipping capacity	mdwt/y	185.69	178.26

For the levels' values it was desired to chose those values thought to force the system to work at higher severity than it was originally (in the base case). It was assumed that increases in the price of crude (namely, AL price), in the supply of heavy crude and in the demand for motor spirit exerted more pressure on the system. The effect of changing the ALP should be readily noticed on the crude and product price structure. Similarly, decreases in the refinery processing units capacities, particularly in the catalytic cracking unit, were to alter the product price structure and the refiner's level of rents. Also the decrease in shipping capacity was expected to influence the price profile and level of refiner's rents for the freight rate is a component of the crude and product prices, thus when freight rates are low more product transportation would take place (more heavy fuel oil exports/imports) and less cracking and vice versa.

Level '0' for all factors corresponded to their base case value. Level '1' value was fixed with different criteria for every factor as explained next and numerically summarized on Table 5-2.

Table 5-2. 2^5 7-area WEM FD factor levels per world area.

Areas	Motor spirit demands (mmt/y) Regular 0	Regular 1 (+5%)	Premium 0	Premium 1 (+5%)	Catalytic crack. capacity (mmt/y) 0	1 (-14%)
B	14.28	14.994	42.095	44.2	32.11	27.615
J	28.235	29.646	17.968	18.866	32.93	28.32
K	27.543	28.920	9.608	10.088	43.81	37.677
M	3.505	3.680	8.543	8.97	7.64	6.57
T	12.473	13.097	29.603	31.083	27.35	23.52
U	210.135	220.642	73.603	77.283	269.77	232.00
V	32.513	34.139	11.735	12.322	36.21	31.141

Ship size	Shipping capacity levels (mdwt) 0	1 (-4%)
≤ 24.999	17.460	2.086
25.000 – 49.999	33.050	23.896
50.000 – 79.999	32.620	31.315
80.000 – 124.999	43.610	14.898
125.000 – 199.999	33.350	7.211
≥ 200.000	164.590	98.856

- for ALP, AHS and MGD a future increasing trend was assumed. Since any change in the supply/demand chemistry is likely to affect the system's feasibility, caution was taken at this stage to keep the balance between demand (now with the additional increase in motor spirit) and supply (with the additional increase in heavy crude). This implied a great computational effort in testing feasibility cases.

The AHS (total, at jetty area) was increased in 20% in order to measure the effect on the system of the heavier supply composition.

The motor gasoline demand was increased evenly in each area and for each type of gasoline to maintain balance. A 5% increase in both, regular and premium in each area, makes a total worldwide increase of 5% in demand: the worldwide motor spirit demand was 521.85 mmt in 1976 (base case level '0') and 547.93 mmt after the 5% increase (level '1').

The ALP was increased in nearly 80% to come to about US$/bl 20.00.[113]

• for the catalytic cracking and shipping capacities, decreases in their worldwide capacities were considered. The CCC was evenly decreased per area in a 14% of its original capacity.

SHC is the only factor whose value at level '0' does not correspond to the base case worldwide availability. The 7-area WEM worldwide total shipping capacity (in the original version) was 324.68 mdwt (see Table C-6, Appendix C). The base case 7-area WEM LP solution accounted for a total surplus capacity of 139. mdwt (summing up all capacity surpluses by type of tanker size). For each tanker size, the surplus was substracted from its available capacity and the difference taken as the *effective capacity* and used as the values of level '0', this gives a total worldwide shipping effective capacity of 185.69 mdwt. Now, for every tanker size, the level '0' value was further reduced by 4% to give the level '1' value, this accounted for a total worldwide shipping capacity of 178.26 mdwt.

[113] Arabian Light prices between 12. and 20. US$/bl occurred in Rotterdam during 1976 and end 1979. The 2^5 7-area WEM FD was designed around 1978 explaining the price range considered.

5.4.2 7-area WEM Orthogonal Designs - Yates - The models

To follow the experimental steps indicated in section 5.3.3, points **1.** to **5.** are now developed. Yates's algorithm is used to construct the orthogonal FD.

1. Coding the factors in such a way as to represent the values ±1 (change of the domain of the variable factors) when they are used at their low and high levels respectively.

The coding x_j^l, j=1,5, is expressed in terms of the original variable factors v_j^l, j=1,5, l=0 or 1 of Table 5-1:

$$x_j^l = \begin{cases} \bullet\ (v_j^l - v_j')/ \delta_j, & j=1,2,3 \\ \bullet\ (v_j' - v_j^l)/ \delta_j, & j=4,5 \end{cases} \qquad (5.8)$$

where,

$v_j' = (v_j^0 + v_j^1)/2$, j=1,5, is the mean value of the two factor levels of variable factor j, and,

$\delta_j = |v_j^0 - v_j^1|/2$, j=1,5, is the absolute value of the mean difference between the two factor levels of variable factor j.

Note for convenience and in order to represent the coded and decoded variable factors in tabular form (Table 5-3), the subscripts and superscripts of variables v_j^l and x_j^l are supressed and the symbols vj and xj, j=1,5, adopted instead -the same applies for Table 5-4.

Table 5-3. 2^5 7-area WEM FD decoded and coded design factors.

Decoded factor vj		Coded factor xj
v1	ALP	x1 = (ALP - 117.425)/32.575
v2	AHS	x2 = (AHS - 48.410)/4.401
v3	MGD	x3 = (MGD - 534.935)/13.096
v4	CCC	x4 = (418.336 - CCC)/31.486
v5	SHC	x5 = (181.977 - SHC)/3.7145

2. The complete 2^5 FD comprises 32 possible ways of combining the five factor levels; each defines a treatment combination and an effect of the model proposed. The linear parametric combinations obtained by using the columns of Yates's design matrix are orthogonal contrasts for they satisfy the properties of orthogonality as in definitions **5.2** to **5.4**.

The construction goes as follows: a column of the Yates's design matrix-X is of the form,

$$F_j = (f_j(1),\ f_j(2),\ \ldots,\ f_j(k))', \qquad j=0,k-1 \tag{5.9}$$

i.e., a column of the design matrix-X, and the desired orthogonal transformation as discussed in section 5.3.2,

where each $f_j(t)$ is defined for all treatment combinations, $t=1,k$, as:

- $f_0(t) = 1$ is the intercept,
- $f_j(t) = X_1(t),\ X_2(t),\ X_3(t),\ X_4(t)$ *or* $X_5(t)$, $j=1,k-1$,

and, the $X_s(t)$, $s=1,2,3,4,5$, symbolic expressions of main effects and interactions, are of the form,

$$X_s(t) = x_1^1 . x_2^1 . \ \ldots \ . x_s^1, \qquad s=1,2,3,4,5$$

for x_j^1 being equal to ±1 (coded variables) for all factors j.

For s=1, $X_1(t)$ is the main effect; for s =2,3,4,5, $X_s(t)$ are the second- to fifth-way interactions. The number of main effects and s-way interactions in a 2^5 FD is given by the combinatorial number $\binom{5}{s}$, s=1,2,3,4,5. The level-1 of each factor is associated with the treatment combination-t applied.

Because the elements in the design matrix-X are equal to ±1, this matrix is conventionally represented by corresponding minus (-) and plus (+) signs and usually reffered to as the *sign matrix*.[114] Table 5-5 represents the 2^5 sign matrix used in the 7-area WEM FD. Note f(t), t=1,k, in (5.1) and F_j, j=0,k-1, in (5.9), are respectively the columns and rows of the design matrix X.

The order in representing and/or applying the treatment combinations has no effect whatsoever on responses if the design matrix is made orthogonal for any effect may be independently worked out. However, the treatment combinations order and the order of the elements $f_j(t)$ in (5.9) must coincide. Davies's standard order of treatment representation was adopted in the 7-area WEM FD to calculate the model's coefficients even though the order in which the computer runs were carried out diferred.

Following, the 32 treatment combinations are presented in standard order in terms of both the coded and the decoded variable factors. The symbolic representation of treatment combinations (in function of 0's and 1's) indicates clearly which are the factors at low and high levels; on the other hand, the coded and decoded variables indicate the factors whose main effect and/or interactions are being measured.

---

[114] Davies, O.L., *The Design and Analysis of Industrial Experiments*, op.cit., presents a table of Yates's matrix of signs for a 2^k, k=2 to 6, FD.

Table 5-4. 2^5 7-area WEM FD treatment combinations, coded and decoded main effects and interactions.

Run number	Treatment combination	Main effects and s-way interactions: Coded factors	Decoded factors
1	00000	(1)	Intercept
2	10000	x1	ALP
3	01000	x2	AHS
4	11000	x1.x2	ALP.AHS
5	00100	x3	MGD
6	10100	x1.x3	ALP.MGD
7	01100	x2.x3	AHS.MGD
8	11100	x1.x2.x3	ALP.AHS.MGD
9	00010	x4	CCC
10	10010	x1.x4	ALP.CCC
11	01010	x2.x4	AHS.CCC
12	11010	x1.x2.x4	ALP.AHS.CCC
13	00110	x3.x4	MGD.CCC
14	10110	x1.x3.x4	ALP.MGD.CCC
15	01110	x2.x3.x4	AHS.MGD.CCC
16	11110	x1.x2.x3.x4	ALP.AHS.MGD.CCC
17	00001	x5	SHC
18	10001	x1.x5	ALP.SHC
19	01001	x2.x5	AHS.SHC
20	11001	x1.x2.x5	ALP.AHS.SHC
21	00101	x3.x5	MGD.SHC
22	10101	x1.x3.x5	ALP.MGD.SHC
23	01101	x2.x3.x5	AHS.MGD.SHC
24	11101	x1.x2.x3.x5	ALP.AHS.MGD.SHC
25	00011	x4.x5	CCC.SHC
26	10011	x1.x4.x5	ALP.CCC.SHC
27	01011	x2.x4.x5	AHS.CCC.SHC
28	11011	x1.x2.x4.x5	ALP.AHS.CCC.SHC
29	00111	x3.x4.x5	MGD.CCC.SHC
30	10111	x1.x3.x4.x5	ALP.MGD.CCC.SHC
31	01111	x2.x3.x4.x5	AHS.MGD.CCC.SHC
32	11111	x1.x2.x3.x4.x5	ALP.AHS.MGD.CCC.SHC

Table 5-5. Sign matrix of a 2^5 Factorial Design.

Signs of values of main effects and interactions																																Treat
1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	3	4	5	6	7	8	9	0	1	2	
									10										20										30			
+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	1
+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	2
+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	3
+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	4
+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	5
+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	6
+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	7
+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	8
+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	9
+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	10
+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	11
+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	12
+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	13
+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	14
+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	15
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	16
+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	17
+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	18
+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	19
+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	20
+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	+	-	-	+	+	-	-	+	-	+	+	-	-	+	+	-	21
+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	+	+	-	-	+	+	-	-	-	-	+	+	-	-	+	+	22
+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	+	-	+	-	+	-	+	-	-	+	-	+	-	+	-	+	23
+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	+	+	+	+	+	+	+	+	-	-	-	-	-	-	-	-	24
+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	25
+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	26
+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	+	-	+	-	-	+	-	+	27
+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	+	+	+	+	-	-	-	-	28
+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	29
+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	+	+	-	-	30
+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	+	-	31
+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	+	32

The heading numbers of this table are the 32 main effects and interactions; the numbers under column 'Treat' are the 32 runs. The symbolic representation of both is respectively that of columns 'Coded factors' and 'Treatment combination' of Table 5-4.

3. For each refinery processing unit (9 in total) and for all 7 areas, a model of the marginal rent-ρ response is defined as in (5.10):

$$\rho_t = b_0 + b_1 x1 + b_2 x2 + \ldots + b_{20} x3.x5 + b_{31} x1.x2.x3.x4.x5 \quad (5.10)$$

where the terms in the coded variables correspond to the elements of vector f'(t) in (5.1) and the elements $(b_0, b_1, \ldots, b_{20}, \ldots, b_{31})$ to the parameter vector B of the same expression.

A particular response ρ_t, t=1,k, is obtained by substituting in (5.10) the factor values at the levels they are set in treatment-t, once the estimators β of B are known. The next step is to calculate the main effects and interactions according to Yates's algorithm:[115]

• the application of the design matrix X (of ±1's) to the response values vector-Y gives the Yates's effects.

From (5.9) a Yates's effect g_j, j=0,k-1, and the matrix of Yates's effects $G=(g_j)$, are expressed are,

$$g_j = Y'F_j$$
$$g_j = (y_1, y_2, \ldots, y_k)\ (f_j(1), f_j(2), \ldots, f_j(k))$$
$$G = Y'X \quad (5.11)$$

• with the Yates's effects (5.11), the solution for β is readily obtained: from (5.7) and (5.11) the estimators β of B are expressed as,

$$\beta = Y'X\ (X'X)^{-1} \quad \text{and} \quad \beta = G\ (X'X)^{-1},$$

where (X'X) is a diagonal matrix (orthogonality property) with diagonal element equal to 2^n (2^5), thus,

[115] Yates's algorithm is in fact a recursive algebraic sum of response values carried out in a special but simple order which Yates found gave the wanted factors effects and interactions. The method relies upon the application of the orthogonality property by making (as already explained) the linear combinations of responses to have coefficients ±1, and this, in turn, is achieved by applying an appropriate transformation to the original factors.

$$\beta = G/2^n = G/2^5 \tag{5.12}$$

is finally the estimator desired.

It is possible to demonstrate that calculating Yates's effects and multiplying them by an appropriate constant is equivalent to carry out an anlysis of variance on the observations and solve the underlying system of least square equations.

The relationships between the estimators β of B (obtained by Yates) and the actual effects (including interactions) of the experimental factors can be stated as:[116]

$$g_j = \text{effect} . K = \text{effect} . 2^{n-1} \qquad (K = 2^{n-1})$$

$$\beta_j = g_j/2^n = (\text{effect} . 2^{n-1})/2^n = (\text{effect})/2 \tag{5.13}$$

and the SS associated with a particular contrast g_j is given by,

$$SS(g_j) = g_j^2 \Big/ \sum_{t=1}^{k} f_j^2(t) = g_j^2/2^n$$

$$= (\text{effect} . (2^{n-1}))^2 / 2^n = (\text{ effect })^2 . 2^{n-2} \tag{5.14}$$

and the mean squares (MS) of effects and interactions for analysis of variance testing purposes, is,

$$MS(g_j) = SS(g_j)/df_j = SS(g_j) \tag{5.15}$$

where df_j (degrees of freedom of the SS of every effect and interaction g_j) is equal to the number of factor's levels minus one.

---

116 Proofs can be found in Davies, O.L., Ibid., Chapter 7, pp.263-273, for the special case of Yates's contrasts; and in Mendenhall, W., *Introduction to Linear Models and the Design and Analysis of Experiments*, op.cit., Chapter 9, pp.231-257, for the general case of orthogonal contrasts.

The sum of squares SS and the mean square MS are usually retained for analysis of variance tests and for checking purposes though no actual summing of squares is required by this method.[117]

4. The *full coded* model must reproduce exactly the responses used in Yates's algorithm. This is, for parameter vector β, with β_j as in (5.12), for all j, and x_j's having values ±1, the model,

$$\rho_t = \beta_0 + \beta_1 x1 + \beta_2 x2 + \ldots + \beta_{20} x3.x5 + \ldots + \beta_{31} x1.x2.x3.x4.x5 + \varepsilon_t \tag{5.16}$$

will give the original observations with $\varepsilon_t = 0$ for all t, t=1,32.

As mentioned at the end of section 5.3.2, the parameters β_j in (5.16) may have no significance in relation to the experimental parameters of interest in estimating the marginal rents-ρ, they have no units whatsoever. From the point of view of rents estimation, the models obtained as function of the original factors v_j are the ones of interest.

A *full decoded* model,

$$\rho_t = \theta_0 + \theta_1 v1 + \theta_2 v2 + \ldots + \theta_{20} v3.v5 + \ldots + \theta_{31} v1.v2.v3.v4.v5 + \varepsilon_t \tag{5.17}$$

is obtained by reversing the transformation of v_j into x_j in (5.8).

A (32×32) lower triangular matrix D, the *decoding matrix*, accounts for all the required transformations on effects and interactions. When applied to the set of coefficients β of the coded model it appropriately transforms them into coefficients θ, estimated parameters of the decoded model: $Y = (\beta D)V = \theta V$.

[117] The constant K varies among orthogonal designs for it depends precisely on the design chosen; provided the design is orthogonal, the value of K is arbitrary. Davies states this point, "It is clear that any function in an orthogonal set may be multiplied by an arbitrary constant without affecting the orthogonality". Ibid., p.272. K is equal to 2^{n-1} when using the particular Yates's method.

Example 5.1

Lack of space does not allow for a representation of the full matrix-D. To illustrate its constructive procedure a simple model of a 2^2 FD (i.e., 2 factors at two levels each) is considered:

$$y = \beta_0 + \beta_1 x1 + \beta_2 x2 + \beta_3 x1.x2, \qquad \text{coded model}$$

with factors-x defined as in (5.8); the reconversion of β into θ must be such that $Y = (\beta D)V$, where $D_{(4x4)}$ is the decoding matrix. For this particular example matrix D is constructed as follows:

$$y = \beta_0 + \beta_1(v1-v1')/\delta_1 + \beta_2(v2-v2')/\delta_2 + \beta_3(v1-v1')/\delta_1(v2-v2')/\delta_2$$

Working out the terms above,

$$\theta_0 = \beta_0 - \beta_1 v1'/\delta_1 - \beta_2 v2'/\delta_2 + \beta_3/\delta_1\delta_2\ v1'v2',$$

$$\theta_1 = \beta_1/\delta_1 - \beta_3 v2'/\delta_1\delta_2,$$

$$\theta_2 = \beta_2/\delta_2 - \beta_3 v1'/\delta_1\delta_2,$$

$$\theta_3 = \beta_3/\delta_1\delta_2;$$

the decoding matrix D_{4x4} to be applied to the set of parameters β in order to get the decoded model above is:

$$\underset{\cdot}{D} \;\underset{\cdot}{=} \begin{bmatrix} 1 & & & \\ -v1'/\delta_1 & 1/\delta_1 & & \\ -v2'/\delta_2 & 0 & 1 & \\ v1'.v2'/\delta_1\delta_2 & -v2'/\delta_1\delta_2 & -v1'/\delta_{12} & 1/\delta_1\delta_2 \end{bmatrix},$$

and $\quad y = \theta_0 + \theta_1 v1 + \theta_2 v2 + \theta_3 v1.v2 \quad$ the decoded model.

The decoded model (5.17) accounts for the real parameter estimators of the rents-ρ model. The parameter vector-θ is a LSQE of the parameter vector-Θ, coefficients of the model of rents to be estimated:

$$\rho_t = \Theta_0 + \Theta_1 v1 + \Theta_2 v2 + \ldots + \Theta_{20} v3.v5 + \ldots + \Theta_{31} v1.v2.v3.v4.v5 \tag{5.18}$$

or, $Y = \theta V$, where $\theta V = \beta DV$, θ is the estimator of Θ in $Y = \Theta V$, and V is the matrix of symbolic representation of main effects an interactions in terms of the original factors-v_j, j=1,32.

Unlike the case of the coded models, the decoded models's coefficients θ have physical/economic units determined by the nature of the variable factor (factorial effects and/or interactions) being measured. For example, if ρ_t is the catalytic cracking rent (irrespective of the area) expressed in terms of the estimators-θ, then:

θ_0, the intercept, is the minimum (expectedly negative) US\$/mt of cracking processing feed if all factors are set to zero,

θ_1 units are mt-AL/mt-cc feed for v1 is in \$/mt-AL,

θ_2 units are (\$/mt-cc feed)/mt-AH for v2 is in mt-AH,

θ_{20} units.are (\$/mt-cc feed)/(mt-mg.dwt) for v3.v5 is in mt-mg.dwt.

In general, θ_j is the rate of change in response per unit change in the level of a factor and/or factorial effects and interactions,

$$\partial\rho_t/\partial v_j = \theta_j \tag{5.19}$$

The 7-area WEM FD full decoded model of refinery unit-u rent in area a is then (for θ estimators of Θ in (5.18)):

$$\rho_u(a) = \theta_0 + \theta_1 ALP + \theta_2 AHS \quad \theta_3 ALP.AHS + \theta_{12} MGD.CCC + \ldots + \\ \theta_{23} ALP.AHS.MGD.SHC. + \ldots + \theta_{31} ALP.AHS.MGD.CCC.SHC \tag{5.20}$$

with, u = ALK, CC, COK, AD, HYD, HYC, REF, RD, VD
a = B, J, K, M, T, U, V.

5. In experimental analysis a full decoded model (as that in (5.17)) is not desirable for practical reasons: many effects and/or interactions do not significantly influence the response thus becoming unworthy to keep all the terms in the model. What it is usually done for significance tests on the effects is to assume that some of them will be zero. If this is true, the calculated response will fit better the actual observation, otherwise, a positive residual ε_t will be accounted for.

Daniel points out that in a 2^5 FD,[118]

> We expect a small number of real effects and low-order interactions. At most all main effects, some two factor interactions, possibly a three factor interaction, and some block effects will be large.

A criterion is needed here for truncating the models on rents-ρ. Such criterion is based on the independence and additivity properties of the mutually orthogonal contrasts. Mendenhall shows[119] that the total sum of squares SST associated with a set of parameters β calculated by means of an orthogonal design with contrasts G (as for instance, the Yates's contrasts) can be decompossed as,

$$SST = SS(g_0) + SS(g_1) + \ldots + SS(g_k) \quad \text{with} \quad SS(g_j) = g_j^2 /2^n.$$

SST can be then partitioned in two sum of squares, SSR, the sum of squares due to regression (i.e., the sum of squares of the effects and/or interactions actually conforming the model), and SSE, the sum of squares due to residuals or error estimation,

$$SST = SSR + SSE \quad \text{so that} \quad SSE = SST - SSR.$$

In a FD with no replication and without previous estimates of error variance (in the 2^5 7-area WEM FD replication will not bring neither, additional information nor the possibility of calculating an estimate of the error variance) the former results provide a useful way of estimating the models error variance.

Either the terms of higher order (high-order interactions) are summed up or consecutively $SS(g_j)$ terms are substracted from the SST and the error variance estimated.

---

[118] See Daniel, C., *Applications of Statistics to Industrial Experiments*, op.cit., p.127.

[119] See Mendenhall, W., *Introduction to Linear Models and the Design and Analysis of Experiments*, op.cit., pp.235-336.

Expression (5.6) on page 174 represents the sum of squares of residuals,

ε^2 = SSE = Y'Y - Y'Xβ where, Y'Y = SST and Y'Xβ = SSR.

For all j, SS(g_j) has one degree of freedom: because of additivity, the sum of squares of p<k (p<32) of p independent contrasts g_j has p degrees of freedom. The estimator of error variance is then,

$$s^2 = \frac{Y'Y - Y'X\beta}{k - p}, \qquad s^2 = \frac{SST - SSR}{k - p}$$

where the degrees of freedom p correspond to the number of estimated parameters β, including β_0, used in the fitted model; s^2 is, in turn, the mean square of the sum of squares of residuals SSE. Finally, the square root of s^2, or standard error, is the models' estimation error s of ε,

$$s = \sqrt{(s^2)} = \sqrt{(SSE/32-p)} \qquad (5.21)$$

5.4.2.a Decoded Models Truncating Criteria

Two criteria have been used in order to reduce the full decoded model (5.20):

• First criterion: a model explaining a prefixed percentage value of the response-ρ is assumed to be satisfactory. A set of percentages, namely, 95% to 99% is fixed a priori and tests for the standard error-s as in (5.21) of the estimated model are carried out.

In order to apply this criteriorn, the SS (5.14) and MS (5.15) are stored when calculating the Yates's effects. The percentage contributions (SSp_j) of every effect and interaction g_j are worked out,

$$SSp_j = \frac{SS(g_j)}{SST} \cdot 100 \qquad (5.22)$$

The partially decoded or reduced models' terms are being selected and added up in a 'stepdown' (decreasing) fashion according to its SSp_j, i.e., according to the SS percentage contributions to the response formation. The selection is done until the sum of percentages SSp_j is greater than or equal to the prefixed percentage value.

The SS of the remaining terms (usually the terms of higher order) are added up to form the SSE as discussed before.

• Second criterion: a reduced model is formed by considering only main effects terms, i.e., the single factors alone. The percentage contribution to explain the rent response is calculated as the sum of main effects (five plus the θ_0 term) percentage contributions of SS. The 26 remaining SS (i.e., 32 minus the six estimated coefficients) are added up to make up the estimation error.

A reduced model in terms of main effects has clear advantages upon reduced models with high order interaction terms for the former are simpler to explain and save computational work. Though this criterion was applied in the study, the reader will not find the corresponding results reported since in every case (i.e., for every refinery unit rent in each area) the percentange sum of squares of explained response was at most 75 to 80. This result confirms once more the complexity of interactions in a refinery system.

Example 5.2

As an instance let assume a model like the one following explains 98% of the response $\rho_u(a)$,

$$\rho_u(a) = \theta_0 + \theta_1 v1 + \theta_2 v2 + \theta_7 v1.v2.v3 + \varepsilon_u,$$

this is to say, the percentage sum of squares of the 4 terms out of the total sum of squares is 98% when applying the first criterion thus the estimation error s of ε_u explaining the remaining 2% of the fitted model would be,

$$s = \sqrt{(SSE/28)}, \quad \text{where} \quad SSE = \Sigma\, SS(g_j), \quad j \neq 0,1,2,7.$$

The models $\rho_u(a)$ to be presented in following sections, are decoded reduced models of the general estimation model in (5.20). It will be seen that even with a decoded reduced model it is possible to get interactions which though having non-zero values are negligible as compared to the effects their component factors alone have on response. In this case it is sensible to conclude the factors act independently and conclusions drawn without the presence of interactions are legitimate.

• Other criteria might well be applied. For instance, it would be possible to consider a standard error SE in terms of \$/mt-processing feed, this is to say, a net difference in the rent value. This latter would obviously differ among refinery processing units within an area and among areas as well. So far this criterion has not been applied here.

Another way of selecting the models' coefficients is by checking the significance of main effects and/or interactions using the ratio given by,

$$F' = \frac{MS(g_j)}{MS(SSE)} \tag{5.23}$$

for all j, j=1,32. F' is contrasted against the statistical number F_α of the Fisher distribution at the significance level α, and with ϕ_i, i=1,2, the degrees of freedom of the numerator and denominator respectively, (i.e., 1 degree of freedom for every effect and interaction, p degrees of freedom for all regression terms, and 32-p degrees of freedom for the residual). If for a particular effect g_j the number F' is greater than the Fisher number F_α, then it is said the effect g_j is significant at the level of significance-α considered and hence should be included in the rent model. In general, the Fisher value F_α, with ϕ_1, ϕ_2 degrees of freedom and the MS(SSE) give the critical (minimum) value υ_α,

$$\upsilon_\alpha = F_\alpha \times MS(SSE) \tag{5.24}$$

at which an effect g_j becomes significant at the significance level-α.

Note that checking on effects is equivalent to check on the the values of coefficients β_j since from (5.13) β=(effect)/2. If an effect g_j happens to be insignificant, its value is assumed zero and hence the corresponding parameter β_j does not enter the regression model. Likewise, if a significance test is carried out on β_j under the null hypothesis H_o: $\beta_j=0$ and H_o is accepted, then the corresponding effect will be directly zero.

5.4.3 Summary

What was discussed in sections 5.4.1 and 5.4.2 can be concluded as follows:

For each of the 7-area WEM, for each of the nine refinery processing units, 32 experimental marginal rents $\rho_u(a)$ values are obtained by applying the 32 different treatment combinations of a 2^5 FD. A linear (in the parameters) model is assumed to fit the 32-point of each of the 63 (7 areas x 9 refinery processing units) response surfaces. The linear model is of the form represented by relation (5.20), namely,

$$\rho_u(a) = \Theta_0 + \Theta_1 ALP + \Theta_2 AHS \quad \Theta_3 ALP.AHS + \Theta_{12} MGD.CCC + \ldots +$$

$$\Theta_{23} ALP.AHS.MGD.SHC. + \ldots + \Theta_{31} ALP.AHS.MGD.CCC.SHC$$

with, u = ALK, CC, COK, AD, HYD, HYC, REF, RD, VD
a = B, J, K, M, T, U, V.

The estimation of coefficients Θ is made simpler by applying the theory of orthogonal designs and LSQE. In this work, the Yates's method of constructing orthogonal designs was applied. That entails the transformation of the original factors' values to the values ±1. The models obtained are coded models in terms of the coded variables. The inverse transformation has to be applied to get the original factor values and the decoded (real) model of rents.

Truncating criteria are used by checking the sum of squares of effects and interactions and/or by applying F-significance tests on selected effects and interactions in order to reduce the full 32-term decoded model.

Accordingly, a software was developed to calculate:

- Yates's effects, full and reduced coded and decoded rents models;
- sum of squares and mean squares of effects, interactions, models' regression terms and residuals; and,
- estimated responses by using the estimated decoded models, and differences between the estimated responses and the experimental responses (i.e., individual residuals) in order to check on accurracy of operations.

The reader may refer to section A5.5 (Factorial Design Software) of the Appendix to Chapter 5 for a presentation of the fortran software applied in the analysis of experiments.

In order to make a distinction between the models in (5.20) and the regression models of section 5.8 which though linear, are expressed in terms of a different set of variables, the former are called *the first kind of RR models* and the latter *the second kind of RR models*.

The second kind are derived by applying LR methods too but comprise a relatively smaller number of parameters for estimation; this is put forward in section 5.8 of this Chapter.

5.5 THE FIRST KIND OF REFINERY RENT MODEL

In this section the 2^5 7-area WEM FD's experimental results are presented and analysed. The responses $\rho_u(a)$ are presented in Davies standard order in tables A5-1 to A5-6 of the Appendix to Chapter 5.

In all areas, the crude distillation, vacuum distillation and coking units did not generate a marginal rent at any of the 32 treatment combinations applied; this reduced the number of refinery processing units of model estimation to six instead of nine, and the total number of possible models to 42 (6 refinery processing units x 7 areas). Moreover, since the rent responses for some of the remaining six refinery processing units in particular areas happened to be zero (underutilization of refining capacity), for those refinery processing units yielding more than 16 zero marginal rents, the corresponding reduced model estimation was not accomplished. Responses are not reported either for these latter cases. This reduces the number of models for estimation even further: only 22 reduced models out of the 42 above remaining possibilities were estimated.

Coded and decoded reduced models (estimators of coefficients) at 90%, 95%, 98% and 99% sum of squares of regression are presented in tables A5-7 to A5-30 of the Appendix to this Chapter.

The analysis of the 22 estimated models along with the estimation of the level of rents by using actual time series is a laborious work which for the sake of illustrating the methodology of FED (Factorial Experimental Designs) and RSM (Response Surface Methodology) applied to rent estimation, need not to be undertaken here for every model in each area.

The concentration was reduced to North West Europe to analyse the models and further (upon data availability) to estimate the levels of rents for the refinery processing units of catalytic cracking, catalytic reforming, alkylation and hydrocracking. The remaining areas models readily follow from tables A5-7 to A5-30 appendix to Chapter 5, and a similar treatment to the next applied to area B applies to them.

It is clear the greater the percentage of response explanation, the more the number of terms included in the model therefore the more the number of computations necessary to obtain marginal rents estimates. In consequence, even if the models are presented at four different levels of sum of squares percentage contribution to response, only one, that comprising the least possible number of terms and contributing substantially to the rent response will be chosen for model computation and analysis with actual time series data.

A look at tables A5-7 to A5-30 shows the number of terms included in the models goes from 4 to 11, depending of the percentage sum of squares and of course, on the refinery processing unit being modelled and the area it represents. For area B, the catalytic cracking and catalytic reforming models happen to comprise both 7 terms (excluding the intercept) at 95% sum of squares of regression.

5.5.1 Application to North West Europe

The area B responses on marginal rents indicate that the four refinery processing units contributing to motor spirit (physical) formation generate a marginal rent for a refiner in the area. The experimental results are summarized in Table A5-1, Appendix to Chapter 5.

From Table A5-1 it can be seen that treatment combinations 2, 3, 5, 9 and 17 correspond to those of changing a single factor at a time. The effects they produce on response are different for every unit in area B. Accordingly, the highest levels of rents for alkylation, catalytic cracking, catalytic reforming and hydrocracking are produced respectively when MGD, ALP, CCC and SHC are set at level '1'. For CCC and SHC that means decreases in capacity produce increase in the level of rents of reforming and hydrocracking respectively. A single effect model to explain the rent response is however not desired as interactions are contributing greatly to the rent response.

Combinations of factors ALP, MGD and CCC at their high levels increase considerably the rent response in the four refinery processing units of area B. It then appears that those three factors interact highly with each other and are the main forces driving refinery rents up in North West Europe, fact that confirms assumptions and experimental hypotheses. The AHS factor has no much effect on responses.

The decoded models $\rho_u(B)$, for u=ALK, CC, HYC, REF, were truncated by applying the first criterion, section 5.4.2.a, at 90%, 95%, 98% and 99% sum of squares of regression: the factors and interactions appearing in the models contributed 90 to 98% to the ρ_u formation.

The respective models' estimation errors (or standard errors-s) are also presented: the higher the number of regression terms, the lower the standard error is. The errors are in \$/mt processing feed to the refinery unit. Unlike the case of pricing crudes and products which is usually done in \$/bl (and an error of even .3 \$/bl is already high), an error of about ±2 \$/mt can be considered small in the present case.

5.5.1.a Alkylation

90% $\rho_{ALK}(B) = -3830.77 + .1019ALP + 7.42MGD + 8.16CCC - .016MGD.CCC$

$s = \pm\ 5.29$ \$/mt processing feed

95% $\rho_{ALK}(B) = -3562.59 - 2.182ALP + 6.67MGD + .006MGD.ALP + 8.47CCC - .0027ALP.CCC - .016MGD.CCC$

$s = \pm\ 3.25$ \$/mt processing feed

98% $\rho_{ALK}(B) = 635.6 - 37.93ALP - 1.18MGD + .073ALP.MGD - 1.56CCC + .083ALP.CCC + .003MGD.CCC - .00016ALP.MGD.CCC$

$s = \pm\ 2.21$ \$/mt processing feed

99% $\rho_{ALK}(B) = 692.6 - 37.93ALP - 1.18MGD + .073ALP.MGD - 1.56CCC + .083ALP.CCC + .003MGD.CCC - .00016ALP.MGD.CCC - .31SHC$

$s = \pm\ 1.79$ \$/mt processing feed

5.5.1.b Catalytic Cracking

90% $\rho_{CC}(B) = -1807.12 + .13ALP + 3.68MGD + 4.08CCC - .008MGD.CCC - .52SHC$

$s = \pm\ 2.86$ \$/mt processing feed

95% $\rho_{CC}(B) = -1690.32 - .866ALP + 3.34MGD + .003MGD.ALP + 4.24CCC - .0013ALP.CCC - .008MGD.CCC - .52SHC$

$s = \pm\ 2.09$ \$/mt processing feed

98% $\rho_{CC}(B)$ = 900.62 - 17.51ALP -.315MGD + .034ALP.MGD - 1.955CCC + .0385ALP.CCC _ .0008MGD.CCC - .00007ALP.MGD.CCC - 4.02SHC + .008CCC.SHC

s = ± 1.40 $/mt processing feed

99% $\rho_{CC}(B)$ = -15674. -17.51ALP + 30.67MGD + .034ALP.MGD + 35.CCC + .038ALP.CCC - .068MGD.CCC -.00007ALP.MGD.CCC + 87.07SHC - - .17MGD.SHC - .19CCC.SHC + .0004MGD.CCC.SHC

s = ± 1.04 $/mt processing feed

5.5.1.c Catalytic Reforming

90% $\rho_{REF}(B)$ = -2304.7 + .08ALP + 4.07MGD + 4.52CCC - .009MGD.CCC + 1.14 SHC

s = ± 3.51 $/mt processing feed

95% $\rho_{REF}(B)$ = -2171.27 - 1.06ALP + 3.67MGD + .0033MGD.ALP + 4.70CCC - .002ALP.CCC -.009MGD.CCC + 1.14SHC

s = ± 2.65 $/mt processing feed

98% $\rho_{REF}(B)$ = -26084.22 - 19.ALP + 48.8MGD + .037ALP.CCC - .117MGD.CCC - .00007ALP.MGD.CCC + 142.95SHC + .01ALP.SHC -.27MGD.SHC - .34CCC.SHC + .00064MGD.CCC.SHC

s = ± 1.69 $/mt processing feed

99% $\rho_{REF}(B)$ = -27927.83 - 19.ALP + 52.22MGD + .034ALP.MGD + 62.37CCC - .037ALP.CCC - .117MGD.CCC - .00007ALP.MGD.CCC + 153.SHC + .01ALP.SHC - .28MGD.SHC - .34CCC.SHC + .0006MGD.CCC.SHC

s = ± 1.28 $/mt processing feed

5.5.1.d Hydrocracking

90% $\rho_{HYC}(B) = -26825. + .035ALP - 51.MGD - 58.5ALP.MGD + .11MGD.CCC - 147.61SHC + .28MGD.SHC + .32CCC.SHC - .0006MGD.CCC.SHC$

$s = \pm\ 1.05$ \$/mt processing feed

95% $\rho_{HYC}(B) = 26814.21 + .13ALP - 51.MGD + .001MGD.ALP - 58.54CCC + .111MGD.CCC - 147.18SHC - .004ALP.SHC + .28MGD.SHC + .32CCC.SHC - .0006MGD.CCC.SHC.$

$s = \pm\ 0.79$ \$/mt processing feed

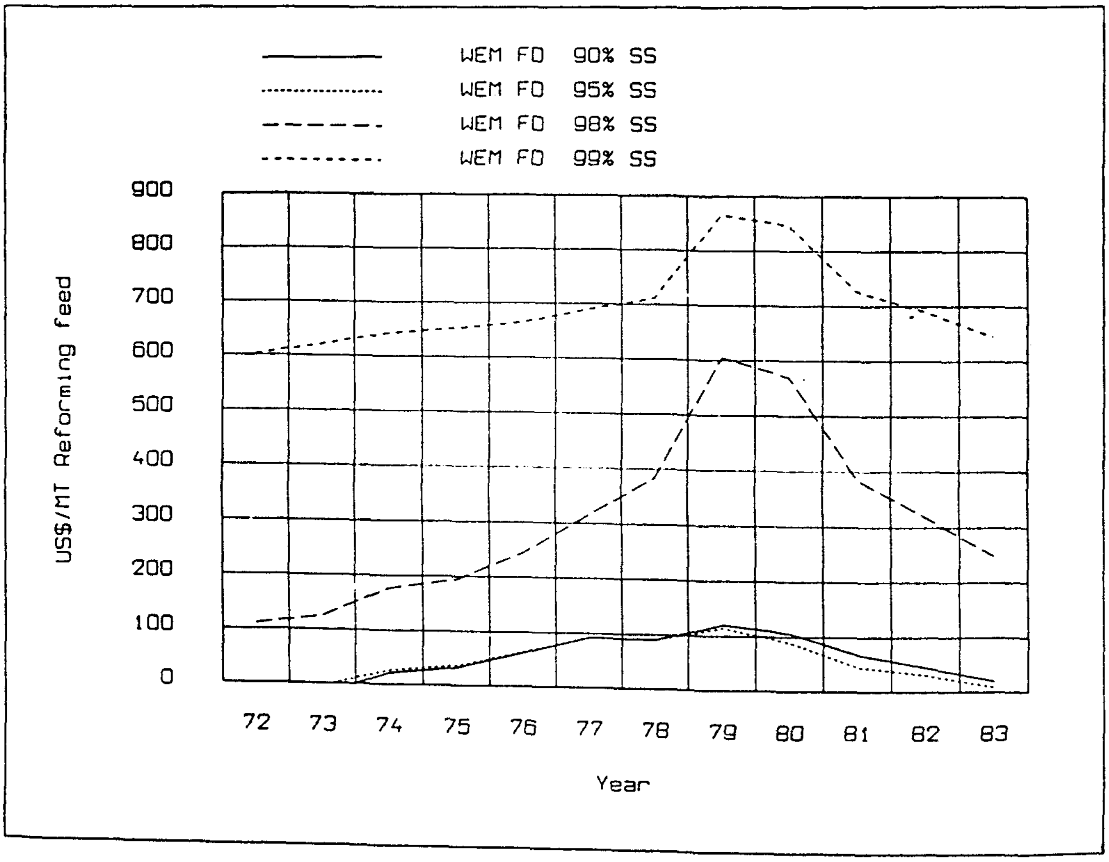


Figure 26. 1972-1983 North West Europe catalytic reforming estimated rents: first kind of Refinery Rent model.

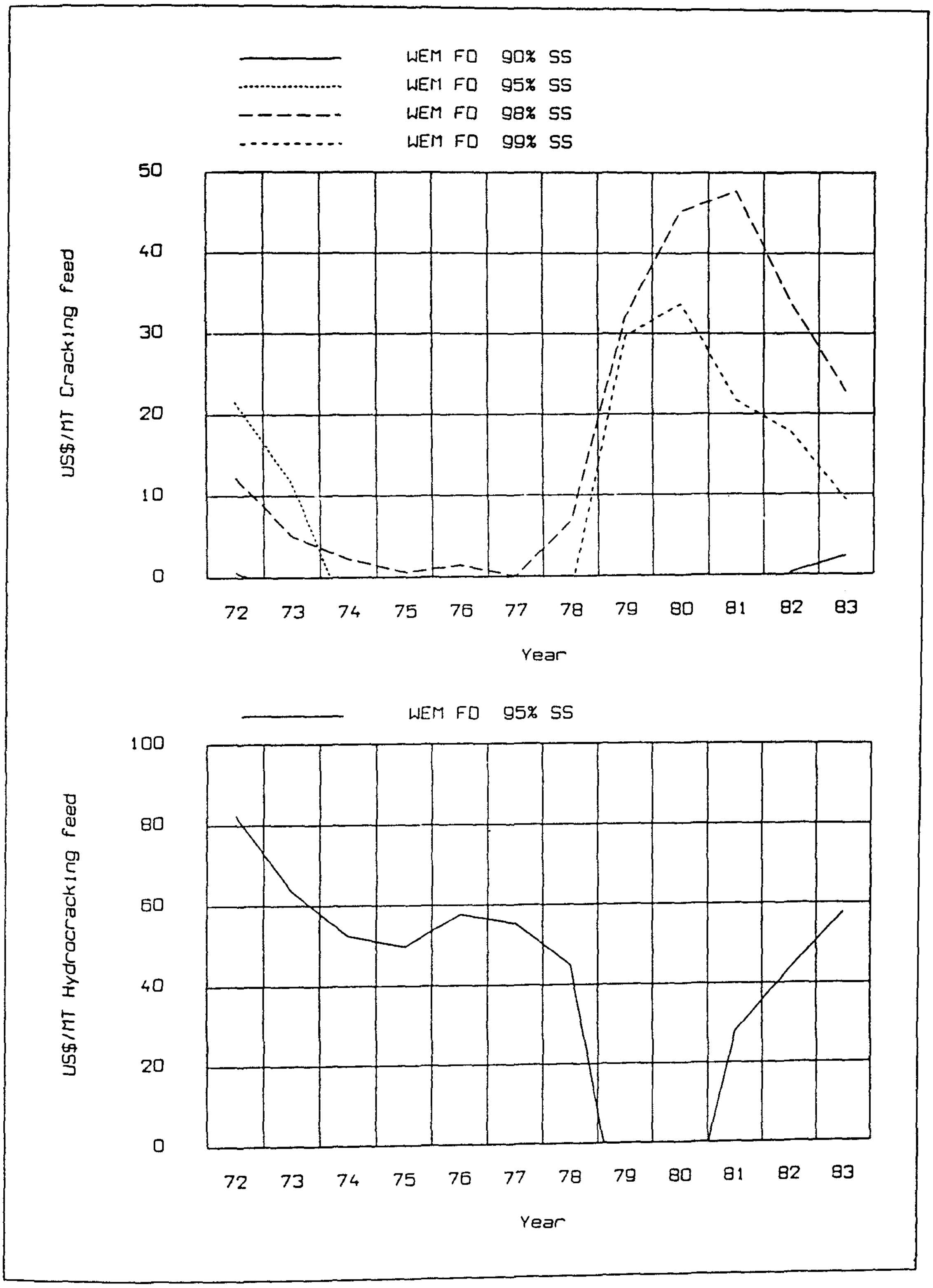


Figure 27. 1972-1983 North West Europe catalytic cracking and hydrocracking estimated rents: first kind of RR models.

5.6 FITTING THE FIRST KIND OF REFINERY RENT MODEL

The first kind of RR models were used to estimate refinery rents in North West Europe with the time series data summarized on Table 5-6 below. Figure 26 on page 202 and Figure 27 on page 203 depict the estimates.

Table 5-6. 1972-1983 Arabian Light price, and worldwide motor gasoline demand, cracking and shipping capacities.

Year	ALP $/mt	MGD mmt/y	CCC mmt/y	SHC mdwt/y
1972	9.16	473.76	431.39	149.70
1973	14.94	498.29	438.84	170.30
1974	81.43	492.13	439.55	202.00
1975	86.65	496.60	439.66	211.00
1976	86.15	520.80	450.00	232.45
1977	92.95	531.00	455.20	258.26
1978	96.31	557.90	465.51	258.40
1979	226.75	548.60	487.66	281.40
1980	268.92	540.42	505.64	267.80
1981	254.30	529.70	517.34	233.50
1982	236.77	536.62	502.00	215.00
1983	213.71	542.34	511.15	198.24

Source: Data from tables D-1, D-4, D-5 and D-8, Appendix D.

So far, the following conclusions can be drawn:

1. No successful results were found for alkylation: the four models give negative and/or zero rents; it could be that unit did not generate in reality a rent during the period considered, or the model does not represent the real determinants of alkylation rent in the area.

2. Figure 26 on page 202 shows the different percentage contribution models of catalytic reforming rent. Because is almost impossible for a refiner to obtain a rent between US$ 200.00 and US$ 900.00 per metric tonne of reformer feed, models at 98% and 99% sum of

squares of regression could be left out and models at 90% and 95% sum of squares used for further comparisons and/or validations.

3. Figure 27 on page 203 shows that models at 98% and 99% sum of squares of regression could explain the behaviour in time of catalytic cracking rent levels in North West Europe. Models at 90% and 95% could be non-sensical since the zero or negative rent would mean a long run non-expansion in the period when indeed an investment in cracking occurred.

4. For hydrocracking only the 95% sum of squares model seems to give reasonable estimates hence the only results depicted in Figure 27 on page 203.

These curves are to be compared in Chapter 6 with those of the second kind of Refinery Rent models and with series of data on capacity utilization and average annual capital charges.

5.6.1 Effects of Factors on Refinery Rents

It was discussed in the theoretical aspects of the 2^5 FD that the following relations hold for θ_j, j=1,32, being any coefficient of a decoded model:

$$\theta_j = (\ \text{effect}_j/2\ ) = \partial\rho_u(B)/\partial v_j,$$

where v_j is factor-j and $\rho_u(B)$ the marginal rent of refinery processing unit-u in area B.

Checking then the effect of any factor-j is equivalent to check on coefficients θ_j consequently on the rates of change of the refinery processing unit rent per unit change of the factor and/or factorial combination.

The effects of the main factors of the models used for rent estimation (depicted in the graphs) are analysed below.

5.6.1.a Catalytic Cracking

With catalytic cracking rent models at 98% and 99% sum of squares of regression the rates of change of $\rho_{CC}(B)$ with respect to every factor, namely, ALP, AHS, MGD, CCC and SHC, are given below.

- With respect to ALP,

$$\partial\rho_{CC}(B)/\partial ALP = -17.51 + .034MGD + .0385CCC - .00007MGD.CCC$$

For both models the change with respect to ALP is the same, and due to interactions of ALP with other factors some other values are required to assess on an absolute positive or negative effect on response when varying ALP. With values of Table 5-6 put in the above expression however, a positive value for the derivative is obtained which means that an increase in ALP brings about an increase in the catalytic cracking rent in North West Europe for any combination of MGD and CCC, i.e., ever full cracking utilization. This is in line with real market developments: the early eighties have shown increases in cracking rents and further expansion due, among other factors, to the high Arabian Light price which brought about a market instability but a short run advantage for the complex conversion type of refinery. Chapter 7 discusses further this effect.

- For $\rho_{CC}(B)$ changes when changing MGD different derivatives are found for each model:

$$\partial\rho_{CC}(B)/\partial MGD = -.315 + .034ALP - .0008CCC - .00007ALP.CCC,$$

98% SS model; and,

$$\partial\rho_{CC}(B)/\partial MGD = 30.67 + .034ALP - .068CCC - .00007ALP.CCC - .17SHC + .0004CCC.SHC,$$

99% SS model.

Fixing values of ALP, CCC and SHC from Table 5-6, and neglecting the intersection term ALP.CCC, the change in catalytic cracking rent is positive for both, the 98% and the 99% sum of squares of regression models when MGD increases, also in line with hypotheses.

- The rates of change of $\rho_{CC}(B)$ when changing CCC are given by:

$$\partial\rho_{CC}(B)/\partial CCC = -1.955 + .0385ALP - .0008MGD - .00007ALP.MGD,$$

98% SS model; and,

$$\partial\rho_{CC}(B)/\partial CCC = 35. + .038ALP - .068MGD - .00007ALP.MGD - .19SHC,$$

99% SS model.

Neglecting again the term .00007ALP.MGD, at any fixed values of ALP, MGD and SHC of the 1972-1983 data, a unit increase in CCC will produce an increase in catalytic cracking rent under the model at 98% SS. When the factor of SHC is added, model at 99% SS, the effect of changing CCC on cracking rent is negative due to the negative effect of shipping capacity. The higher the shipping capacity is, the more pronounced the decrease in cracking rent is.

- For $\rho_{CC}(B)$ changing when changing SHC factor, the rates are as follows:

$$\partial\rho_{CC}(B)/\partial SHC = -4.02, \quad 98\% \text{ SS model; and,}$$

$$\partial\rho_{CC}(B)/\partial SHC = 87.07 - .17MGD - .19CCC, \quad 99\% \text{ SS model.}$$

A SHC unit increase in the short run will produce a negative change in the catalytic cracking rent according to both models, and for a worldwide motor gasoline demand/catalytic cracking capacity pattern of the period 1972-1983.

5.6.1.b Catalytic Reforming

The 90% and 95% SS of regression models of catalytic reforming produce rates of change on catalytic reforming rent as follows:

- With respect to ALP,

$$\partial\rho_{REF}(B)/\partial ALP = .08, \quad 90\% \text{ SS model; and,}$$

$$\partial\rho_{REF}(B)/\partial ALP = -1.06 + .0033MGD - .002CCC, \quad 95\% \text{ SS model.}$$

For the 90% model a unit increase in ALP produces a positive increment in the reforming rent; however because interactions are present in the 95% model, the change in reforming rent no longer depends on ALP only but also on MGD and CCC. With values of Table 5-6, these interactions make the change in reforming rent to be negative with ALP increases. It is apparent that for small cracking capacity relative to MGD the increase in ALP will fall on the reforming unit, that is a sensible result: if motor spirit demand increases more than the available cracking capacity to satisfy it, severity in reforming will increase and the probability of a rent with increasing ALP is higher.

- For $\rho_{REF}(B)$ when changing MGD:

$$\partial\rho_{REF}(B)/\partial MGD = 4.07 - .009CCC, \quad 90\% \text{ SS model; and,}$$

$$\partial\rho_{REF}(B)/\partial MGD = 3.67 + .0033ALP - .009CCC, \quad 95\% \text{ SS model.}$$

Both models give positive and/or negative changes in reforming rent per unit change in MGD. At 90% SS, the change in reforming when changing MGD will depend on the actual value of CCC. At 95% SS, it will depend on the combination of values ALP/CCC.

- For $\rho_{REF}(B)$ changes when changing factor CCC:

$$\partial\rho_{REF}(B)/\partial CCC = 4.52 - .009MGD, \quad 90\% \text{ SS model; and,}$$

$$\partial\rho_{REF}(B)/\partial CCC = 4.70 - .002ALP - .009MGD, \quad 95\% \text{ SS model.}$$

Here again the change in reforming could be either positive or negative depending on the MGD value (90% SS model) or on the combination of values ALP/MGD (95% SS model).

- For $\rho_{REF}(B)$ changes when changing SHC:

$$\partial\rho_{REF}(B)/\partial SHC = 1.14, \quad 90\% \text{ and } 95\% \text{ SS models.}$$

Increasing SHC by a unit, increases in general the catalytic reforming rent by 1.14 US$/mt. And this is in line with reality. The hydroskimming units are considered part of the transport system so that increasing shipping could be interpreted also as decreasing reforming (hydroskimming) capacity and hence the possibility of increasing rents.

5.6.1.c Hydrocracking

The 95% SS of regression model of hydrocracking rent produces coefficients or rates of change as follows:

- With respect to ALP,

$$\partial\rho_{HYC}(B)/\partial ALP = .13 + .001MGD - .004SHC$$

the change of hydrocracking rent is positive at any of the level of worldwide motor gasoline demand and shipping capacity presented on Table 5-6.

- With respect to MGD unit change,

$$\partial\rho_{HYC}(B)/\partial MGD = -51. + .001ALP + .111CCC + .32SHC - .0006CCC.SHC$$

the change in hydrocracking rent is invariably positive for fixed values of the other factors; the interaction CCC.SHC can be neglected since its coefficient approaches zero.

- With respect to CCC:

$$\partial\rho_{HYC}(B)/\partial CCC = -58.54 + .111MGD + .32SHC - .0006MGD.SHC$$

the change in the hydrocracking rent would be positive or negative depending on the combination values of MGD/SHC. By neglecting interaction MGD.SHC, the series of values of MGD, and SHC of Table 5-6 substituted above produce in every case a positive increase in hydrocracking rent.

- With respect to SHC:

$$\partial\rho_{HYC}(B)/\partial SHC= -147.18 - .004ALP + .28MGD + .32CCC - .0006MGD.CCC$$

The hydrocracking rent increases with the per unit increase of shipping capacity for values of all other factors as in Table 5-6.

5.7 CONCLUDING REMARKS

Although no factor in isolation has an absolute effect on refinery rents since all interact highly with each other, the table below shows the effects of factors on the catalytic cracking, catalytic reforming and hydrocracking units rents due consideration to interaction effects as analysed in sections 5.6.3.a to 5.6.3.c (assuming changes in rents per unit increase of the factor's level, i.e., right side partial derivatives, and the series of worldwide statistics of Table 5-6).

The following summarizes the direction of changes of factors on rent responses:

Change in rent for small increases in factor value

Refinery unit	Factors				
	ALP	AHS	MGD	CCC	SHC
Cracking	+	none	+	±	-
Reforming	±	none	±	±	+
Hydrocracking	+	none	+	+	+

It is seen the AHS factor has no effect whatsoever on refinery rents responses in North West Europe for the levels assigned.

The combination of MGD and CCC factors' values determines in the case of catalytic reforming, the positive or negative change in reforming rent.

The impact of such a choice of factors to this particular case is certainly not to be the same as in the case of pricing crudes and refined products, to which the 7-area WEM FD was in principle designed.

Indeed, a more sensible choice of factors to estimate refinery rents is sound: there is empirical evidence (SR and 7-area WEM experiments, other than a 2^5 FD) on top of a general theory on rent formation, Chapter 3, to believe that factors such as capacities of other refinery processing units (for instance, the catalytic reforming capacity whose rent is not independent on that of the catalytic cracking unit); the demand levels of the products jointly produced with catalytic cracked spirit (as is shown that gas oil has a determinant influence in the performance of both catalytic cracking and catalytic reforming units for it is a product competing with the catalytic cracked spirit at the former unit). Also a more severe perturbation in crude supply (e.g., higher crude levels) would in principle alter the system in a more appreciable way for the sake of rents estimation. Another chemistry of supply would indeed be required along with changes in the chemistry of demand particularly in the levels of middle distillates.

A hypothetical new 7-area WEM FD setting up would then take consideration of the following factors:

- the Arabian Light Price;

- a greater increase in the supply of Arabian Heavy *or* of any other heavy crude *or* of a combination of heavy crudes;

- catalytic cracking and catalytic reforming capacities;

- motor gasoline and gas oil demands, particularly in North West Europe; and,

- shipping capacity.

This setting up would result in a 2^7 FD. Such a FD, in turn, would increase the number of treatment combinations to 128 (instead of the 32 treatment combinations of the 2^5 FD). Running the 7-area WEM 128 times consumes great deal of computer time (an average of 4 minutes per case) only, but finding out the levels of all combined factors generating feasible solutions could be indeed a hard task, as was already for fixing the 2^5 FD levels. But perhaps a work worth undertaking if unrestricted computer time is available once the 7-area WEM has been updated along the lines expressed in section 5.1 of this Chapter.

The factors and interactions forming the refinery rent models were selected by using the first truncating criterion (i.e., by setting a percentage SS of regression contribution of the model's terms to the response formation, see section 5.4.2.a). In order to compare such a selection to that obtained by using significance tests on coefficients, the test using the Fisher ratio F' and the Fisher distribution was also applied at various significance levels-α (.10, .05 and .01). It is worth noting that the same model coefficients resulted from this truncating criterion, this is to say, to the choice of model resulting when setting a particular significance level-α corresponded in the first criterion a model between 90 to 99% SS of regression. In view of this equivalence, only one selection criterion was subsequently applied, the one reported here.

5.8 THE SECOND KIND OF REFINERY RENT MODEL

The second kind of Refinery Rent model as mentioned earlier in this Chapter is estimated by applying LR analysis directly to refinery rents responses obtained either from the FD or from successive PP experiments with the 7-area WEM.

While in a full decoded model in terms of the five factors/factorial effects, 32 parameter needed to be estimated, here a reduced number of parameters is estimated when reducing the number of independent variables.

Accordingly, refinery rents are estimated in terms of prices only. It could be argued this oversimplifies the complex oil market interactions. Nonetheless prices are a reflection of product demand levels, input prices and seasonality movements on top of strategic policies of the parties involved (i.e., oil companies, governments, shipowners, etc.). On the other hand, there exists for them a spot market from which more reliable (day to day) data are accounted in relation to the availability of other oil statistics. Thus the error which may be made by leaving out determinant factors is overcome by the more accurate data, the less cumulative error in computational operations and the less effort in gathering and keeping updated the series of required oil statistics.

The general LR method was already put forward in section 5.3.2. Here, the problem changes slightly to become,

$Y = (y_t),\ \ t=1,k,$ the k responses, realizations of the variable-y

either when applying the treatment combinations of the 2^5 FD to 7-area WEM, *or* when carrying out the PP set of experiments on the 7-area WEM. These responses are clearly the refinery unit rents (LP refinery processing units marginal values).

$X = [\ f_j(t)\ ],\ \ j=0,k-1,\ \ t=1,k,$ is a much simpler matrix than that defining the FD for it comprises the direct marginal values associated with premium motor gasoline, heavy fuel oil, gas oil, kerosine, naphtha and straight-run benzine corresponding to every treatment combination or parametric programming experiment.

The problem is again to find an estimation vector β of B assuming a linear model of Y in terms of price values,

$$Y = XB, \quad \text{with} \quad E[Y] = \mu,\ \ V[Y] = 0.$$

The parameter vector-β must then minimize the difference,

$\varepsilon^2 = (Y - X\beta)' (Y - X\beta)$ with $E[\beta] = B$ and $Cov[\beta] = \sigma^2(X'X)^{-1}$.

The LSQ estimators β of B provide BLUE estimators by solving the expression given in (5.7), i.e., $\beta = Y'X(X'X)^{-1}$, where now matrix X does not comprise ±1 elements since the independent variables are not longer coded thus making the computations of β more laborious, the inversion of matrix (X'X) is now required. Also β and θ denote the same vector parameters for the variables do not move any longer between two different reference systems therefore decoding matrix-D is no longer required.

A well-known software, SPSS is particularly developed for computing statistical approximating techniques, among others, all kind of regressions, and reporting on relevant statistical indicators. The simplicity of the models makes it easier to develop an ad hoc software for calculating vector parameter-θ, estimators of the actual models' parameters-Θ, the Linear Regression Software developed is presented in the Appendix to Chapter 5, section A5.6.

The equivalent to model (5.20), first kind of RR model, is here,

$$\rho_u(a) = \theta_0 + \theta_1 p_{p1} + \theta_2 p_{p2} + \dots + \theta_n p_{pn} \tag{5.25}$$

with, u = CC, HYC, REF;

a = B, North West Europe;

n number of independent variables, less than or equal to 3 for most of the cases;

p_{pj}, j=1,n, price of product pj, where,
pi = pms, dfo, hfo, srb, naphtha or jetkero;

$\theta_j = \partial\rho_u/\partial p_{pj}$, j=1,n, is the rate of change of the refinery unit rent per unit change of price p_{pj}; and,

θ_0 is the intercept.

The intercept is expressed as $\theta_0 = E[\rho_u(a)] - \theta E[p_{pj}]$, $j\neq0$, i.e., θ_0 assumes the regression lines pass by the point of means.[120]

[120] Here $E[\rho_u(a)] = \rho_u(a)$, for all units-u, and $E[p_{pj}] = p_{pj}$, for all products-pj, for the $\rho_u(a)$'s are deterministic responses, so that the intercept θ_0 is equal to $\rho_u(a) - \theta p_{pj}$, $j\neq0$.

The number of refinery processing units reduces to three since the prices available at spot markets refer to marketable products hence that for a refinery unit as the alkylation unit which requires a gas input from intermediate processes, an estimation of rents becomes difficult from product spot prices.

Similarly, the number of areas has been reduced to North West Europe for which the time series of prices 1972-1983 where easier to gather. A similar analysis should follow for the remaining areas, having at hand the corresponding area spot market prices.

Tables D-7, D-10 and D-11 in appendix D show the 1972-1983 time series of premium motor gasoline, gas oil, heavy fuel oil, naphtha, jetkero and Arabian Light prices at Rotterdam. Prices in table D-7 are in nominal terms whereas deflated prices according to the US$ deflator for the time, are presented in US$ 1976 and US$ 1980 basis in the other tables respectively.

Figure 28 on page 215 depicts the price time series of table D-7. The two major oil price increases in the period have a clear incidence on product prices at Rotterdam, that is levels at the end of 1973 and 1979. It is then expected levels of rents to be higher at those points than at previous years'.

A kind of seasonality in the level of rents is expected to appear as a reflect of the oil products demand seasonality. It is clear this effect will be only noticed if referring to monthly levels of rents for the year the seasonal trend is smoothened and the mid-point (average) year value shown.

Generally, and as a result of correlating refinery rents to spot prices, the trends of both variables become comparable. This is particularly noticeble when the model comprises only two price variables. If more than two, the effects of prices on rents are somehow changed because of the respective end uses of products in the market and the already mentioned seasonal effect; and on the other hand, due to the attached product logistics at the refinery level (which implies the use of diverse processes and capacity utilizations).

Various alternative regresssion models of the second kind (5.25) have been constructed to estimate catalytic reforming, catalytic cracking and hydrocracking rents in North West Europe as explained below.

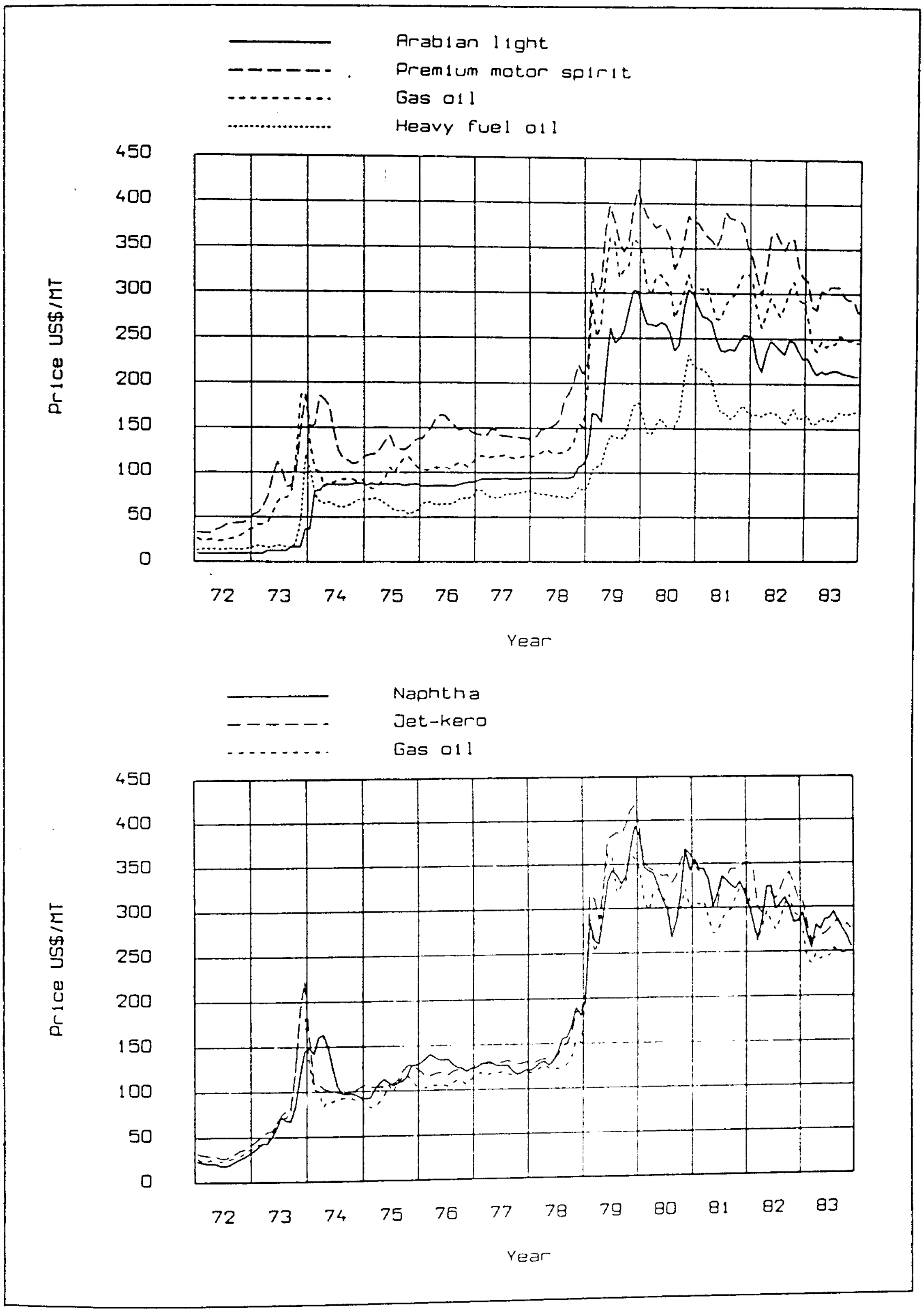


Figure 28. 1972-1983 Arabian Light and main product prices at Rotterdam, prices in US$ per metric tonne per month.

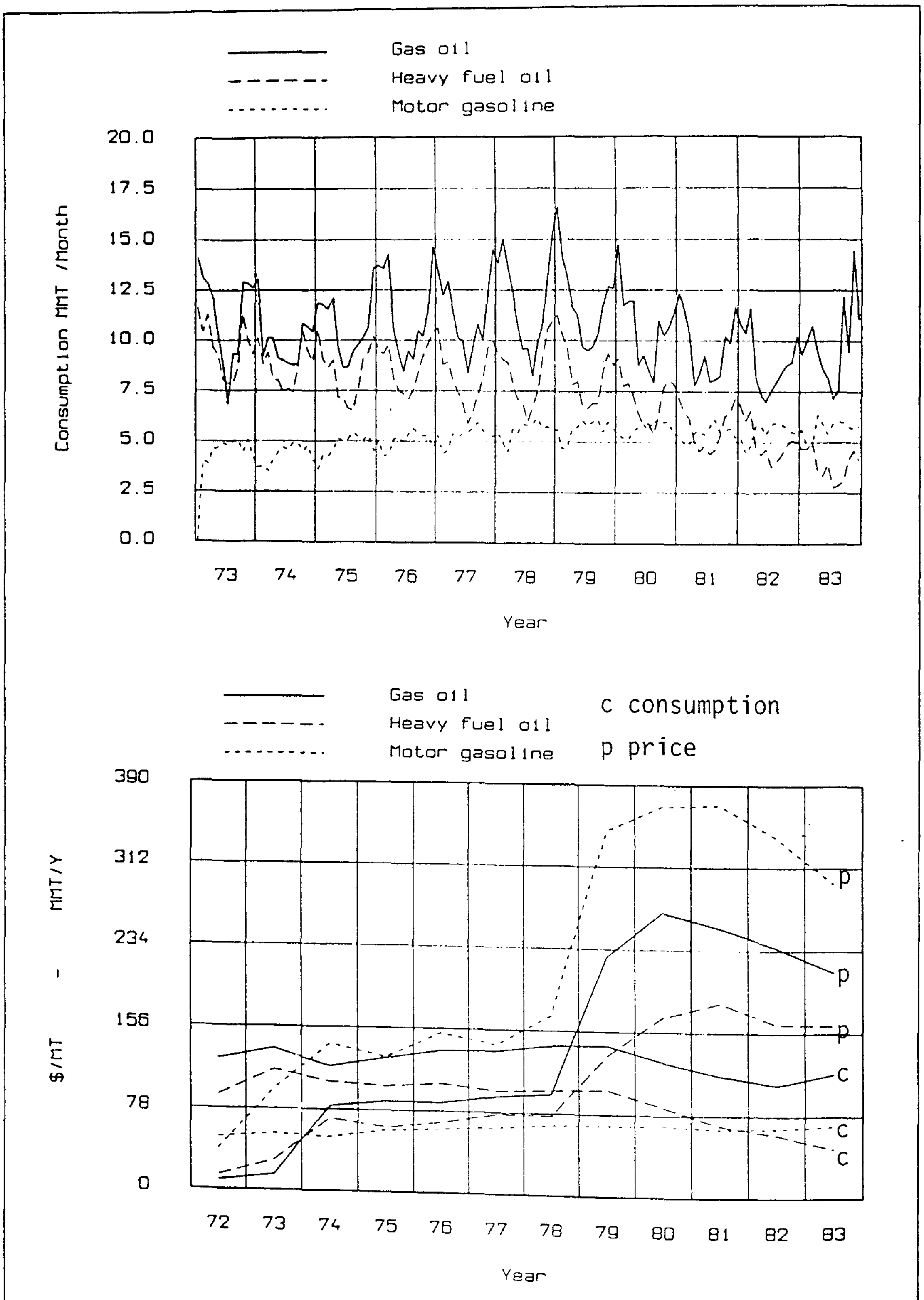


Figure 29. 1972-1983 North West Europe main oil products consumption and prices.

5.8.1 Catalytic Cracking

1 $\rho_{CC}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{hfo}$

2 $\rho_{CC}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{dfo} + \theta_3 p_{hfo}$

3 $\rho_{CC}(B) = \theta_0 + \theta_1 p_{dfo} + \theta_2 p_{hfo}$

The three models would expectedly give comparable level of rents. The reason for correlating a catalytic cracking rent with motor spirit and gas oil prices in alternative models rests on the fact that cracking in North West Europe is directed either to satisfy motor gasoline demand or gas oil depending on seasonal movements. Since the demand for middle distillates in that region is greater than the demand for light products, (see Figure 29 on page 216) the catalytic cracking rent will be clearly affected by the current level and price of middle distillates, even though premium motor gasoline is a more valuable product. Heavy fuel oils (in form of waxies) are inputs to cracking.

5.8.2 Catalytic Reforming

1 $\rho_{REF}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{ker} + \theta_3 p_{srb}$

2 $\rho_{REF}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{hfo}$

3 $\rho_{REF}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{naph} + \theta_3 p_{srb}$

Unlike the catalytic cracking unit, it is not always expected the catalytic reforming unit generates a rent; the operational flexibility of the catalytic cracking unit permits it to be easily adapted to the demand by controlling operation modes. The reformers, whose main function is to increase the octane grade of gasolines by taking in the light ends of crude distillation, have less flexibility of operation showing underutilization of capacity as an effect of the seasonality of product demands, i.e., low motor gasoline demand.

In recent years (1980 onwards) not only the seasonal effect influenced the use of reformers but the fact that less of the light crudes and more of the heavier crudes have been produced, has been causing surpluses in capacity to the extent of making refinery units either to work inefficiently or to become iddle with consequent profit losses leading to closures of excess hydroskimming capacity.

5.8.3 Hydrocracking

1 $p_{HYC}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{dfo} + \theta_3 p_{ker}$

2 $p_{HYC}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{dfo} + \theta_3 p_{naph}$

3 $p_{HYC}(B) = \theta_0 + \theta_1 p_{pms} + \theta_2 p_{dfo} + \theta_3 p_{ker} + \theta_4 p_{naph}$

The hydrocracker type in the 7-area WEM converts middle distillates and kerosines in more valuable products as gasolines, naphthas and gases, thus the three main products involved in the regression models.

Another type of hydrocracker is widely used in North West Europe, namely that having vacuum residues as feedstock to be converted mostly in middle distillates. This type is not represented in the 7-area WEM; further additions to the model should consider the introduction of such hydrocrackers specially now when the chemistry of supply is becoming a heavier one.

5.9 PARAMETRIC PROGRAMMING ON THE 7-AREA WEM

Parametric Programming is carried out on the 7-area WEM to test the effect of changing the pattern of demand in North West Europe on the level of rents.

Unlike the FD, PP is a one at a time experiment, particularly designed to measure the absolute effect on the system of varying key variables rather than measuring interactions though the information obtained from a FD indicates on which variables a further search should be conducted.

The rationale behind choosing two variables of the demand side as PP variables is explained by:

- the 2^5 7-area WEM FD showed that CCC and MGD are, as expected, key determinants of the level of rents in a refinery. Leaving CCC fixed and changing the pattern of demand has a similar effect than reducing capacity keeping demand fixed though the price profile may be altered;

- two of the refinery units under study, namely, catalytic cracking and hydrocracking, are producing gasoline and either using gas oil as feedstock (long gas oil in the catalytic cracking case) or yielding gas oil for intermediate/final consumption; and,

- the demand for gas oil in North West Europe is higher than the demand for any other product (as already seen), and the premium motor spirit is the most valuable product at Rotterdam (and has been with few exceptions).

The demand for gas oil was varied within the range 95. mmt/y - 135. mmt/y; the demand for premium motor spirit within the range 25. mmt/y - 67 mmt/y. These levels are in line with real ones. From Table D-6, Appendix D:

- the average 1973-1983 gas oil demand was 127.96 mmt/y, while the average of the 3-year period 1981-1983 was 114. mmt/y; and the gas oil average demand of the gas oil runs is 115. mmt/y;

- for premium motor spirit demands, comparisons are not direct since the demands in Table D-6 are of total motor gasoline (premium and regular). Premium motor gasoline consumption is about 66% of total motor gasoline consumption, so that the average premium motor spirit level of the premium motor spirit runs, 46. mmt/y, is in line with the average of the 3-year period 1981-1983, 44. mmt/y (i.e., .66 times 66.75 mmt/y).

In other words, the ranging levels of demands being set up for the PP runs could well correspond at any particular time to real demands in North West Europe.

5.9.1 Application to North West Europe

A summary table of the 32 price responses attached to the 32 FD treatment combinations is presented in the Appendix to Chapter 5, this is Table A5-13. Prices correspond to product final demands, those compared to the spot market prices.

Tables A5-32 to A5-34 are summary tables of product prices and refinery units marginal rents generated by the PP cases (43 in total) applied to the 7-area WEM: 21 PP runs on gas oil changes, and 22 on premium motor spirit demand changes, i.e., PP on the right hand side of the respective final demand balance equations of area B (North West Europe) of the 7-area WEM.

The information of tables A5-1 and A5-13, (FD), and A5-32 to A5-34, (PP), is used to calculate the set of estimators θ_j for every rent estimation model. The regression lines are obtained by using the Linear Regression Software (main program REGRES and soubroutine MATINV presented in section A5.6 of the Appendix to Chapter 5), and the results reported in tables 5-7 to 5-10 in following pages.

The estimation of the regression lines (parameters-θ_j) proposed in sections 5.8.1-5.8.3 is carried out on four kind of responses:

- the 2^5 7-area WEM FD responses (coefficients in Table 5-7);
- PP: changes in the demand of gas oil responses (coefficients in Table 5-8);
- PP: changes in the demand of premium motor spirit responses (coefficients in Table 5-9); and,
- all the above responses combined (coefficients in Table 5-10).

With the second kind of refinery rents models derived by using the parameters on tables 5-7 to 5-10 it is possible to estimate refinery rents from Rotterdam spot prices.

It is important to clarify the difference in product designation in this model with respect to Rotterdam's product denomination. The light crude cuts 70^{o}-150^{o} in the 7-area WEM (that is the straight-run benzine) are *naphthas* in Rotterdam; and the light crude cuts 150^{o}-185^{o} in the 7-area WEM (that is, naphtha and kerosine) represent a single product at Rotterdam, the *jetkero* or *aviation kerosine*. Hereafter, whenever the term straight-run benzine appears it refers then to naphtha in Rotterdam; and whenever naphtha and/or kero appear they refer to the single jetkero product at Rotterdam. Generally, the solutions of the 7-area WEM route naphtha to blend to aviation kerosine reflecting in the model, the use of naphtha as jetkero in Rotterdam.

Table 5-7. 2^5 7-area WEM FD: regression parameters, second kind of Refinery Rent model.

Var Pr.	Catalytic Cracking			Catalytic Reforming			Hydrocracking		
Mod	1	2	3	1	2	3	1	2	3
Int	-3.28	-1.27	2.23	-2.17	1.65	-2.32	-1.27	3.32	3.48
PMS	.55	.38		.89	.61	.89	.15	.12	.12
DFO		.5	1.36				- .29	-.58	-.50
HFO	-.72	-1.07	-1.55		-.89	.03			
KER				.013			.13		-.11
NAP								.39	.42
SRB				-1.		-1.01			

Table 5-8. 7-area WEM gas oil demand cases: regression parameters, second kind of Refinery Rent model.

Var Pr.	Catalytic Cracking			Catalytic Reforming			Hydrocracking		
Mod	1	2	3	1	2	3	1	2	3
Int	1261.7	-235.86	44.71	-416.5	7927.4	141.5	839.	311.8	332.
PMS	-6.95	1.71		1.6	-51.2	.9	-1.16	-.32	-.33
DFO		.25	.21				-2.40	-1.27	-1.4
HFO	4.73	-1.70	-.38		40.8	-.54			
KER				.5			.57		.08
NAP								.42	.40
SRB				-.75		-1.06			

Table 5-9. 7-area WEM premium motor spirit demand cases: regression parameters, second kind of Refinery Rent model.

Var Pr.	Catalytic Cracking			Catalytic Reforming			Hydrocracking		
Mod	1	2	3	1	2	3	1	2	3
Int	84.20	51.	1728.	108.4	-717.	-3.88	389.	73.91	48.8
PMS	.38	.36		.9	.9	.91	.24	.18	.18
DFO		.11	-5.81				79.77	-1.11	-8.
HFO	-.84	-.85	-1.14		2.11	.20			
KER				-.15			-80.77		6.88
NAP								.56	.58
SRB				-1.27		-1.17			

Table 5-10. 2^5 7-area WEM FD and PP cases: regression parameters, kind of Refinery Rent model.

Var Pr.	Catalytic Cracking			Catalytic Reforming			Hydrocracking		
Mod	1	2	3	1	2	3	1	2	3
Int	2.60	-.50	- 1.0	-3.48	-21.4	-2.75	5.42	.23	2.60
PMS	.57	.40		.90	.5	.91	.21	.20	.16
DFO		.45	.99				-12.60	-.77	-.29
HFO	-.81	-1.06	-1.06		-.44	.08			
KER				.08			12.16		-.56
NAP								.49	.61
SRB				-1.06		-1.06			

5.9.2 Significance of Coeefficients in the Second Kind of RR Model

The coefficients in the second kind of RR models are simple terms: since no interactions form the models, the coefficients units are basically the physical (metric tonnes) input/output exchange ratios. Because the models comprise less number of products than actually are used or jointly produced in a particular refinery unit, a particular output coefficient necessarily carries some fractional production of the other products jointly produced at the same unit. In other words, the coefficients are not the *absolute* marginal productivities of a refinery process.

With the latter in mind, the unit of a coefficient θ_j of such models is interpreted as follows:

$$\rho_u(a) = \theta_0 + \theta_1 p_{p1} + \ \ldots + \theta_j p_{pj} \qquad \text{(second kind of RR model)}$$

$$\$/\text{mt-feed} = \$/\text{mt-feed} + (\text{mt-p1/mt-feed})\text{x}(\$/\text{mt-p1}) + \ldots +$$

$$+ (\text{mt-pj/mt-feed})\text{x}(\$/\text{mt-pj}),$$

i.e., θ_j units are in mt-pj/mt-feed, the quantity in metric tonnes of product(s)-pj used or produced per metric tonne of refinery processing feed.

Since interactions are not present, the effect of a change in a product price on the rent response is directly positive or negative depending on the sign of the coefficient. As regards the coefficients of tables 5-7 to 5-10, those presented in Table 5-10 can be considered at this stage the best results for they agree with theory and behavioural assumptions.

A preliminary check for every refinery technology shows that:

Catalytic cracking: premium motor spirit and gas oil price increases bring about increases in cracking rent, while increase of the heavy fuel oil price have the opposite effect. Except in the case of gas oil demand changes (Table 5-8) the above pattern prevailed. The former exception is explained by gas oil being the force driving cracking to produce more of it at the expense of catalytic cracked spirit due to the former steadily increasing demand. As more gas oil is produced, the room is created for gas oil to blend to heavy fuel oil thus improving its usually lower value and deteriorating the premium motor spirit one as now cracking is not the factor producing gasoline but gas oil.

Catalytic reforming: for catlaytic reforming there is less general agreement. Premium motor spirit price increases have always a positive effect on reforming rents (except in one case, model 2 of Table 5-8, gas oil demand changes) and straight-run benzine, the main reforming feedstock, has a negative effect on rents when its price (hence crude price) or the demand for premium motor spirit increase. Heavy fuel oil and jetkero (kerosine) have variable effects depending on the behaviour of the demand, i.e., changing gas oil or premium motor spirit.

Hydrocracking: increases in premium motor spirit bring about increases in hydrocracking rent except in the case of gas oil demand changes (Table 5-8). Gas oil price increases have always a negative effect on hydrocracking rent except also in the same former case. Kerosine and naphtha price increases have nearly always positive effects reflecting both the use of hydrocrackates to blend to naphtha and this in turn to jetkero, and the use of gas oil as feedstock to hydrocracking rather than kerosine though both are possible hydrocracking feeds.

However no conclusion is drawn now, a verification of the models' levels of rents against the refinery rent proxies will decide on the adequacy of the particular model.

5.10 CONCLUDING REMARKS

The conclusions to be drawn at this point relate solely to the methodological and empirical aspects in estimating the RR models.

Although the experimental approaches employed in this chapter, namely a 2^5 Factorial Design and (right hand side) Parametric Programming with the 7-area WEM were designed to fulfil a common aim, that of producing sets of responses whereby to build the RR models, it is noted that the information obtained from both differs both qualitatively and quantitatively. The Factorial Design provides the basis for an extensive interpretive analysis of results as regards the international oil trade functioning is concerned. The complexities of the oil system are particularly reflected in the results and vice versa. The fact that the Arabian Light price appears always in the models obtained as an independent variable shows the price Unidimensionality extends to refinery rents by their being related to the product spot prices and these in turn to the Arabian Light spot price.

Parametric Programming results produced rather simple models whose interpretation in terms of coefficients and independent variables (product prices) is made more evident than in the Factorial Design case.

From the experimental setting up viewpoint, performing a FD or PP with a deterministic device (e.g. a LP model) entails basically the same: once the FD is designed, carrying out the experiments (i.e., applying the treatment combinations to the model) corresponds to performing PP on the set of factorial variables considered together, i.e., it is a case for Multiparametric Programming. The problem either in a FD or in PP remains as for selecting the appropriate variables of experimentation and the degree of perturbation one expects to cause in the system. As was for example noted that the Arabian Heavy supply, the second factor of the 2^5 7-area WEM FD, had no effect on rents in any refinery processing unit in any area.

The verification following in Chapter 6 will decide upon the choice of the kind of model to produce the estimated level of rents.

APPENDIX TO CHAPTER 5. EXPERIMENTAL RESULTS AND COMPUTER SOFTWARE

A5.1 2^5 7-area WEM FD Marginal Values on Refinery Capacity

Table A5-1. Area B rent responses $\rho_u(B)$ of the 2^5 7-area WEM FD.

Treatment combination		ALK	CC	REF	HYC
1	00000	0.08	5.77	8.71	0.16
2	10000	0.00	10.79	10.86	0.44
3	01000	0.17	5.76	8.84	0.08
4	11000	0.00	10.71	11.22	0.16
5	00100	6.33	8.48	9.90	2.84
6	10100	8.27	14.07	16.36	3.71
7	01100	6.31	8.44	9.86	2.77
8	11100	8.30	13.90	16.72	3.36
9	00010	5.70	8.03	11.47	1.79
10	10010	7.48	13.55	19.03	1.52
11	01010	5.74	8.04	11.21	1.94
12	11010	7.54	13.56	18.14	2.10
13	00110	28.37	17.98	21.60	6.13
14	10110	48.14	31.30	35.97	10.66
15	01110	28.25	17.84	21.66	6.02
16	11110	47.90	30.93	36.04	10.80
17	00001	0.24	7.58	0.91	3.60
18	10001	0.00	12.92	0.44	5.72
19	01001	0.16	7.29	0.48	3.37
20	11001	0.00	12.97	0.30	5.79
21	00101	6.56	9.61	4.64	4.91
22	10101	9.39	16.58	2.70	9.00
23	01101	6.71	9.73	3.52	4.95
24	11101	8.73	16.43	3.56	7.90
25	00011	7.23	10.34	3.73	5.58
26	10011	10.42	18.15	3.78	8.56
27	01011	7.30	10.51	3.63	5.19
28	11011	10.51	18.26	3.71	8.23
29	00111	31.80	23.01	20.02	2.90
30	10111	54.63	39.88	33.28	5.65
31	01111	33.15	24.27	18.12	3.50
32	11111	58.98	43.29	29.25	8.14

Table A5-2. Area J rent responses $\rho_u(J)$ of the 2^5 7-area WEM FD.

Treatment combination		CC	RD	HYC
1	00000	4.26	0.00	1.90
2	10000	8.11	0.00	5.11
3	01000	4.42	0.00	2.17
4	11000	8.27	0.00	5.56
5	00100	5.62	0.00	4.62
6	10100	9.73	0.00	8.98
7	01100	5.69	0.00	4.71
8	11100	10.41	0.00	10.12
9	00010	4.83	0.00	2.82
10	10010	8.80	0.00	6.85
11	01010	4.81	0.00	2.94
12	11010	8.70	0.00	7.01
13	00110	8.14	0.62	5.48
14	10110	14.24	1.40	10.50
15	01110	7.92	0.80	5.08
16	11110	14.05	1.64	9.61
17	00001	5.30	0.11	2.82
18	10001	10.03	0.27	6.50
19	01001	5.81	0.04	3.46
20	11001	10.68	0.20	7.41
21	00101	6.14	0.07	4.08
22	10101	11.32	0.29	8.41
23	01101	6.32	0.06	4.34
24	11101	11.51	0.26	8.69
25	00011	6.82	0.50	3.00
26	10011	13.10	0.92	6.73
27	01011	7.25	0.52	3.32
28	11011	13.59	0.99	7.02
29	00111	13.34	1.81	7.80
30	10111	23.60	3.43	13.93
31	01111	15.50	2.12	15.61
32	11111	28.19	4.14	18.70

Table A5-3. Area K rent responses $\rho_u(K)$ of the 2^5 7-area WEM FD.

Treatment combination		ALK	CC	REF	HYC
1	00000	0.00	1.76	15.99	1.06
2	10000	0.00	2.70	26.35	2.02
3	01000	0.00	1.77	15.98	1.08
4	11000	0.00	2.70	26.35	2.02
5	00100	5.68	3.18	19.25	2.68
6	10100	7.01	4.90	31.45	5.05
7	01100	5.67	3.17	19.22	2.67
8	11100	7.02	4.91	31.47	5.06
9	00010	5.12	4.47	21.99	2.75
10	10010	6.70	7.15	36.15	4.70
11	01010	5.15	4.48	22.00	2.76
12	11010	6.74	7.15	36.14	4.71
13	00110	56.36	17.69	58.20	12.22
14	10110	100.04	31.43	103.09	22.08
15	01110	56.86	17.78	58.44	12.23
16	11110	100.32	31.48	102.64	22.44
17	00001	0.00	1.88	17.31	1.10
18	10001	0.00	2.93	28.81	2.21
19	01001	0.00	1.88	16.78	1.19
20	11001	0.00	3.03	28.57	2.49
21	00101	5.90	3.66	20.24	3.80
22	10101	7.33	5.48	32.98	6.34
23	01101	5.69	3.55	20.00	3.70
24	11101	7.62	5.46	32.84	6.20
25	00011	6.03	5.24	23.48	3.62
26	10011	7.45	8.37	38.63	7.08
27	01011	6.06	5.25	23.48	3.64
28	11011	7.86	8.40	38.93	6.88
29	00111	59.28	18.99	61.20	4.27
30	10111	105.07	33.61	108.08	25.58
31	01111	66.80	21.25	67.70	15.61
32	11111	118.85	38.06	120.46	28.98

Table A5-4. Area M rent responses $\rho_u(M)$ of the 2^5 7-area WEM FD.

Treatment combination		ALK	CC	REF
1	00000	0.00	0.58	8.49
2	10000	0.00	0.14	12.14
3	01000	0.00	0.57	8.47
4	11000	0.00	0.00	11.61
5	00100	2.56	1.30	11.26
6	10100	0.00	0.69	13.77
7	01100	2.84	1.36	11.45
8	11100	0.00	0.60	13.64
9	00010	2.08	1.45	11.23
10	10010	0.00	1.05	14.84
11	01010	2.10	1.46	11.25
12	11010	0.65	1.28	15.47
13	00110	29.90	13.34	30.13
14	10110	53.24	24.00	52.51
15	01110	29.66	12.96	30.04
16	11110	54.29	23.75	53.69
17	00001	1.92	1.74	11.22
18	10001	0.36	1.92	16.02
19	01001	1.57	1.38	10.40
20	11001	0.00	1.57	14.70
21	00101	3.78	1.93	11.50
22	10101	3.60	2.20	16.17
23	01101	4.92	2.16	12.06
24	11101	3.59	2.15	15.10
25	00011	1.67	1.46	9.73
26	10011	0.78	1.56	14.32
27	01011	1.68	1.33	9.75
28	11011	0.68	1.40	14.28
29	00111	53.06	22.79	42.01
30	10111	90.07	38.66	71.87
31	01111	63.73	26.14	49.17
32	11111	114.74	45.87	89.74

Table A5-5. Area T rent responses $\rho_u(T)$ of the 2^5 7-area WEM FD.

Treatment combination		ALK	CC	REF	HYC
1	00000	0.00	5.18	8.84	0.00
2	10000	0.00	8.85	12.09	0.00
3	01000	0.00	5.17	8.86	0.00
4	11000	0.00	8.83	12.16	0.00
5	00100	3.32	6.20	11.03	0.00
6	10100	3.68	10.58	17.72	0.00
7	01100	3.31	6.16	10.98	0.00
8	11100	4.01	10.54	17.57	0.00
9	00010	2.91	6.49	12.00	0.00
10	10010	3.08	11.23	19.06	0.00
11	01010	2.92	6.52	11.88	0.00
12	11010	3.07	11.29	18.71	0.00
13	00110	24.12	14.53	22.72	2.41
14	10110	40.68	25.06	38.40	4.23
15	01110	24.57	14.71	22.37	2.56
16	11110	41.09	25.15	38.29	4.56
17	00001	0.00	4.48	3.22	0.00
18	10001	0.00	7.89	3.59	0.00
19	01001	0.00	4.59	2.53	0.00
20	11001	0.00	7.97	3.74	0.00
21	00101	3.10	6.32	6.56	0.99
22	10101	2.94	10.71	7.36	1.66
23	01101	2.99	6.56	5.88	0.82
24	11101	2.78	11.14	7.29	1.06
25	00011	3.07	6.95	6.55	1.12
26	10011	3.51	12.31	8.24	1.13
27	01011	2.95	7.25	6.50	0.63
28	11011	3.71	12.29	7.87	0.92
29	00111	23.74	12.81	17.12	6.22
30	10111	40.93	22.28	29.06	10.99
31	01111	24.68	13.41	15.33	6.75
32	11111	44.37	24.25	25.58	12.74

Table A5-6. Areas U and V rent responses $\rho_u(U)$ and $\rho_u(V)$ of the 2^5 7-area WEM FD.

Treatment combination		Rents area U CC	Rents area U HYC	Rents area V CC	Rents area V HYD
1	00000	0.11	5.38	0.59	2.16
2	10000	0.41	10.29	0.98	3.66
3	01000	0.12	5.41	0.75	1.73
4	11000	0.41	10.29	0.99	3.20
5	00100	2.15	8.77	0.95	4.54
6	10100	3.89	15.86	0.19	10.05
7	01100	2.15	8.75	0.97	4.86
8	11100	3.89	15.87	0.01	10.35
9	00010	4.08	11.15	1.43	1.04
10	10010	7.22	19.92	1.86	2.45
11	01010	4.09	11.16	1.25	1.69
12	11010	7.21	19.91	1.69	3.58
13	00110	10.66	25.54	14.79	14.87
14	10110	18.95	45.67	27.48	26.43
15	01110	10.66	25.55	14.59	14.45
16	11110	18.96	45.70	28.13	26.54
17	00001	0.00	6.46	0.00	5.40
18	10001	0.24	12.82	0.00	8.21
19	01001	0.00	6.47	0.00	4.89
20	11001	0.43	13.44	0.00	7.56
21	00101	2.87	11.55	0.00	6.84
22	10101	4.66	19.95	0.00	8.64
23	01101	2.72	11.30	0.00	10.03
24	11101	4.55	19.74	0.00	8.83
25	00011	4.40	12.09	0.00	4.50
26	10011	7.42	22.08	0.00	7.16
27	01011	4.41	12.09	0.00	4.50
28	11011	7.36	22.05	0.00	7.11
29	00111	12.51	31.13	23.82	19.57
30	10111	21.97	54.89	42.43	34.27
31	01111	12.92	32.38	28.87	22.93
32	11111	23.32	58.58	54.21	42.66

A5.2 Estimated Coefficients of the First Kind of RR model

Table A5-7. Alkylation 90% SS of regression.

1 * MARGINAL RENT REFINERY UNIT: ALKYLATION * $/TON * AREAS B K M T *

PARTIALLY CODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	14.199687	23.956562	16.358750	9.860625
X1	3.118437	6.169062		
X3	10.299063	20.762187	15.515000	8.284375
X4	10.371563	20.711562	14.787500	8.226875
X1.X4			4.394375	
X3.X4	6.542187	17.517187	14.425000	6.650625
X1.X3.X4			4.536875	
X5			5.276250	
X3.X5			5.036250	
X3.X4.X5			4.621250	
RESIDUAL SS	756.514893	3374.601807	2226.607178	623.277100

PARTIALLY DECODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	-3830.773330	-10077.973304	-132273.419150	-3828.695516
ALP	0.101871	0.109380	-73.793555	
MGD	7.422984	19.357274	247.757977	7.379915
ALP.MGD			0.141299	
CCC	8.157935	22.067605	291.242098	8.366706
ALP.CCC			0.176399	
MGD.CCC	-0.015966	-0.042483	-0.544382	-0.016129
ALP.MGD.CCC			-0.000338	
SHC			729.155296	
MGD.SHC			-1.365731	
CCC.SHC			-1.614009	
MGD.CCC.SHC			0.003017	
SE	5.293303	11.179680	9.839157	4.718019

Table A5-8. Catalytic cracking 90% SS of regression.

2 * MARGINAL RENT REFINERY UNIT: CATALYTIC CRACKING * $/TON * AREAS B J K M T U V *

PARTIALLY CODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
GRAND MEAN	15.624062	9.890625	9.805000	7.462187	10.865625	6.398125	7.686875
X1	4.206562	2.880000	2.555000		2.907500	1.782500	2.186250
X3	4.734687	2.091875	5.482500	6.281562	2.910000	3.403750	7.090625
X4	4.934688	2.164375	6.495000	6.194062	1.292500	4.610625	7.347500
X1.X4				1.824687			2.254375
X3.X4	1.269063	1.475625	4.503750	6.000937	1.956875	1.831250	7.165000
X1.X3.X4							2.239375
X5	1.927188	1.890625		2.179063			
X3.X5				1.814688			2.242500
X4.X5		1.228125					
X3.X4.X5		0.799375		1.866563			2.168125
RESIDUAL SS	211.965138	69.086594	289.809570	436.500078	67.118042	109.548569	475.193604

PARTIALLY DECODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
INTERCEPT	-1807.122530	-21219.671743	-2581.297064	-56663.647670	-1136.757035	-1071.647838	-61941.168010
ALP	0.129135	0.088411	0.078434	0.744207	0.009256	0.054720	-36.322148
MGD	1.678135	41.388114	4.987875	106.110338	2.207535	2.117788	115.772364
ALP.MGD							0.069744
CCC	4.084297	50.740445	5.636527	126.429952	2.434125	2.229287	136.646517
ALP.CCC				-0.001779			0.086987
MGD.CCC	-0.007928	-0.098554	-0.010922	-0.236324	-0.004746	-0.004441	-0.255399
ALP.MGD.CCC							-0.000167
SHC	-0.518828	111.891676		292.085142			341.436716
MGD.SHC		-0.218332		-0.547118			-0.638278
CCC.SHC		-0.268688		-0.651912			-0.757235
MGD.CCC.SHC		0.000522		0.001219			0.001416
SE	2.855259	1.696646	3.276231	4.265071	1.576659	2.014287	4.545391

Table A5-9. Hydrofining 90% SS of regression.

```
-----------------------------------------------------------------------
4   *        MARGINAL  RENT  REFINERY UNIT:  HYDROFINING
-----------------------------------------------------------------------


   ------------------------------------------------------------
      PARTIALLY   CODED    MODEL   FOR 90% SUM OF SQUARES
   ------------------------------------------------------------


   VARIABLE                AREA  V


   GRAND MEAN             10.459375
   X1                      2.709375
   X3                      6.156875
   X1.X3                   1.645625
   X4                      4.150000
   X3.X4                   4.448750
   X5                      2.234375

   RESIDUAL SS           282.729492


   ------------------------------------------------------------
      PARTIALLY  DECODED   MODEL   FOR 90% SUM OF SQUARES
   ------------------------------------------------------------


   VARIABLE                AREA  V


   INTERCEPT           -2258.271212
   ALP                    -1.980423
   MGD                     4.530586
   ALP.MGD                 0.003858
   CCC                     5.639649
   MGD.CCC                -0.010789
   SHC                    -0.601528

   SE                      3.362912
```

Table A5-10. Catalytic reforming 90% SS of regression.

6 * MARGINAL RENT REFINERY UNIT: CATALYTIC REFORMING *

PARTIALLY CODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	12.489375	40.756250	22.438437	13.721875
X1	2.595625	10.677500	5.053437	2.948750
X3	5.210625	14.697500	10.318438	4.606875
X4	5.675625	16.781875	10.063438	5.008125
X1.X4			3.284688	
X3.X4	3.616875	12.740625	9.574687	2.771875
X1.X3.X4			3.218437	
X5	-4.235000			-3.945625
X3.X4.X5			3.064063	
RESIDUAL SS	320.494141	2077.257080	1208.483398	212.554871

PARTIALLY DECODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	-2304.699620	-7289.611441	-80807.559691	-1816.179208
ALP	0.079682	0.327782	-52.125414	0.090522
MGD	4.067347	14.048164	151.024428	3.163960
ALP.MGD			0.100237	
CCC	4.511989	15.995682	192.168615	3.436953
ALP.CCC			0.124974	
MGD.CCC	-0.008772	-0.030898	-0.359132	-0.006722
ALP.MGD.CCC			-0.000240	
SHC	1.140127		447.677457	1.062223
MGD.SHC			-0.836884	
CCC.SHC			-1.070149	
MGD.CCC.SHC			0.002001	
SE	3.510940	8.771285	7.096018	2.859228

Table A5-11. Residue desulphurization 90% SS of regression.

7 * MARGINAL RENT REFINERY UNIT: RESIDUE DESULPHURIZATION

PARTIALLY CODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA J
GRAND MEAN	0.630937
X1	0.215313
X3	0.409062
X1.X3	0.139688
X4	0.549687
X1.X4	0.169063
X3.X4	0.405312
X5	0.352188
X4.X5	0.270938
RESIDUAL SS	2.622815

PARTIALLY DECODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA J
INTERCEPT	-23.443737
ALP	-0.099604
MGD	0.403990
ALP.MGD	0.000327
CCC	0.106164
ALP.CCC	-0.000165
MGD.CCC	-0.000983
SHC	-1.063892
CCC.SHC	0.002317
SE	0.337691

Table A5-12. Hydrocracking 90% SS of regression.

9 * MARGINAL COST REFINERY UNIT: HYDROCRACKING * $/TON * AREAS B J K T U *

PARTIALLY CODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA T	AREA U
GRAND MEAN	4.608417	6.546562	7.444375	1.837187	19.757500
X1	1.125112	2.274062	2.170625	0.493438	5.683750
X3	1.219062	1.882813	4.362500	1.599687	7.194375
X1.X3				0.474688	
X4	0.935937	0.991563	4.402500	1.554063	8.360625
X3.X4		0.694063	2.966875	1.316563	4.617500
X5	1.233438	0.705313		0.977187	
X3.X5	-1.162187			0.739687	
X4.X5	-0.779062	0.546563		0.694063	
X3.X4.X5	-0.939687	0.859062			
RESIDUAL SS	26.365082	31.102936	176.001068	26.564392	553.126953

PARTIALLY DECODED MODEL FOR 90% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA T	AREA U
INTERCEPT	26825.094193	-22892.360513	-1730.244653	-1669.896651	-2689.497643
ALP	0.034545	0.069810	0.066635	-0.580105	0.174482
MGD	-50.959947	43.545871	3.343133	4.094393	5.234001
ALP.MGD				0.001113	
CCC	-58.541057	54.617747	3.709165	0.578750	5.724846
MGD.CCC	0.111646	-0.103750	-0.007195	-0.003193	-0.011198
SHC	-147.611986	123.369256		5.388842	
MGD.SHC	0.280548	-0.234635		-0.015206	
CCC.SHC	0.321533	-0.295361		0.005934	
MGD.CCC.SHC	-0.000614	0.000561			
SE	1.043115	1.138400	2.553146	1.074697	4.526166

Table A5-13. Alkylation 95% SS of regression.

1 * MARGINAL RENT REFINERY UNIT: ALKYLATION * $/TON * AREAS B K M T *

PARTIALLY CODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	14.199687	23.956562	16.350750	9.860625
X1	3.318437	6.169062		2.255625
X3	10.289063	20.762187	15.515000	8.284375
X1.X3	2.735312	5.769687	4.300625	
X4	10.371563	20.711562	14.787500	8.226875
X1.X4	2.810313	5.791562	4.394375	2.211875
X3.X4	6.542187	17.517187	14.425000	6.650625
X1.X3.X4			4.536875	
X5			5.276250	
X3.X5			5.036250	
X4.X5			4.380000	
X3.X4.X5			4.621250	
RESIDUAL SS	264.363525	1235.995117	1020.854736	303.910400

PARTIALLY DECODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	-3562.591908	-9505.760204	-128789.277137	-3942.668348
ALP	-2.181983	-4.683630	-79.186492	0.971368
MGD	6.670043	17.769070	246.574157	7.379915
ALP.MGD	0.006412	0.013525	0.151381	
CCC	8.479651	22.730647	284.427227	8.619931
ALP.CCC	-0.002740	-0.005647	0.176399	-0.002156
MGD.CCC	-0.015866	-0.042483	-0.544382	-0.016129
ALP.MGD.CCC			-0.000338	
SHC			713.489106	
MGD.SHC			-1.365731	
CCC.SHC			-1.576560	
MGD.CCC.SHC			0.003017	
SE	3.251852	7.031344	6.972239	3.418898

Table A5-14. Catalytic cracking 95% SS of regression.

2 * MARGINAL RENT REFINERY UNIT: CATALYTIC CRACKING * $/TON * AREAS B J K M T U V *

PARTIALLY CODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
GRAND MEAN	15.624062	9.890625	9.805000	7.462187	10.865625	6.398125	7.686875
X1	4.206562	2.880000	2.555000	1.715312	2.907500	1.782500	2.186250
X3	4.734687	2.091875	5.482500	6.281562	2.910000	3.403750	7.090625
X1.X3	1.232188		1.573750	1.780937			2.092500
X4	4.934688	2.164375	6.495000	6.194062	3.292500	4.610625	7.347500
X1.X4	1.349688	0.598750	1.851250	1.824687	0.916875	1.260000	2.254375
X3.X4	3.269063	1.475625	4.503750	6.000937	1.956875	1.831250	7.165000
X1.X3.X4				1.010312			2.239375
X5	1.927188	1.890625		2.179063			
X1.X5		0.591250					
X3.X5		0.616875		1.814688			2.242500
X4.X5		1.228125					1.985625
X3.X4.X5		0.799375		1.866563			2.168125
RESIDUAL SS	105.087021	34.251007	100.889130	136.061249	40.216934	58.745377	208.914734

PARTIALLY DECODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
INTERCEPT	-1690.323023	-22587.259257	-2438.223588	-52866.004608	-1180.668316	-1131.992174	-60340.634503
ALP	-0.865539	1.221819	-1.139990	-31.596810	0.463207	0.568617	-38.946121
MGD	3.338954	43.695870	4.554674	99.019503	2.207535	2.117788	115.196368
ALP.MGD	0.002888		0.003689	0.060556			0.074650
CCC	4.238814	50.808992	5.848465	117.963985	2.539092	2.373537	133.557070
ALP.CCC	-0.001316	-0.000584	-0.001805	0.070318	-0.000894	-0.001228	0.086987
MGD.CCC	-0.007928	-0.098554	-0.010922	-0.220498	-0.004746	-0.004441	-0.255399
ALP.MGD.CCC				-0.000135			-0.000167
SHC	-0.518828	119.249286		292.085342			334.334620
ALP.SHC		-0.004886					
MGD.SHC		-0.231014		-0.547118			-0.638278
CCC.SHC		-0.268688		-0.651912			-0.740258
MGD.CCC.SHC		0.000522		0.001219			0.001416
SE	2.092517	1.277106	2.008871	2.545409	1.243706	1.503142	3.154096

Table A5-15. Hydrofining 95% SS of regression.

4 * MARGINAL RENT REFINERY UNIT: HYDROFINING

PARTIALLY CODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA V
GRAND MEAN	10.459375
X1	2.709375
X3	6.156875
X1.X3	1.645625
X4	4.150000
X1.X4	1.456250
X3.X4	3.448750
X1.X3.X4	1.448750
X5	2.234375
RESIDUAL SS	147.704514

PARTIALLY DECODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA V
INTERCEPT	506.230875
ALP	-25.523133
MGD	-0.767724
ALP.MGD	0.048978
CCC	-0.968745
ALP.CCC	0.056278
MGD.CCC	0.001876
ALP.MGD.CCC	-0.000108
SHC	-0.601528
SE	2.534154

Table A5-16. Catalytic reforming 95% SS of regression.

6 * MARGINAL RENT REFINERY UNITS CATALYTIC REFORMING * $/TON * AREAS B K M T *

PARTIALLY CODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	12.489375	40.756250	22.438437	13.721875
X1	2.595625	10.677500	5.053437	2.948750
X3	5.210625	14.697500	10.318438	4.606875
X1.X3	1.439375		3.000938	1.381250
X4	5.675625	16.781875	10.063438	5.008125
X1.X4	1.639375	4.799375	3.284688	1.472500
X3.X4	3.616875	12.740625	9.574687	2.771975
X1.X3.X4			3.218437	
X5	-4.215000		3.064063	-3.945625
X3.X4.X5			3.064063	
RESIDUAL SS	168.196121	1340.169189	619.873291	82.119476

PARTIALLY DECODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	-2171.265630	-7519.464696	-80215.561711	-1683.312071
ALP	-1.056652	2.285230	-55.888557	-1.040985
MGD	3.671134	14.048164	150.198369	2.783747
ALP.MGD	0.003374		0.107272	0.003238
CCC	4.699671	16.545134	192.168615	3.605531
ALP.CCC	-0.001598	-0.004679	0.124974	-0.001436
MGD.CCC	-0.008772	-0.030898	-0.359132	-0.006722
ALP.MGD.CCC			-0.000240	
SHC	1.140127		446.852565	1.062223
MGD.SHC			-0.836884	
CCC.SHC			-1.070149	
MGD.CCC.SHC			0.002001	
SE	2.647295	7.179482	5.308112	1.849769

Table A5-17. Residue desulphurization 95% SS of regression.

```
----------------------------------------------------------------------------------------
7   *       MARGINAL  RENT  REFINERY UNIT:  RESIDUE DESULPHURIZATION      *  $/TON  *   AREAS J
----------------------------------------------------------------------------------------


 ---------------------------------------------------
    PARTIALLY   CODED    MODEL  FOR 95% SUM OF SQUARES
 ---------------------------------------------------


 VARIABLE              AREA  J


 GRAND MEAN             0.630937
 X1                     0.215313
 X3                     0.409062
 X1.X3                  0.139688
 X4                     0.549687
 X1.X4                  0.169063
 X3.X4                  0.405312
 X1.X3.X4               0.133438
 X5                     0.352188
 X3.X5                  0.130313
 X4.X5                  0.270938

 RESIDUAL SS            1.509639


 ---------------------------------------------------
    PARTIALLY  DECODED   MODEL  FOR 95% SUM OF SQUARES
 ---------------------------------------------------


 VARIABLE              AREA  J


 INTERCEPT            -23.177670
 ALP                   -2.322716
 MGD                    0.403492
 ALP.MGD                0.004483
 CCC                   -0.517860
 ALP.CCC                0.005149
 MGD.CCC                0.000184
 ALP.MGD.CCC           -0.000010
 SHC                    0.369166
 MGD.SHC               -0.002679
 CCC.SHC                0.002317

 SE                     0.268119
```

Table A5-18. Hydrocracking 95% SS of regression.

9 * MARGINAL RENT REFINERY UNIT: HYDROCRACKING * $/TON * AREAS D J K T U *

PARTIALLY CODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA D	AREA J	AREA K	AREA T	AREA U
STAND MEAN	[illegible]	6.546562	7.444375	1.817187	19.757500
X1	1.125312	2.274062	2.170625	0.493438	5.683750
X2		0.325938			
X3	1.219062	1.982813	4.362500	1.599687	7.194375
X1.X3	0.449687	0.818063	1.238750	0.474688	
X4	0.915937	0.991563	4.402500	1.554063	8.360625
X1.X4			1.288750	0.436563	2.298125
X3.X4		0.694063	2.966875	1.316563	4.617500
X5	1.273438	0.705313		0.977187	1.931250
X1.X5	0.416563				
X3.X5	-1.162187	0.336563		0.739687	
X4.X5	-0.779062	0.546563		0.694063	
X3.X4.X5	-0.939687	0.859062		0.456563	
RESIDUAL SS	13.795410	18.592361	73.749039	13.795307	264.773438

PARTIALLY DECODED MODEL FOR 95% SUM OF SQUARES

VARIABLE	AREA D	AREA J	AREA K	AREA T	AREA U
INTERCEPT	26814.213559	-23508.507926	-1609.560377	-13829.807440	-2704.946674
ALP	0.127206	-0.449420	-0.961122	-0.402051	1.111783
ANS		0.074060			
MGD	-51.083732	44.690988	3.002146	26.786898	5.234001
ALP.MGD	0.001054	0.000971	0.002904	0.001113	
CCC	-58.541057	54.617747	3.856706	29.646357	5.987945
ALP.CCC			-0.001256	-0.000426	-0.002241
MGD.CCC	0.111646	-0.103750	-0.007195	-0.057438	-0.011198
SHC	-147.188322	127.070463		72.095295	-0.519922
ALP.SHC	-0.003608				
MGD.SHC	0.280548	-0.241554		-0.139907	
CCC.SHC	0.321533	-0.295361		-0.153524	
MGD.CCC.SHC	-0.000614	0.000561		0.000298	
SE	0.791874	0.940431	1.717545	0.810506	3.254372

Table A5-19. Alkylation 98% SS of regression.

1 • REGIONAL BENT REFINERY UNIT: ALKYLATION • $/TON • AREAS S K M T •

PARTIALLY CODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA S	AREA K	AREA M	AREA T
GRAND MEAN	14.199687	23.956562	16.358750	9.860625
X1	1.318437	6.169062	3.766875	2.255625
X3	10.289063	20.762187	15.515000	8.284375
X1.X3	2.735312	5.769687	4.300625	2.160625
X4	10.371563	20.711562	14.787500	8.226875
X1.X4	2.810313	5.791562	4.394375	2.211875
X3.X4	6.542187	17.517187	14.425000	6.650625
X1.X3.X4	2.145937	5.392187	4.536875	2.116875
X5			5.276250	
X3.X5			5.036250	
X1.X3.X5			1.419375	
X4.X5			4.380000	
X3.X4.X5			4.621250	
RESIDUAL SS	117.002411	305.574463	502.328857	11.129590

PARTIALLY DECODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA S	AREA K	AREA M	AREA T
INTERCEPT	635.588081	1043.183399	-118563.651465	516.806854
ALP	-37.934003	-94.519237	-166.268691	-37.005863
MGD	-1.177993	-1.951020	227.433100	-0.956583
ALP.MGD	0.073297	0.181463	0.314387	0.070994
CCC	-1.555874	-2.486039	284.427227	-1.279683
ALP.CCC	0.082723	0.209101	0.176399	0.082149
MGD.CCC	0.002894	0.004657	-0.544382	0.002377
ALP.MGD.CCC	-0.000160	-0.000401	-0.000338	-0.000158
SHC			657.222444	
ALP.SHC			0.479171	
MGD.SHC			-1.260547	
ALP.MGD.SHC			-0.000896	
CCC.SHC			-1.576560	
MGD.CCC.SHC			0.003017	
SE	2.207963	3.568231	5.141824	0.680979

Table A5-20. Catalytic cracking 98% SS of regression.

2 * MARGINAL RENT REFINERY UNIT: CATALYTIC CRACKING * $/TON * AREAS B J K M T U V *

PARTIALLY CODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
GRAND MEAN	15.624062	9.890625	9.805000	7.462187	10.865625	6.398125	7.686875
X1	4.206562	2.880000	2.555000	1.715312	2.907500	1.782500	2.186250
X3	4.734687	2.091875	5.482500	6.281562	2.910000	3.403750	7.090625
X1.X3	1.232188	0.518750	1.573750	1.780937	0.780625	0.939375	2.092500
X4	4.934688	2.164375	6.495000	6.194062	3.292500	4.610625	7.347500
X1.X4	1.349688	0.598750	1.851250	1.824687	0.916875	1.260000	2.254375
X3.X4	3.269063	1.475625	4.503750	6.000937	1.956875	1.831250	7.165000
X1.X3.X4	0.999063	0.400000	1.378750	1.810312		0.574375	2.239375
X5	1.927188	1.890625		2.179063			1.646250
X1.X5		0.591250					
X3.X5		0.616875		1.814688			2.242500
X4.X5	0.977812	1.228125		1.565938			1.985625
X1.X4.X5		0.376250					
X3.X4.X5		0.799375		1.866563			2.168125
RESIDUAL SS	42.551270	15.989794	40.058685	57.592133	20.717102	19.950882	122.190292

PARTIALLY DECODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
INTERCEPT	900.624856	-22611.127825	259.078016	-51846.759328	-1065.721550	130.002359	-60259.983357
ALP	-17.510247	1.425079	-24.110416	-31.596810	-0.515688	-10.178625	-38.946121
MGD	-0.319777	42.090212	-0.487635	99.019503	1.992655	-0.241372	115.196368
ALP.MGD	0.034004	0.013674	0.046630	0.060556	0.001830	0.020091	0.074650
CCC	-1.954709	51.048644	-0.599290	115.527533	2.539092	-0.312541	133.557070
ALP.CCC	0.038472	-0.002625	0.053105	0.070318	-0.000894	0.021646	0.086987
MGD.CCC	0.000806	-0.095057	0.001131	-0.220498	-0.004746	0.000580	-0.255399
ALP.MGD.CCC	-0.000074	-0.000030	-0.000103	-0.000135		-0.000043	-0.000167
SHC	-4.016225	124.100408		286.484366			333.891424
ALP.SHC		-0.046199					
MGD.SHC		-0.231014		-0.547118			-0.638278
CCC.SHC	0.009360	-0.280284		-0.638523			-0.740258
ALP.CCC.SHC		0.000099					
MGD.CCC.SHC		0.000522		0.001219			0.001416
SE	1.390737	0.942508	1.291941	1.696940	0.910321	0.911749	2.471744

Table A5-21. Hydrofining 98% SS of regression.

* *	MARGINAL RENT REFINERY UNIT: HYDROFINING	* $/TON *	AREAS V *

PARTIALLY CODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA V
GRAND MEAN	10.459375
X1	2.709375
X3	6.156875
X1.X3	1.645625
X4	4.150000
X1.X4	1.456250
X3.X4	4.448750
X1.X3.X4	1.448750
X5	2.234375
X4.X5	0.993750
X1.X4.X5	0.645000
X3.X4.X5	1.043750
X1.X3.X4.X5	0.678750
RESIDUAL SS	53.186798

PARTIALLY DECODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA V
INTERCEPT	36941.836291
ALP	-566.633354
MGD	-70.500047
ALP.MGD	1.094616
CCC	-88.066153
ALP.CCC	1.349772
MGD.CCC	0.168567
ALP.MGD.CCC	-0.002583
SHC	-200.323202
ALP.SHC	2.973518
MGD.SHC	0.333194
ALP.MGD.SHC	-0.005691
CCC.SHC	0.478619
ALP.CCC.SHC	-0.007108
MGD.CCC.SHC	-0.000916
ALP.MGD.CCC.SHC	0.000014
SS	1.673112

Table A5-22. Catalytic reforming 98% SS of regression.

6 • MARGINAL RENT REFINERY UNIT: CATALYTIC REFORMING • $/TON • AREAS B K M T •

PARTIALLY CODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	12.409375	40.756250	22.438437	13.721875
X1	2.595625	10.677500	5.053437	2.948750
X3	5.210625	14.647500	10.318438	4.606875
X1.X3	1.439375	4.245000	3.000938	1.381250
X4	5.675625	16.781875	10.063438	5.008125
X1.X4	1.639375	4.799375	3.284688	1.472500
X3.X4	3.616875	12.740625	9.574687	2.771875
X1.X3.X4	0.968125	3.869375	3.218437	
X5	-4.235000		3.064063	-3.945625
X1.X5	-1.222500		0.969063	-1.133750
X3.X5			2.631563	
X4.X5			2.042813	
X3.X4.X5	0.978750		3.064063	
RESIDUAL SS	51.724991	284.420316	234.681702	40.987030

PARTIALLY DECODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	-26084.223362	675.417108	-84323.374626	-1483.091747
ALP	-19.024498	-67.503008	-54.431146	-2.746076
MGD	48.777313	-1.271266	160.043156	2.783747
ALP.MGD	0.033526	0.130461	0.107272	0.003238
CCC	62.378379	-1.550086	188.990190	3.605531
ALP.CCC	0.036958	0.149421	0.124974	-0.001436
MGD.CCC	-0.116596	0.002929	-0.359132	-0.006722
ALP.MGD.CCC	-0.000072	-0.000288	-0.000240	
SHC	142.954845		469.425879	-0.038032
ALP.SHC	0.010103		-0.008009	0.009370
MGD.SHC	-0.267325		-0.890983	
CCC.SHC	-0.341836		-1.052683	
MGD.CCC.SHC	0.000639		0.002001	
SE	1.686410	3.442530	3.514494	1.334932

Table A5-23. Residue desulphurization 98% SS of regression.

7 * MARGINAL RENT REFINERY UNIT: RESIDUE DESULPHURIZATION * $/TON * AREAS J

PARTIALLY CODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA J
GRAND MEAN	0.630937
X1	0.215313
X3	0.409062
X1.X3	0.139688
X4	0.549687
X1.X4	0.169063
X3.X4	0.405312
X1.X3.X4	0.133438
X5	0.352188
X1.X5	0.114063
X3.X5	0.130313
X4.X5	0.270938
X3.X4.X5	0.126563
RESIDUAL SS	0.580734

PARTIALLY DECODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA J
INTERCEPT	-3408.342423
ALP	-2.151173
MGD	6.694022
ALP.MGD	0.004483
CCC	7.526041
ALP.CCC	0.005149
MGD.CCC	-0.014854
ALP.MGD.CCC	-0.000010
SHC	18.971381
ALP.SHC	-0.000943
MGD.SHC	-0.037247
CCC.SHC	-0.041886
MGD.CCC.SHC	0.000083
SE	0.174828

Table A5-24. Hydrocracking 98% SS of regression.

9 * PRINCIPAL RENT REFINERY UNIT: HYDROCRACKING * $/TON * AREAS B J K T U *

PARTIALLY CODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA T	AREA U
GRAND MEAN	4.608437	6.546562	7.448375	1.837187	19.757500
X1	1.125312	2.274062	2.170625	0.493438	5.683750
X2		0.125938			
X3	1.219062	1.882813	4.362500	1.599687	7.194375
X1.X3	0.449687	0.414063	1.238750	0.474688	1.896875
X4	0.935937	0.491563	4.402500	1.554063	8.360625
X1.X4	0.287813	0.231563	1.288750	0.436563	2.298125
X3.X4		0.694063	2.966875	1.316563	4.617500
X1.X3.X4	0.224688		0.895625	0.417813	1.401250
X5	1.203438	0.705313	0.848750	0.977187	1.931250
X1.X5	0.436563			0.254688	
X2.X5		0.267188			
X3.X5	-1.162187	0.336563		0.719687	
X1.X3.X5	-0.207812				
X4.X5	-0.779062	0.546563		0.694063	
X2.X4.X5		0.258438			
X3.X4.X5	-0.939687	0.859062		0.456563	
X1.X3.X4.X5	-0.295312	0.257813			
X2.X3.X4.X5		0.289688			
RESIDUAL SS	5.356500	7.642419	25.028442	6.133452	86.801697

PARTIALLY DECODED MODEL FOR 98% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA T	AREA U
INTERCEPT	-2562.665967	83919.261927	184.166188	-13057.402620	315.686744
ALP	250.303031	-210.782410	-15.882527	-6.979910	-24.612161
ABS		-1708.823608			
RGD	3.807350	-159.882261	-0.273297	25.258891	-0.412741
ALP.RGD	-0.466402	0.394341	0.030798	0.014125	0.048088
ABS.RGD		3.271604			
CCC	0.099208	-202.526711	-0.331704	27.692447	-0.565033
ALP.CCC	-0.567514	0.502790	0.034412	-0.016214	0.053565
ABS.CCC		4.092137			
RGD.CCC	-0.012869	0.385271	0.000635	-0.053786	0.001052
ALP.RGD.CCC	0.001060	-0.000940	-0.000067	-0.000031	-0.000104
ABS.RGD.CCC		-0.007821			
SHC	16.583814	-463.329533	-0.228496	72.342458	-0.519922
ALP.SHC	-1.398304	1.156344		-0.002105	
ABS.SHC		9.390763			
RGD.SHC	-0.025606	0.802621		-0.139907	
ALP.RGD.SHC	0.002607	-0.002162			
ABS.RGD.SHC		-0.017978			
CCC.SHC	-0.050261	1.117849		-0.153524	
ALP.CCC.SHC	0.003166	-0.002764			
ABS.CCC.SHC		-0.022487			
RGD.CCC.SHC	0.000082	-0.002126		0.000298	
ALP.RGD.CCC.SHC	-0.000006	0.000005			
ABS.RGD.CCC.SHC		0.000043			
SE	0.545512	0.691123	1.043165	0.568167	1.942676

Table A5-25. Alkylation 99% SS of regression.

1 * MARGINAL RENT REFINERY UNIT: ALKYLATION * $/TON * AREAS B K M T *

PARTIALLY CODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	14.199687	23.956562	16.358750	9.860625
X1	1.118437	6.169062	3.766875	2.255625
X2			1.169375	
X3	10.289063	20.762187	15.515000	8.284375
X1.X3	2.735312	5.769687	4.300625	2.160625
X2.X3			1.178125	
X4	10.371563	20.711562	14.787500	8.226875
X1.X4	2.810313	5.791562	4.394375	2.211875
X3.X4	6.542187	17.517187	14.425000	6.650625
X1.X3.X4	2.145937	5.392187	4.536875	2.116875
X5	1.163438		5.276250	
X1.X5			1.326875	
X3.X5			5.036250	
X1.X3.X5			1.419375	
X2.X3.X5			1.151875	
X4.X5			4.380000	
X1.X4.X5			1.279375	
X3.X4.X5			4.621250	
RESIDUAL SS	73.687653	305.574463	262.981689	11.129590

PARTIALLY DECODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	692.585844	1043.183399	-95926.647839	516.806854
ALP	-37.934003	-94.519237	-138.709801	-37.005863
ARS			-534.446990	
MGD	-1.177993	-1.951020	179.042151	-0.956583
ALP.MGD	0.073247	0.181463	0.314387	0.070994
ARS.MGD			0.999586	
CCC	-1.555874	-2.486039	291.602815	-1.279683
ALP.CCC	0.082723	0.209101	0.115292	0.082149
MGD.CCC	0.002894	0.004657	-0.544382	0.002377
ALP.MGD.CCC	-0.000160	-0.000401	-0.000338	-0.000158
SHC	-0.313215		535.665490	
ALP.SHC			0.327729	
ARS.SHC			2.878272	
MGD.SHC			-1.000066	

Table A5-26. Catalytic cracking 99% SS of regression.

2 * MARGINAL RENT REFINERY UNIT: CATALYTIC CRACKING * $/TON * AREAS B J K M T U V *

PARTIALLY CODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
GRAND MEAN	15.624062	9.890625	9.805000	7.462187	10.865625	6.398125	7.686875
X1	4.206562	2.800000	2.555000	1.715312	2.907500	1.782500	2.186250
X2		0.304375					0.529375
X3	4.734687	2.091875	5.482500	6.281562	2.910000	3.403750	7.090625
X1.X3	1.232188	0.518750	1.573750	1.780937	0.780625	0.939375	2.092500
X2.X3							0.540625
X4	4.934688	2.164375	6.495000	6.194062	3.292500	4.610625	7.347500
X1.X4	1.349688	0.598750	1.851250	1.824687	0.916875	1.260000	2.254375
X2.X4							0.528750
X3.X4	3.269063	1.475625	4.503750	6.000937	1.956875	1.831250	7.165000
X1.X3.X4	0.999063	0.400000	1.378750	1.810312	0.555000	0.574375	2.239375
X2.X3.X4							0.561250
X5	1.927188	1.890625	0.635000	2.179063		0.463125	1.646250
X1.X5		0.591250		0.559688			0.560625
X2.X5		0.270625					
X3.X5	0.564063	0.616875		1.814688			2.242500
X1.X3.X5				0.426563			0.654375
X4.X5	0.977812	1.228125		1.565938			1.985625
X1.X4.X5		0.376250					
X2.X4.X5		0.241875					
X3.X4.X5	0.580938	0.799375		1.866563			2.168125
RESIDUAL SS	21.570358	8.809458	27.155548	41.745560	10.860342	13.087431	61.083206

PARTIALLY DECODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA M	AREA T	AREA U	AREA V
INTERCEPT	-15674.046607	-24492.077793	290.187190	-48868.428462	20.046276	152.691235	-52128.022740
ALP	-17.510247	1.425079	-24.110416	-56.960500	-9.762170	-10.178625	-78.303826
AHS		38.853754					-72.512080
MGD	30.669715	42.090212	-0.487635	93.267072	-0.037069	-0.241372	99.654148
ALP.MGD	0.034004	0.013674	0.046630	0.109545	0.019115	0.020091	0.149801
AHS.MGD							0.138762
CCC	34.967789	55.188327	-0.599290	115.527533	-0.056378	-0.312541	125.732463
ALP.CCC	0.038472	-0.002625	0.053105	0.070318	0.021209	0.021646	0.086987
AHS.CCC		-0.085511					0.161629
MGD.CCC	-0.068217	-0.095057	0.001131	-0.220498	0.000106	0.000580	-0.240427
ALP.MGD.CCC	-0.000074	-0.000030	-0.000103	-0.000135	-0.000041	-0.000043	-0.000167
AHS.MGD.CCC							-0.000309
SHC	87.065187	134.418239	-0.170952	270.117788		-0.124680	308.494845
ALP.SHC		-0.046199		0.139379			0.216279
AHS.SHC		-0.213130					
MGD.SHC	-0.170267	-0.231014		-0.515507			-0.589785
ALP.MGD.SHC				-0.000269			-0.000413
CCC.SHC	-0.194537	-0.303033		-0.638523			-0.740258
ALP.CCC.SHC		0.000099					
AHS.CCC.SHC		0.000470					
MGD.CCC.SHC	0.000379	0.000522		0.001219			0.001416
SE	1.038517	0.766353	1.086589	1.522891	0.672692	0.754333	2.088800

Table A5-27. Hydrofining 99% SS of regression.

4 * MARGINAL RENT REFINERY UNIT: HYDROFINING

PARTIALLY CODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA V
GRAND MEAN	10.459375
X1	2.709375
X2	0.472500
X3	6.156875
X1.X3	1.645625
X2.X3	0.492500
X4	4.150000
X1.X4	1.456250
X3.X4	4.448750
X1.X3.X4	1.448750
X5	2.234375
X2.X3.X5	0.528750
X4.X5	0.993750
X1.X4.X5	0.645000
X3.X4.X5	1.043750
X1.X3.X4.X5	0.678750
RESIDUAL SS	29.334427

PARTIALLY DECODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA V
INTERCEPT	48797.481699
ALP	-566.633354
AHS	-244.895742
MGD	-92.672576
ALP.MGD	1.084616
AHS.MGD	0.458006
CCC	-88.066153
ALP.CCC	1.349772
MGD.CCC	0.168567
ALP.MGD.CCC	-0.002583
SHC	-264.785049
ALP.SHC	2.973518
AHS.SHC	1.321225
MGD.SHC	0.502764
ALP.MGD.SHC	-0.005691
AHS.MGD.SHC	-0.002470
CCC.SHC	0.478619
ALP.CCC.SHC	-0.007108
MGD.CCC.SHC	-0.000916
ALP.MGD.CCC.SHC	0.000014
SE	1.354032

Table A5-28. Catalytic reforming 99% SS of regression.

6 * MARGINAL RENT REFINERY UNIT: CATALYTIC REFORMING * $/TON * AREAS B K M T

PARTIALLY CODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
GRAND MEAN	12.489375	40.756250	22.438437	13.721875
X1	2.595625	10.677500	5.053437	2.948750
X3	5.210625	14.697500	10.318438	4.606875
X1.X3	1.439375	4.245000	3.000938	1.381250
X2.X3			0.866562	
X4	5.675625	16.781875	10.063438	5.008125
X1.X4	1.639375	4.799375	3.284688	1.472500
X2.X4			0.934062	
X3.X4	3.616875	12.740625	9.574687	2.771875
X1.X3.X4	0.968125	3.869375	3.218437	0.921250
X5	-4.235000	1.711875	3.064063	-3.945625
X1.X5	-1.222500		0.969063	-1.133750
X3.X5	0.921250		2.631563	
X1.X3.X5			0.744063	
X2.X3.X5			0.800938	
X4.X5			2.042813	
X2.X4.X5			0.794687	
X3.X4.X5	0.978750		3.064063	
RESIDUAL SS	32.566559	190.648499	124.279785	13.828728

PARTIALLY DECODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA K	AREA M	AREA T
INTERCEPT	-27927.839230	759.283277	-66761.385673	319.185027
ALP	-19.024498	-67.503008	-100.141856	-18.094402
AHS			-251.893500	
MGD	52.223749	-1.271266	116.321412	-0.585411
ALP.MGD	0.033526	0.130461	0.192723	0.031930
AHS.MGD			0.695868	
CCC	62.378379	-1.550086	202.917557	-0.702715
ALP.CCC	0.036958	0.149421	0.124974	0.035254
AHS.CCC			-0.287690	
MGD.CCC	-0.116596	0.002929	-0.359132	0.001331
ALP.MGD.CCC	-0.000072	-0.000288	-0.000240	-0.000069
SHC	153.085912	-0.460863	374.308477	-0.038032
ALP.SHC	0.010103		0.243182	0.009370
AHS.SHC			1.355505	
MGD.SHC	-0.286264		-0.654722	
ALP.MGD.SHC			-0.000470	
ABS.MGD.SHC			-0.003741	
CCC.SHC	-0.341836		-1.127424	
AHS.CCC.SHC			0.001544	
MGD.CCC.SHC	0.000639		0.002001	
SE	1.276059	2.879073	2.979451	0.792829

Table A5-29. Residue desulphurization 99% SS of regression.

7 * MARGINAL RENT REFINERY UNIT: RESIDUE DESULPHURIZATION * $/TON * AREAS J *

PARTIALLY CODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA J
GRAND MEAN	0.630937
X1	0.215313
X3	0.409062
X1.X3	0.139688
X2.X3	0.045313
X4	0.549687
X1.X4	0.169063
X2.X4	0.053438
X3.X4	0.405312
X1.X3.X4	0.133438
X5	0.352188
X1.X5	0.114063
X3.X5	0.130313
X4.X5	0.270938
X1.X4.X5	0.067813
X3.X4.X5	0.126563
RESIDUAL SS	0.276500

PARTIALLY DECODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA J
INTERCEPT	-3554.898943
ALP	-0.796203
AHS	-0.259258
MGD	6.655961
ALP.MGD	0.009483
AHS.MGD	0.000786
CCC	7.925047
ALP.CCC	0.001910
AHS.CCC	-0.000386
MGD.CCC	-0.014854
ALP.MGD.CCC	-0.000010
SHC	19.845711
ALP.SHC	-0.008389
MGD.SHC	-0.037247
CCC.SHC	-0.043977
ALP.CCC.SHC	0.000018
MGD.CCC.SHC	0.000083
SE	0.131459

Table A5-30. Hydrocracking 99% SS of regression.

9 * MARGINAL RENT REFINERY UNIT: HYDROCRACKING * $/TON * AREAS B J K T U

PARTIALLY CODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA T	AREA U
GRAND MEAN	4.608437	6.546562	7.444375	1.837187	19.757500
X1	1.125312	2.274062	2.170625	0.493438	5.683750
X2		0.325938			
X3	1.219062	1.882813	4.362500	1.599687	7.194375
X1.X3	3.449687	0.414063	1.238750	0.474688	1.896875
X4	0.935937	0.991563	4.402500	1.554063	8.360625
X1.X4	0.287813	0.231563	1.288750	0.436563	2.298125
X2.X4	0.160313				
X3.X4		0.694063	2.966875	1.316563	4.617500
X1.X3.X4	0.224688	0.150313	0.895625	0.417813	1.401250
X2.X3.X4		0.159688			
X5	1.203438	0.705313	0.848750	0.977187	1.931250
X1.X5	0.436563	0.147813		0.254688	
X2.X5		0.267188			
X3.X5	-1.162187	0.336563		0.739687	1.056875
X1.X3.X5	-0.207812			0.235938	
X2.X3.X5		0.194688			
X4.X5	-1.779062	0.546563	0.511875	0.694063	
X1.X4.X5	-0.173437	0.151563		0.197813	
X2.X4.X5		0.258438			
X3.X4.X5	-0.939687	0.859062		0.456563	
X1.X3.X4.X5	-0.295312	0.257813			
X2.X3.X4.X5	0.167187	0.289688			
RESIDUAL SS	2.677068	3.456280	16.643967	3.099973	51.058197

PARTIALLY DECODED MODEL FOR 99% SUM OF SQUARES

VARIABLE	AREA B	AREA J	AREA K	AREA T	AREA U
INTERCEPT	46717.510655	89070.635107	517.337963	-11819.501724	-1799.343401
ALP	246.837555	-210.035976	-15.882527	-17.521970	-24.612161
AMS	-1009.548433	-1817.043317			
MGD	-87.599558	-170.225751	-0.273297	22.077143	3.541074
ALP.MGD	-0.466402	0.399023	0.030798	0.041221	0.048088
AMS.MGD	1.888142	3.473908			
CCC	-109.702370	-204.658410	-1.128134	28.801911	-0.565033
ALP.CCC	-0.559229	0.501537	0.034412	0.006766	0.053565
AMS.CCC	2.413271	4.139209			
MGD.CCC	0.205635	0.390845	0.000635	-0.053786	0.001052
ALP.MGD.CCC	0.001060	-0.000952	-0.000067	-0.000031	-0.000104
AMS.MGD.CCC	-0.004514	-0.007909			
SMC	-254.350259	-484.782904	-2.059348	65.539922	11.102625
ALP.SMC	-1.379260	1.138481		0.055826	
AMS.SMC	5.550347	9.877242			
MGD.SMC	0.476695	0.926647		-0.122422	-0.021727
ALP.MGD.SMC	0.002607	-0.002162		-0.000149	
AMS.MGD.SMC	-0.010376	-0.018888			
CCC.SMC	0.597390	1.113178	0.004377	-0.159621	
ALP.CCC.SMC	0.003121	-0.002724		0.000052	
AMS.CCC.SMC	-0.013268	-0.022487			
MGD.CCC.SMC	-0.001119	-0.002126		0.000298	
ALP.MGD.CCC.SMC	-0.000004	0.000005			
AMS.MGD.CCC.SMC	0.000025	0.000043			
SE	0.422458	0.560542	0.869795	0.427026	1.523426

A5.3 2^5 7-area WEM FD and Parametric Programming cases: Marginal Values on Oil Products

Table A5-31. 2^5 7-area WEM FD marginal values on oil products area B.

Treatment combination		Product prices - US$ / mt PMS	DFO	HFO	JETKERO	NAPHTHA
1	00000	127.24	99.75	84.56	102.30	103.99
2	10000	219.21	176.44	148.11	181.08	184.93
3	01000	127.34	99.83	84.57	102.35	103.94
4	11000	134.25	102.31	85.87	105.70	109.08
5	00100	131.28	99.70	84.10	102.64	104.83
6	10100	133.37	106.12	88.69	120.14	117.29
7	01100	219.23	176.49	148.13	181.03	184.57
8	11100	232.07	180.91	151.69	186.31	191.02
9	00010	227.01	176.85	148.54	181.38	183.79
10	10010	231.96	188.25	156.85	195.61	206.92
11	01010	134.07	102.24	85.75	105.62	108.95
12	11010	131.29	99.70	84.08	102.71	105.10
13	00110	133.29	106.49	88.65	110.80	117.61
14	10110	152.65	106.78	85.84	110.82	113.82
15	01110	137.99	106.46	88.20	110.98	117.66
16	11110	138.26	106.51	88.04	111.28	118.75
17	00001	232.22	181.18	151.78	186.46	190.80
18	10001	226.74	176.56	148.32	181.33	184.43
19	01001	232.35	188.43	156.59	195.83	207.31
20	11001	265.37	188.97	151.70	195.96	201.27
21	00101	238.70	189.18	155.45	197.44	210.52
22	10101	239.43	189.61	155.51	197.68	210.09
23	01101	152.48	106.69	85.68	110.71	113.65
24	11101	137.88	106.94	88.02	111.57	118.62
25	00011	138.16	106.88	87.96	111.56	118.75
26	10011	170.58	126.03	96.21	129.88	131.30
27	01011	265.39	201.32	151.66	195.82	201.32
28	11011	238.68	189.73	155.78	197.59	209.60
29	00111	239.33	189.90	155.41	197.89	210.07
30	10111	294.87	220.40	168.32	227.27	230.11
31	01111	174.83	130.09	98.35	134.40	136.98
32	11111	310.44	233.43	176.55	241.77	248.13

Table A5-32. 7-area WEM gas oil PP cases: oil product prices area B.

Gas oil demand mmt/y	PMS	DFO	HFO	JETKERO	SRB	NAPHTHA
	Product Prices US$ / mt					
95.00	327.86	243.47	217.71	84.56	269.06	276.92
97.00	327.85	244.15	217.78	247.55	267.81	275.66
99.00	327.93	244.43	217.86	247.84	267.20	275.04
101.00	328.21	247.40	218.13	250.02	262.21	270.04
103.00	328.10	247.59	218.14	250.21	261.82	269.65
105.00	328.13	247.74	218.20	250.36	261.45	269.27
107.00	328.10	248.01	218.23	250.64	261.14	268.96
109.00	328.06	248.87	218.21	251.50	260.18	268.00
111.00	328.08	249.62	218.23	252.25	259.26	267.08
113.00	327.96	250.86	218.09	253.50	257.88	265.72
115.00	327.67	251.21	217.70	253.84	258.08	265.93
117.00	327.65	251.63	217.68	254.27	257.49	265.35
119.00	327.68	251.72	217.73	254.36	257.28	265.13
121.00	327.71	252.16	217.76	254.81	256.63	264.48
123.00	327.71	252.24	217.76	254.88	256.52	264.38
125.00	327.71	252.24	217.76	254.88	256.52	264.38
127.00	327.70	252.32	217.74	254.97	256.45	264.30
129.00	327.72	252.54	217.80	255.18	256.04	263.89
131.00	327.71	252.54	217.80	255.19	256.04	263.89
133.00	327.43	252.69	217.54	255.34	256.25	264.10
135.00	327.44	252.72	217.56	255.36	256.18	264.03

Table A5-33. 7-area WEM premium motor spirit PP cases: oil product prices area B.

PMS demand mmt/y	PMS	DFO	HFO	JETKERO	SRB	NAPHTHA
	Product Prices US$/mt					
25.00	318.12	255.22	216.61	254.86	259.17	266.00
27.00	319.97	252.49	216.54	255.13	258.90	265.97
29.00	320.85	252.42	216.44	255.06	259.02	266.22
31.00	324.56	252.28	216.76	254.91	258.36	265.96
33.00	326.44	252.04	216.89	254.63	258.36	266.17
35.00	326.53	251.96	216.96	254.60	258.31	266.13
37.00	326.79	252.19	217.14	254.83	257.66	265.49
39.00	327.29	252.33	217.55	254.97	256.78	264.61
41.00	327.71	252.24	217.76	254.88	256.52	264.38
43.00	327.71	252.24	217.76	254.88	256.52	264.37
45.00	327.83	252.18	217.86	254.82	256.42	264.27
47.00	328.01	252.08	218.03	254.72	256.25	264.10
49.00	328.16	252.04	218.15	254.69	256.11	263.96
51.00	328.17	252.03	218.17	254.68	256.09	263.95
53.00	328.30	251.97	218.29	254.62	255.97	263.82
55.00	328.49	251.91	218.44	254.56	255.84	263.70
57.00	328.61	251.92	218.48	254.56	255.76	263.62
59.00	328.97	251.95	218.42	254.60	255.78	263.70
61.00	330.51	251.97	218.42	254.62	255.67	263.77
63.00	335.22	251.10	218.25	253.74	256.92	265.62
65.00	340.74	251.27	217.64	253.91	257.61	267.08
67.00	353.07	251.61	216.88	254.25	258.09	269.17

A5.4 7-area WEM Parametric Programming cases: Marginal Values on Refinery Capacity

Table A5-34. 7-area WEM PP cases: marginal values on refinery processing units area B.

DFO-d PP case	CC	REF	HYC	PMS-d PP case	CC	REF	HYC
	(US$ / mt feed)				(US$ / mt feed)		
95.00	12.76	26.10	16.99	25.00	12.26	27.20	0.00
97.00	12.46	27.31	15.59	27.00	13.20	29.18	0.00
99.00	12.51	28.00	14.97	29.00	13.33	29.81	0.15
101.00	13.06	33.14	9.01	31.00	13.89	34.01	0.85
103.00	13.13	33.52	8.59	33.00	14.31	35.75	1.57
105.00	13.13	33.94	8.24	35.00	14.25	35.90	1.65
107.00	13.22	34.28	7.76	37.00	14.27	36.93	1.09
109.00	13.41	35.36	6.31	39.00	14.31	38.49	0.55
111.00	13.60	36.46	5.01	41.00	14.34	39.21	0.60
113.00	14.02	37.93	2.88	43.00	14.34	39.22	0.60
115.00	14.23	37.35	2.57	45.00	14.29	39.47	0.63
117.00	14.25	38.01	1.78	47.00	14.20	39.86	0.67
119.00	14.21	38.30	1.57	49.00	14.20	40.18	0.70
121.00	14.32	39.09	0.74	51.00	14.20	40.22	0.70
123.00	14.34	39.21	0.60	53.00	14.20	40.50	0.75
125.00	14.34	39.21	0.60	55.00	14.28	40.85	0.82
127.00	14.39	39.29	0.46	57.00	14.31	41.06	0.80
129.00	14.36	39.79	0.01	59.00	14.44	41.35	0.87
131.00	14.35	39.79	0.00	61.00	15.00	42.88	1.18
133.00	14.31	39.24	0.00	63.00	16.57	45.67	3.92
135.00	14.31	39.34	0.00	65.00	19.29	49.76	5.58
				67.00	24.50	60.25	8.68

```
C
C    PROGRAM RENT1 IS THE MAIN PROGRAM FOR THE ANALYSIS OF
C    EXPERIMENTAL RESPONSES OF THE 2**5 7-AREA WEM FD. RENT1
C    READS THE RESPONSES AND ARRANGES THEM IN A THREE-
C    DIMENSIONAL ARRAY FOR USE BY SUBROUTINES YATES AND TESTEO.
C
C    IT READS MARGINAL RENT RESPONSES FROM THE 2^5 7-AREA
C    WEM FD OUTPUT FILE, AND CREATES THE RENT RESPONSE MATRIX
C    RRESP(I,J1,K) ACCORDING TO  DAVIES STANDARD ORDER BY USING
C    ORDER BY USING INDEX VECTOR NDPLOT(J), J=1,32, WITH
C    J1=NDPLOT(J), SINCE THE ORIGINAL RUNS FOLLOW A DIFFERENT
C    ORDER. NDPLOT(J) IS IN DAVIES STANDARD ORDER.
C
C    THE DATA FILE INPUT ORDER IS CASE-AREA-REF UNIT; THE THREE
C    LOOPS  (1,2,3)  BELOW FOLLOW THAT ORDER; MATRIX RRESP HOW_
C    EVER IS LATER REARRAGEND TO THE ORDER REF UNIT-CASE-AREA.
C
C    STATEMENT READ(1,105) SKIPS THE 'FILE NO.' CARD, HEADING
C    INDICATOR OF EACH OF THE 32 SUB-FILES. THE SUB_FILES ARE
C    SUB-SETS OF INFORMATION OF THE FULL 32 CASES OR PARTITIONS
C    OF TAPE ASB002 WERE THE SOLUTION CASES ARE STORED.
C
         PROGRAM RENT1
         DOUBLE PRECISION RRESP,D,SURFAC
         COMMON /DPAC/RRESP(9,32,7),D(32,32),SURFAC(10,63,33)
         COMMON /SPAC/NR,NC,NA,MSIGNS(32,32),NMVARU(9,7)
         DIMENSION NAREAS(7),NDPLOT(32)
         DATA NDPLOT/1,2,3,5,9,17,4,6,10,18,7,11,19,13,21,25,8,12,
     +               20,14,22,26,15,23,27,29,16,24,28,30,31,32/
         DATA NAREAS/'B','J','K','M','T','U','V'/
C
         READ(1,100) NR,NC,NA
 100     FORMAT(3I3)
         DO 1 J=1,NC
            READ(1,105) AFILE
 105        FORMAT(A80)
            DO 2 K=1,NA
               DO 3 I=1,NR
                  J1=NDPLOT(J)
                  READ(1,110) A,B,RRESP(I,J1,K)
 110              FORMAT(A8,A58,F14.2)
 3             CONTINUE
 2          CONTINUE
 1       CONTINUE
C
C             WRITING MATRIX RRESP(9,32,7) IN 9-SUB MATRIX
C             FORM, EACH ACCOUNTING FOR THE RENT RESPONSES
C             PER AREA PER TREATMENT COMBINATION.
C
         WRITE(2,200)
 200     FORMAT(T21,'****    REFINERY UNITS RENTS RESPONSES  PER AREA',
     +          '    ****',//,T21,'UNIT: $-TONNE',////)
         DO 4 I=1,NR
```

```
          WRITE(2,210) (NAREAS(K),K=1,7)
210       FORMAT(T5,'TREATMENT',/,T5,'STANDARD',/,T5,'ORDER',
   +           T22,7(A1,8X),//)
          DO 5 J=1,NC
             J2=J1+4
             WRITE(2,220) (RRESP(I,J,K),K=1,NA)
220          FORMAT(7(F8.2))
5         CONTINUE
          WRITE(2,230)
230       FORMAT(6(/))
4       CONTINUE
        CALL YATES
        STOP
        END
```

```
C
C   SUBROUTINE YATES ESTIMATES THE RENT MODELS PER REFINERY
C   PROCESSING UNIT AND AREA, FROM THE 32-EXPERIMENTAL
C   RESPONSES OF THE 2**5 7-AREA WEM FD. FULL AND REDUCED
C   CODED AND DECODED MODELS ARE WORKED OUT ONCE THE SET OF
C   YATES EFFECTS AND CODED COEFFICIENTES ARE CALCULATED.
C
C    FORTRAN VSF NAME - VSF.FDSUB2
C
      SUBROUTINE YATES
      DOUBLE PRECISION RRESP,D
      DOUBLE PRECISION FLEVEL,FDM,FACT,PCM,PDM,SSMOD,SURFAC
      DOUBLE PRECISION SSP,SSPP,SSRFAR,UNIT,Y,SS
      COMMON /DPAC/RRESP(9,32,7),D(32,32),SURFAC(10,63,33)
      COMMON /SPAC/NR,NC,NA,MSIGNS(32,32),NMVARU(9,7)
      COMMON /COMUN/FLEVEL(2,5),FDM(32),IR,K2,FACT(32),PCM(33),
     +        PDM(33),SSMOD(5),MODEL,MSS,SW,IP(6),IPRU(9),
     +        IPAR(9),NARM(9,7),NRUM(9),NRUMA(7),NRUA(7,9),
     +        JIND,SSRFAR(9,7)
      DIMENSION UNIT(32),Y(32),SS(32),NAREAS(7)
      DIMENSION SSP(32),SSPP(32),JMAX(32)
      REAL MSI,MSRES
      DATA NAREAS/'B','J','K','M','T','U','V'/
C
C   ***** SECTION 1: READING DATA, CREATING DECODING MATRIX
C                    BY CALLING SUBROUTINE TESTEO           *****
C
      DO 100 I=1,NR
         READ(4,200) (NARM(I,J),J=1,NA)
 100  CONTINUE
      DO 110 I=1,NA
         READ(4,200) (NRUA(I,J),J=1,NR)
 110  CONTINUE
      READ(4,200) (NRUM(I),I=1,NR),(NRUMA(I),I=1,NA)
 200  FORMAT(16I2)
      DO 120 I=1,NC
         UNIT(J)=0.
```

```
         DO 121 I2=1,10
            DO 121 I3=1,63
               SURFAC(I2,I3,I)=0.
 121     CONTINUE
 120  CONTINUE
      READ(4,200) (IPRU(I),I=1,9),(IPAR(I),I=1,7)
      READ(4,210) MSS,ILAB,NLABEL
 210  FORMAT(3I2)
      PRINT 210,MSS,ILAB,NLABEL
      IF(MSS.NE.0) PRINT  211
 211  FORMAT(///,5X,'PLEASE ENTER SS OF REGRESSION, FORMAT F6.2')
      IF(MSS.NE.0) READ 212,(SSMOD(MODEL), MODEL=1,MSS)
      PRINT 212,(SSMOD(I),I=1,MSS)
 212  FORMAT(3F6.2)
      NAUX=NLABEL
      ILABEL=1
      CALL TESTEO(ILABEL,NLABEL)
C
C              END SECTION 1
C
C     ******   SECTION 2:  GENERAL MODELS COMPUTATIONS      ******
C
C              DO LOOP 1200 SCANNING PER REFINREY UNIT
C
      DO 1199 IR1=1,NR
         IF(IPRU(IR1).EQ.0) GOTO 2220
         DO 1200 IR=1,NR
            IF(IPRU(IR1).NE.IR) GOTO 1200
            SW=0.
            K1=NRUM(IR)
C
C             DO LOOP 1210 SCANNING PER AREA WITHIN
C             REFINERY UNIT IR
C
            DO 1210 IA=1,K1
               K2=NARM(IR,IA)
               IF(K2.EQ.0) GO TO 9999
C
C              DO LOOP 1220 ASSIGNING RESPONSES PER AREA K2
C              WITHIN REFINERY UNIT IR TO TRANSITIONAL VECTOR
C              UNIT(32)
C
               DO 1220 J=1,NC
                  UNIT(J)=RRESP(IR,J,K2)
                  IF(ILABEL.EQ.3) GOTO 1220
                  WRITE(5,1221) IR,NAREAS(K2),(RRESP(IR,J,K2))
 1221             FORMAT(2X,I2,A1,2X,F15.5)
 1220          CONTINUE
C
C             YATES EFFECTS COMPUTATIONS
C
               DO 1230 J=1,NC
                  Y(J)=0.
                  SS(J)=0.
                  FACT(J)=0.
                  FDM(J)=0.
 1230          CONTINUE
C
               DO 1240 J=1,NC
```

```
                DO 1250 K=1,NC
                   Y(J)=Y(J)+UNIT(K)*MSIGNS(K,J)
 1250           CONTINUE
                Y(J)=Y(J)/16.
 1240         CONTINUE
C
C         CALCULATING SUM OF SQUARES
C         SS = (2**(N-2)*(YATES)**2)
C         N=5, SO  SS= 8*(YATES**2)
C
              DO 1270 J=1,NC
                 SS(J)=8.*(Y(J)**2)
 1270         CONTINUE
C
C         ALLOCATION OF YATES EFFECTS AND SS TO MATRIX
C         SURFAC(I,J,K), O.L. DAVIES ORDER
C
              JUMP=(K2-1)*9+IR
              DO 1280 J=1,NC
                 SURFAC(1,JUMP,J)=Y(J)
                 SURFAC(2,JUMP,J)=SS(J)
                 FACT(J)=SURFAC(1,JUMP,J)/2.
 1280         CONTINUE
C
C       ASSIGNING CODED MODEL COEFFICIENTS TO
C            SURFAC MATRIX
C
              DO 1290 J=1,NC
                 SURFAC(3,JUMP,J)=FACT(J)
                 FDM(J)=0.
 1290         CONTINUE
C
C       DECODING CODED MODEL COEFFICIENTS
C
              DO 1300 J=1,NC
                 DO 1310 K=1,NC
                    FDM(J)=FDM(J)+FACT(K)*D(K,J)
 1310            CONTINUE
                 SURFAC(4,JUMP,J)=FDM(J)
 1300         CONTINUE
              ILABEL=ILAB
C
C   FULL CODED/DECODED MODELS TREATMENT WHEN MSS=0.
C
              IF(NLABEL.EQ.5) MSS=1
              IF(MSS.EQ.0.AND.ILABEL.EQ.3) GO TO 1210
              IF(MSS.EQ.0.AND.ILABEL.EQ.2) GO TO 1211
C
C   PARTIALLY CODED/DECODED MODELS TREATMENT: DO LOOP 1510
C   ACCORDING TO INPUT CRITERIA IN SSMOD(MODEL);
C   SSMOD(MODEL) IS A VECTOR WITH PREFIXED PERCENTAGE
C   VALUES FOR REDUCING THE DECODED MODELS.
C
C
 2222         DO 1510 MODEL=1,MSS
                 IND=0
                 RES=0.
                 SSPT=0.
                 SSTI=0.
```

```
                  DO 1380 J=1,NC
                     SSP(J)=0.
                     SSPP(J)=0.
                     PCM(J)=0.
                     PDM(J)=0.
 1380             CONTINUE
                  SST=0.
                  DO 1390 J=2,NC
                     SST=SST+SS(J)
 1390             CONTINUE
                  SSPT1=0.
                  DO 1400 J=2,NC
                     SSP(J)=(SS(J)/SST)*100.
                     SSPT1=SSPT1+SSP(J)
 1400             CONTINUE
                  PCM(1)=FACT(1)
                  IF(NLABEL.EQ.5)GOTO 1442
                  NC1=NC-1
C
 1420             DSS=SSMOD(MODEL)-SSPT
                  IF(DSS.LT..01)GO TO 1421
                  IND=IND+1
                  IMAX=32
                  SSPMAX=SSP(IMAX)
                  DO 1410 J=2,NC1
                     IF(SSP(J).LE.SSPMAX) GO TO 1410
                     IMAX=J
                     SSPMAX=SSP(IMAX)
 1410             CONTINUE
                  JMAX(IND)=IMAX
                  SSPP(IMAX)=SSP(IMAX)
                  SSTI=SSTI+SS(IMAX)
                  SSP(IMAX)=0.
                  SSPT=SSPT+SSPMAX
                  IF(SSPT.LT.SSMOD(MODEL))GO TO 1420
 1421             JIND=IND
C
C   COMPUTING PARTIALLY CODED AND DECODED MODELS
C
                  DO 1430 J=1,JIND
                     DO 1440 J1=2,NC
                        IF(J1.EQ.JMAX(J)) PCM(J1)=FACT(J1)
C        1.EQ.6.OR.J1.EQ.10.OR.J1.EQ.13.OR.J1.EQ.17) PCM(J1)=0.
 1440                CONTINUE
 1430             CONTINUE
C
                  DO 1450 J=2,NC
                     DO 1460 J1=1,JIND
C           IF(J.EQ.6.OR.J.EQ.10.OR.J.EQ.13.OR.J.EQ.17) GO TO 1460
                        IF(J.EQ.JMAX(J1)) GOTO 1450
 1460                CONTINUE
                     RES=RES+SS(J)
 1450             CONTINUE
C
C   CODED MODEL STATISTICAL VARIANCE INDICATORS:
C   RY    COEFFICIENT OF DETERMINATION OF RESPONSE,
C         SUM OF SQUARES OF REAL EFFECTS DIVIDED BY
C         TOTAL SUM OF SQUARES (IT MUST BE GE TO INPUT(SSMOD)
C         PERCENTAGE/100.)
```

```
C   MSI   MEAN SQUARE OF THE SUM OF SQUARES OF REAL EFFECTS
C   MSRES MEAN SQUARE OF RESIDUAL
C   FRAT  RATIO FOR F-TEST AT F1=JIND (FOR MSI) AND F2=31-JIND
C         (FOR MSRES) DEGREES OF FREEDOM RESPECTIVELY
C
C   THE NUMBER OF DEGREES OF FREEDOM OF RESIDUAL ERROR IS EQUAL
C   TO THE TOTAL NO. OF OBSERVATIONS (32) MINUS 1 (TOTAL SUM
C   OF SQUARES) MINUS NO. OF TERMS IN MODEL FORMATION
C
          GOTO 1441
C
C   ***  SELECTED CRITERION:  ALL MAIN FACTORS CONSIDERED  ***
C
 1442     JIND=5
          DO 1443 J=2,NC
   IF(J.NE.2.AND.J.NE.3.AND.J.NE.5.AND.J.NE.9.AND.J.NE.17)GOTO 1444
             SSTI=SSTI+SS(J)
             SSPT=SSPT+SSP(J)
             SSPP(J)=SSP(J)
             PCM(J)=FACT(J)
             GOTO 1443
 1444        RES=RES+SS(J)
 1443     CONTINUE
C
 1441     PCM(32)=RES
          DF=31-JIND
          SE=SQRT(RES/DF)
          RY=SSTI/SST
          SSRFAR(IR,K2)=RY*100.
          MSI=SSTI/JIND
          IF(JIND-31) 445;446,446
 446      FRAT=0.
          GO TO 447
 445      MSRES=RES/DF
          FRAT=MSI/MSRES
C
C   PARTIALLY DECODED COEFFICIENTS
C
 447      CONTINUE
          DO 1470 J=1,NC
             DO 1470 I=1,31
                PDM(J)=PDM(J)+PCM(I)*D(I,J)
                IF(I.EQ.31) PRINT 8088,J,PCM(J),PDM(J),FDM(J)
 8088           FORMAT(2X,I3,3F20.10)
 1470     CONTINUE
C
          I=3+MODEL*2
          I1=I+1
          SURFAC(I,JUMP,33)=SE
          DO 1480 J=1,NC
             SURFAC(I,JUMP,J)=PCM(J)
             SURFAC(I1,JUMP,J)=PDM(J)
 1480     CONTINUE
          IF(NAUX.EQ.5) NLABEL=2
          IF(NAUX.EQ.5) SSMOD(MODEL)=SSPT
          IF(ILABEL.EQ.3) GO TO 1510
          CALL TESTEO(ILABEL,NLABEL)
          IF(NAUX.EQ.5) NLABEL=NAUX
```

```
          SW=1.
C
          J2=4
          J1=1
          IF(NLABEL.EQ.5) SSMOD(MODEL)=SSPT
          WRITE(5,1326)IR,NAREAS(K2),SST,SSMOD(MODEL),RY,MSI,MSRES,
     +               FRAT,JIND,DF,SE
 1326     FORMAT(///,4X,'PARTIALLY CODED AND DECODED MODELS',2X,I2,
     +        A1,3X,'SST= ',F15.5,/,4X,'PERCENT SS-',F6.2,2X,'RY=',
     +        F6.4,2X,'MSEFF=',F9.3,2X,'MSRES=',F9.3,/,4X,'FRAT=',
     +     F7.2,2X,'DF-EFF=',I3,2X,'DF-RES=',F6.2,2X,'SE=',2X,F8.4)
          DO 1320 K=1,8
             WRITE(5,1330)K,(SURFAC(I,JUMP,J),J=J1,J2)
             WRITE(5,1340)K,(SURFAC(I1,JUMP,J),J=J1,J2)
             WRITE(5,1350)K,(SSPP(J),J=J1,J2)
 1330        FORMAT(/,2X,'CDEST',I2,2X,4F15.5)
 1340        FORMAT(2X,'UCEST',I2,2X,4F15.5)
 1350        FORMAT(2X,'SSP  ',I2,2X,4F15.5)
 1360        FORMAT(2X,'FISH ',I2,2X,4F15.5)
             J1=J1+4
             J2=J2+4
 1320        CONTINUE
 1510     CONTINUE
C
          IF(ILABEL.EQ.3) GO TO 1210
          GO TO 1213
C
 1211     J2=4
          J1=1
          DO 1301 K=1,8
             WRITE(5,1302) K,(SURFAC(1,JUMP,J),J=J1,J2)
             WRITE(5,1303) K,(SURFAC(2,JUMP,J),J=J1,J2)
             WRITE(5,1304) K,(SURFAC(3,JUMP,J),J=J1,J2)
             WRITE(5,1305) K,(SURFAC(4,JUMP,J),J=J1,J2)
 1302        FORMAT(/,2X,'YATES',I2,2X,4F15.5)
 1303        FORMAT(2X,'SS   ',I2,2X,4F15.5)
 1304        FORMAT(2X,'CDEST',I2,2X,4F15.5)
 1305        FORMAT(2X,'UCEST',I2,2X,4F15.5)
             J1=J1+4
             J2=J2+4
 1301     CONTINUE
          CALL TESTEO(ILABEL,NLABEL)
          SW=1.
C
 1213     WRITE(5,1370)
 1370     FORMAT(////)
C
 1210     CONTINUE
 9999     PRINT 1500,IR
 1500     FORMAT(2X,'REF. UNIT NO.',I35)
 1200     CONTINUE
C
 1199     CONTINUE
C
C   *****   SECTION 3: ONLY PRINTING BY CALLING SUB. TESTEO  *****
C
 2220     IF(ILABEL-3) 1214,1215,1215
 1215     IP(1)=1
          DO 1216 JK=1,MSS
```

```
          IP(JK+1)=JK
 1216     CONTINUE
          CALL TESTEO(ILABEL,NLABEL)
 1214     RETURN
          END
```

```
C
C    TESTEO IN VSF.FDSUB3   IS A TWO PARAMETER SUBROUTINE
C    DIVIDED IN THREE SECTIONS,
C
C    - IN SECTION 1 DECODING MATRIX D(32,32) IS BEING
C      CREATED ACCORDING TO O.L. DAVIES ORDER
C      SIGN AND FACTOR LEVELS MATRICES ARE READ
C
C    - IN SECTION 2, FULL AND/OR PARTIALLY CODED AND/OR
C      DECODED MODELS ARE WORKED OUT; MODEL RESPONSES AND
C      RESIDUALS ARE PRINTED AS REQUIRED FOR PLOTTING
C      PURPOSES
C
C    - IN SECTION 3 ONLY PRINTING ROUTINES ARE CARRIED OUT
C
C    ACCESSING ANY OF THE THREE SUBROUTINE SECTIONS DEPENDS
C    ON VALUE OF PARAMETER JLABEL (1,2 OR 3)
C
C    FORMING CODED OR DECODED MODELS DEPENDING ON VALUE OF
C    PARAMETER KLABEL (1,2,3 OR 4)
C
C    IF KLABEL IS EQUAL TO 5, CODED AND DECODED MODELS ARE
C    WORKED OUT FOR SPECIAL TRUNCATING CRITERIA
C
C    SUBROUTINE TESTEO CALLS SUBROUTINE ORDER WHENEVER AN
C    ARRAY REQUIRES AN ASCENDING ORDER OF ITS ELEMENTS
C
      SUBROUTINE TESTEO(JLABEL,KLABEL)
      DOUBLE PRECISION RRESP,D,DIV,DIV1,DR,DR1,FX,SVDIF
      DOUBLE PRECISION FLEVEL,FDM,FACT,PCM,PDM,SSMOD,SURFAC
      DOUBLE PRECISION V,XM,DM,T,SUM,SUM1,DIF,DIFM,TDIF,VDIF
C
      COMMON /DPAC/RRESP(9,32,7),D(32,32),SURFAC(10,63,33)
      COMMON /SPAC/NR,NC,NA,MSIGNS(32,32),NMVARU(9,7)
      COMMON /COMUN/FLEVEL(2,5),FDM(32),IR,K2,FACT(32),PCM(33),
     + PDM(33),SSMOD(5),MODEL,MSS,SW,IP(6),IPRU(9),IPAR(9),
     + NARM(9,7),BRUM(9),NRUMA(7),NRUA(7,9),JIND,SSRFAR(9,7)
      COMMON /FREE/IV(32)
C
      DIMENSION NAREAS(7),X(5),V(32),XM(5),DM(5),SSN(7),
     +  IFACT(32,5),NAR(7),SUM(32),SUM1(32),T(32),DIF(32)
      REAL*8 KFACT1(32,6),NREFU(9,5),FEATM(6),PCOEF(32,7),
     +       PVAR(32,3)
      CHARACTER*8 NBLANK
```

```
      DATA NAREAS/'B','J','K','M','T','U','V'/
C
C
      NBLANK='        '
      N1=NC
      GO TO(150,160,165),JLABEL
C
C   * * *  SECTION 1:  READING, DECODING MATRIX CREATION  * * *
C                      MATRIX D(32x32): DECODING MATRIX
C
150   CONTINUE
      DO 100 I=1,32
         READ(3,200) (KFACT1(I,J),J=1,6)
200      FORMAT(6A8)
100   CONTINUE
      DO 110 I=1,32
         READ(3,130) (MSIGNS(I,J),J=1,32)
130      FORMAT(32I2)
110   CONTINUE
      DO 120 I=1,2
         READ(3,140) (FLEVEL(I,J),J=1,5)
140      FORMAT(5F8.3)
120   CONTINUE
      DO 121 I=1,9
         READ(3,141) (NREFU(I,J),J=1,4)
141      FORMAT(4A8)
121   CONTINUE
      READ(3,142) (FEATM(I),I=1,5)
142   FORMAT(5A8)
      DO 143 I=1,NC
         READ(3,144) (IFACT(I,J),J=1,5)
144      FORMAT(5I1)
143   CONTINUE
C
C                FACTORS MEANS
C
      XM(1)=-117.425
      XM(2)=-48.411
      XM(3)=-534.9343
      XM(4)=-418.3322
      XM(5)=-181.97640007300
C
C                FACTORS MEANS DISTANCES
C
      DM(1)=32.575
      DM(2)=4.401
      DM(3)=13.0955
      DM(4)=-31.487
      DM(5)=-3.7145
C
        DO 21 I=1,NC
        DO 21 J=1,NC
           D(I,J)=0.
  21    CONTINUE
C
       DO 22 J=1,NC
          DIV1=1.
          DR1=1.
C
```

```
          DO 23 K=1,5
             IF(IFACT(J,K).EQ.0) GOTO 23
             DIV1=DIV1*XM(K)
 23       CONTINUE
C
           DO 24 I=J,NC
              DIV=1.
              DR=1.
              DO 25 K=1,5
                 IF(IFACT(I,K).EQ.0.AND.IFACT(J,K).EQ.1) GOTO 24
                 IF(IFACT(I,K).EQ.0) GOTO 25
                 DIV=DIV*XM(K)
                 DR=DR*DM(K)
 25           CONTINUE
C
              DIV2=DIV/DIV1
              D(I,J)=DIV2/DR
 24        CONTINUE
 22    CONTINUE
C
C                    END OF SECTION 1
  GO TO 205
C
C      * * * SECTION 2: RESPONSES COMPUTATIONS FROM MODELS   * * *
C
160    CONTINUE
       IF(SW.EQ.1.) GO TO 166
       GO TO(410,412,414,416),KLABEL
C
410    WRITE(6,411) FEATM(3),FEATM(4),FEATM(1)
       GO TO 166
412    WRITE(6,411) FEATM(3),FEATM(4),FEATM(2)
       GO TO 166
414    WRITE(6,413) FEATM(5),FEATM(1)
       GO TO 166
416    WRITE(6,413) FEATM(5),FEATM(2)
411    FORMAT(//,10X,'***',2X,2A8,2X,A8,2X,'***',//)
413    FORMAT(//,10X,'***',2X,A8,2X,A8,2X,'***',//)
C
166    CONTINUE
       DO 211 I=1,NC
          DIF(I)=0.
          V(I)=0.
          SUM(I)=0.
          SUM1(I)=0.
          T(I)=0.
          TDIF=0.
          VDIF=0.
          DIFM=0.
 211   CONTINUE
       DO 170 I=1,NC
          JUMP=(K2-1)*9+IR
          GOTO(424,426,425,427),KLABEL
424       DO 175 I3=1,31
             SUM(I) = SUM(I)+MSIGNS(I,I3)*PCM(I3)
175       CONTINUE
          GOTO 210
425       DO 180 I3=1,NC
```

```
             SUM(I) = SUM(I)+MSIGNS(I,I3)*FACT(I3)
 180      CONTINUE
          GOTO 210
 426      DO 182 I3=1,NC
             T(I3) = PDM(I3)
 182      CONTINUE
          GOTO 428
 427      DO 184 I3=1,NC
             T(I3) = SURFAC(4,JUMP,I3)
 184      CONTINUE
 428      J=0
          DO 187 I3=1,5
             J=J+1
             IF(IFACT(I,I3).EQ.0) X(J)=FLEVEL(1,J)
             IF(IFACT(I,I3).EQ.1) X(J)=FLEVEL(2,J)
 187           CONTINUE
             DO 185 I3=1,NC
                FX=1.
                DO 186 I4=1,5
                   IF(IFACT(I3,I4).EQ.0) GOTO 186
                   FX=FX*X(I4)
 186            CONTINUE
                FX=FX*T(I3)
                SUM(I)=SUM(I)+FX
               IF(I.EQ.1.OR.I.EQ.32) PRINT 8089,I,FX,SUM(I)
 185           CONTINUE
 8089           FORMAT(2X,I3,2F20.10)
 210         CONTINUE
             DIF(I) = RRESP(IR,I,K2) -SUM(I)
             TDIF = TDIF+DIF(I)
             V(I)=DIF(I)
             SUM1(I)=SUM(I)
 170         CONTINUE
C
C     COMPUTING MEAN AND VARIANCE OF RESIDUALS. COMPARING THE
C     STANDARD DEVIATION OF RESIDUALS WITH THE STANDARD ERROR
C     CALCULATED BY THE SUM OF SQUARES OF ERROS IN SUB. YATES
C
       DIFM=TDIF/32.
       DO 213 I=1,NC
          VDIF=VDIF+(DIF(I)-DIFM)**2
 213   CONTINUE
       DF1=32.-(JIND+1.)
       VDIF=VDIF/DF1
       SVDIF=SQRT(VDIF)
C
       IF(MSS.EQ.0.AND.KLABLE.NE.5) SSMOD(MODEL)=0.0
       WRITE(6,190) IR,NAREAS(K2),SSMOD(MODEL),DIFM,SVDIF
190    FORMAT(//,2X,'CASE',2X,I2,A1,4X,'RESP.   ','MODEL RESP.',
     +          2X,'ERROR',6X,'ERR-ASC',3X,'RESP-ASC',4X,'RESP-ASC',
     +          4X,'RESP-ERR',/,2X,'SS=',F7.2,T16,
     +       'MEAN OF RESIDUALS= ',F7.4,2X,'SE OF RESIDUALS= ',F8.5,/)
C
       CALL ORDER(DIF,V,N1)
       DO 222 JJJ=1,NC
          IV(JJJ)=0
 222   CONTINUE
C
```

```
      CALL ORDER(SUM,SUM1,N1)
C
      DO 212 I=1,NC
         I1=(I-1)*5+1
         I2=I*5
         J=IV(I)
 2222    FORMAT(2X,I2,3X,F10.6)
         WRITE(6,202) (IFACT(I,I3),I3=1,5),I,RRESP(IR,I,K2),
     +                SUM(I),DIF(I),V(I),SUM1(I),DIF(J)
202      FORMAT(2X,5I1,1X,I2,2X,F8.4,2X,F8.4,2X,F10.5,2X,F10.5,2X,
     +          F84.4,2X,F10.5,1X,I2)
212      CONTINUE
         GO TO 205
C
C            END OF SECTION 2
C
165   CONTINUE
C
C  * * *      SECTION 3:   PRINTING ROUTINES       * * *
C
         IX=IP(1)
         GOTO(500,501,500), IX
 500     DO 1 I=1,NR
            IF(IPRU(I).EQ.0) GOTO 62
C       DO 63 J=1,NA
C          A(J)=NBLANK
C63     CONTINUE
            DO 2 J=1,MSS
               IF(IP(J+1).EQ.0.) GOTO 1
               JK=IP(J+1)
               K3=IPRU(I)
               MT=NRUM(K3)
               WRITE(7,700) K3,(NREFU(K3,I1),I1=1,4)
               DO 20 I2=1,MT
                  KR=NARM(K3,I2)
                  NAR(I2)=KR
                  GOTO (900,901,902,903,904,905,906),I2
 900              WRITE(7,910) NAREAS(KR)
 910              FORMAT(1H+,T99,A1)
                  GOTO 20
 901              WRITE(7,911) NAREAS(KR)
 911              FORMAT(1H+,T101,A1)
                  GOTO 20
 902              WRITE(7,912) NAREAS(KR)
 912              FORMAT(1H+,T103,A1)
                  GOTO 20
 903              WRITE(7,913) NAREAS(KR)
 913              FORMAT(1H+,T105,A1)
                  GOTO 20
 904              WRITE(7,914) NAREAS(KR)
 914              FORMAT(1H+,T107,A1)
                  GOTO 20
 905              WRITE(7,915) NAREAS(KR)
 915              FORMAT(1H+,T109,A1)
                  GOTO 20
 906              WRITE(7,916) NAREAS(KR)
 916              FORMAT(1H+,T111,A1)
 20            CONTINUE
C              WRITE(7,710) (A(I2),I2=1,MT)
```

```
 710            FORMAT(1H+,T99,7(A4))
 700            FORMAT(2X,115('-'),/,2X,I2,3X,'*',T17,'MARGINAL  ',
     +         'RENT  ','REFINERY UNIT:',4A8,'*  $/TON  *   AREAS')
                WRITE(7,720)
 720            FORMAT(1H+,T117,'*',/,2X,115('-'))
C
                ISW=1
                MODEL=3+2*JK
                N2=2
                N3=3
 10             J1=1
                DO 3 K1=1,NC
                   IF(K1.EQ.32.AND.ISW.EQ.2) GOTO 33
                   DO 4 K=1,NA
                      PCOEF(K1,K)=NBLANK
 4                 CONTINUE
 33                DO 5 K=1,3
                      PVAR(K1,K)=NBLANK
 5                 CONTINUE
 3              CONTINUE
C
                IF(SSMOD(JK)-0.) 712,712,715
 712            WRITE(7,711) FEATM(3),FEATM(4),FEATM(ISW),SSMOD(JK)
                GOTO 716
 715            IF(SSMOD(JK).EQ.90.00) INMOD=90
                IF(SSMOD(JK).EQ.95.00) INMOD=95
                IF(SSMOD(JK).EQ.98.00) INMOD=98
                IF(SSMOD(JK).EQ.99.00) INMOD=99
                WRITE(7,701) FEATM(3),FEATM(4),FEATM(ISW),INMOD
 701            FORMAT(///,3X,58('-'),/,2A8,2X,A8,2X,'MODEL  FOR',
     +              I3,'%',' SUM OF SQUARES',/,3X,58('-'),///)
 711            FORMAT(///,30X,53('-'),/,T24,2A8,2X,A8,2X,'MODEL  FOR',
     +              ' *** ','% SUM OF SQUARES',/,T31,53('-'),///)
C
 716            CONTINUE
                DO 810 K=1,MT
                   K1=NAR(K)
                   GOTO(740,741,742,743,744,745,746),K
 740               WRITE(7,750) NAREAS(K1)
 750               FORMAT(3X,'VARIABLE',T25,'AREA',2X,A1)
                   GOTO 810
 741               WRITE(7,751) NAREAS(K1)
 751               FORMAT(1H+,T41,'AREA',2X,A1)
                   GOTO 810
 742               WRITE(7,752) NAREAS(K1)
 752               FORMAT(1H+,T57,'AREA',2X,A1)
                   GOTO 810
 743               WRITE(7,753) NAREAS(K1)
 753               FORMAT(1H+,T73,'AREA',2X,A1)
                   GOTO 810
 744               WRITE(7,754) NAREAS(K1)
 754               FORMAT(1H+,T89,'AREA',2X,A1)
                   GOTO 810
 745               WRITE(7,755) NAREAS(K1)
 755               FORMAT(1H+,T105,'AREA',2X,A1)
                   GOTO 810
 746               WRITE(7,756) NAREAS(K1)
 756               FORMAT(1H+,T121,'AREA',2X,A1)
 810            CONTINUE
```

```
              WRITE(7,757)
 757          FORMAT(//)
C
            DO 7 K1=1,NC
               IZ=0
               DO 8 K=1,MT
                  JNA=NARM(K3,K)
                  SSN(K)=SSRFAR(K3,JNA)
                  JUMP=(JNA-1)*9+K3
                  IF(SURFAC(MODEL,JUMP,K1).EQ.0.) IZ=IZ+1
                  IF(SURFAC(MODEL,JUMP,K1).NE.0.) THEN
                    PCOEF(J1,K)=SURFAC(MODEL,JUMP,K1)
                  ENDIF
                  IF(NC.EQ.32.AND.ISW.EQ.1) THEN
                    PCOEF(32,K)=SURFAC(MODEL,JUMP,33)
                  ENDIF
 8             CONTINUE
               IF(IZ.EQ.MT) GOTO 7
               DO 9 IN=1,N2
                  N4=IN+N3
                  PVAR(J1,IN)=KFACT1(K1,N4)
 9             CONTINUE
               J1=J1+1
 7          CONTINUE
C
            IF(ISW.EQ.1) J1=J1-2
            IF(ISW.EQ.2) J1=J1-1
            DO 6 J2=1,J1
               WRITE(7,703) (PVAR(J2,IN),IN=1,N2)
 703           FORMAT(3X,3A8)
               DO 800 K=1,MT
                  IF(PCOEF(J2,K).EQ.NBLANK) GOTO 800
                  IF(PCOEF(J2,K)-0.) 761,800,761
 761              GOTO(763,764,765,766,767,768,769),K
 763              WRITE(7,780) PCOEF(J2,K)
 780              FORMAT(1H+,T20,F14.6)
                  GOTO 800
 764              WRITE(7,781) PCOEF(J2,K)
 781              FORMAT(1H+,T36,F14.6)
                  GOTO 800
 765              WRITE(7,782) PCOEF(J2,K)
 782              FORMAT(1H+,T52,F14.6)
                  GOTO 800
 766              WRITE(7,783) PCOEF(J2,K)
 783              FORMAT(1H+,T68,F14.6)
                  GOTO 800
 767              WRITE(7,784) PCOEF(J2,K)
 784              FORMAT(1H+,T84,F14.6)
                  GOTO 800
 768              WRITE(7,785) PCOEF(J2,K)
 785              FORMAT(1H+,T100,F14.6)
                  GOTO 800
 769              WRITE(7,786) PCOEF(J2,K)
 786              FORMAT(1H+,T116,F14.6)
 800           CONTINUE
   6        CONTINUE
C
            J1=J1+1
            IF(ISW.EQ.1) WRITE(7,721) (PCOEF(J1,K),K=1,MT)
```

```
 721           FORMAT(/,3X,'RESIDUAL SS',T20,7(F14.6,2X))
               IF(ISW.EQ.2) GOTO 61
               ISW=2
               MODEL=MODEL+1
               N2=3
               N3=0
               GOTO 10
C
 61            CONTINUE
               WRITE(7,722) (PCOEF(32,K),K=1,MT)
 722           FORMAT(/,3X,'SE ',T20,7(F14.6,2X))
               IF(SSMOD(JK).EQ.0.) WRITE(7,713) (SSN(K),K=1,MT)
 713           FORMAT(/,3X,'*** % SS',T20,7(F14.6,2X))
             WRITE(7,704)
 704         FORMAT(1H1)
C
   2       CONTINUE
   1    CONTINUE
C
  62    IF(IP(1).EQ.3) GOTO 501
        RETURN
C
C        ****           PRINTING REFINERY UNITS PER AREA  ****
C
 501    DO 11 K=1,NA
           IF(IPAR(K).EQ.0) GOTO 205
           K3=IPAR(K)
           MAT=NRUMA(K3)
           DO 12 J=1,MSS
              IF(IP(J+1).EQ.0) GOTO 11
              JK=IP(J+1)
              MODEL=3+2*JK
              MODEL1=MODEL+1
              DO 17 I=1,NC
                 DO 18 II=1,7
                    PCOEF(I,II)=NBLANK
  18             CONTINUE
                 DO 19 II=1,5
                    PVAR(I,II)=NBLANK
  19             CONTINUE
  17          CONTINUE
              DO 13 I=1,MAT
                 JNR=NRUA(K3,I)
                 WRITE(7,700) JNR,(NREFU(JNR,I1),I1=1,4)
                 WRITE(7,710) K3
                 WRITE(7,720)
                 IF(SSMOD(JK)-0.) 717,717,718
 717             WRITE(7,709) FEATM(3),FEATM(4),FEATM(1),FEATM(2)
                 GOTO 719
 718             WRITE(7,705)FEATM(3),FEATM(4),FEATM(1),FEATM(2),
    +            SSMOD(JK)
 705             FORMAT(T26,69('-'),/,T26,2A8,1X,A8,2X,'AND',2X,A8,2X,
    +            'MODELS  FOR  ',F6.2,'  SUM  OF  SQUARES',/,T26,69('-'),
    +            ///,T8,'CODED',T35,'COEFF.',T51,'DECODED',T79,'COEFF.',
    +            /,T8,'VARIABLE',T32,'ESTIMATOR',T51,'VARIABLE',T76,
    +            'ESTIMATOR',//)
 709             FORMAT(T26,69('-'),/,T26,2A8,1X,A8,2X,'AND',2X,A8,2X,
    +            'MODELS  FOR  ***  ','  SUM  OF  SQUARES',/,T26,69('-'),
    +            ///,T8,'CODED',T35,'COEFF.',T51,'DECODED',T79,'COEFF.',
```

```
     +         /,T8,'VARIABLE',T32,'ESTIMATOR',T51,'VARIABLE',T76,
     +         'ESTIMATOR',//)
 719           J1=1
               JUMP=(K3-1)*9+JNR
               DO 14 K1=1,NC
                  IF(SURFAC(MODEL,JUMP,K1).EQ.0.AND.
     +               SURFAC(MODEL1,JUMP,K1).EQ.0) GOTO 14
                  PCOEF(J1,1)=SURFAC(MODEL,JUMP,K1)
                  PCOEF(J1,2)=SURFAC(MODEL1,JUMP,K1)
                  DO 15 IN=1,5
                     PVAR(J1,IN)=KFACT1(K1,IN)
  15              CONTINUE
                  J1=J1+1
  14           CONTINUE
               J1=J1-1
               DO 16 J2=1,J1
                  IF(PCOEF(J2,1).EQ.0.) THEN
                     WRITE(7,706) (PVAR(J2,IN),IN=4,5),
     +                            (PVAR(J2,IN),IN=1,3),PCOEF(J2,2)
                  ENDIF
                  IF(PCOEF(J2,2).EQ.0.) THEN
                     WRITE(7,707) (PVAR(J2,IN),IN=4,5),
     +                            PCOEF(J2,1),(PVAR(J2,IN),IN=1,3)
                  ENDIF
                  IF(PCOEF(J2,1).NE.0.AND.PCOEF(J2,2).NE.0) THEN
                  WRITE(7,708) (PVAR(J2,IN),IN=4,5),PCOEF(J2,1),
     +                         (PVAR(J2,IN),IN=1,3),PCOEF(J2,2)
                  ENDIF
 706              FORMAT(T8,2A8,T51,3A8,T76,F9.6,/)
 707              FORMAT(T8,2A8,T32,F9.3,T51,3A8,/)
 708              FORMAT(T8,2A8,T32,F9.3,T51,3A8,T76,F9.6,/)
  16           CONTINUE
               IF(SSMOD(JK).EQ.0.) WRITE(7,714) SSRFAR(JNR,K3)
 714           FORMAT(T8,'*** % SS',T32,F6.2)
               WRITE(7,704)
  13        CONTINUE
  12     CONTINUE
  11  CONTINUE
C
 205  RETURN
      END
```

```
C
C        THIS SUBROUTINE ORDERS ANY UNIDIMENSIONAL ARRAY IN
C        ASCENDING ORDER KEEPING THE ORIGINAL ELEMENTS
C        POSITIONS IN THE ARRAY IN A SEPARATE INDEX ARRAY IV(N)
C
C        N         IS THE ARRAY DIMENSION
C        VECTOR(N) IS THE ASCENDING ORDERED ARRAY
C        TORD(N)   IS THE ORIGINAL ARRAY
C
C        THREE PARAMETERS ARE REQUIRED BY THE SUBROUTINE, NAMELY,
```

```
C         ARRAYS TORD AND VECTOR, ANDTHE ARRAYS DIMENSION N
C
C         THE INDEX ARRAY IV(N), WILL BE KEPT IN A COMMON AREA
C
      SUBROUTINE ORDER(TORD,VECTOR,N)
C
      DOUBLE PRECISION TORD,VECTOR,TEMP,RRESP,D,SURFAC
      COMMON /DPAC/RRESP(9,32,7),D(32,32),SURFAC(10,63,33)
      COMMON /SPAC/NR,NC,NA,MSIGNS(32,32),NMVARU(9,7)
      COMMON /COMUN/FLEVEL(2,5),FDM(32),IR,K2,FACT(32),PCM(33),PDM(33),
     +   SSMOD(5),MODEL,MSS,SW,IP(6),IPRU(9),IPAR(9),NARM(9,7),NRUM(9),
     +   NRUMA(7),NRUA(7,9),JIND,SSRFAR(9,7)
      COMMON /FREE/IV(32)
      DIMENSION TORD(32),VECTOR(32),TEMP(32)
C
C         ORDER OF VECTOR ELEMENTS IN ASCENDING ORDER
C
      DO 1 I=1,N
         TEMP(I)=TORD(I)
1     CONTINUE
      M =N-1
      DO 10 I=1,M
         M1=I+1
         DO 11 J=M1,N
            IF(VECTOR(I).LE.VECTOR(J)) GO TO 11
            AUX=VECTOR(I)
            VECTOR(I)=VECTOR(J)
            VECTOR(J)=AUX
            IV(I)=J
11       CONTINUE
10    CONTINUE
2222  FORMAT(2X,F10.6)
C
C            ASSIGNMENT OF ORIGINAL INDEXES TO INCREASING
C            ORDERED VECTOR ELEMENTS
C
      DO 12 I=1,N
         DO 13 J=1,N
            IF(VECTOR(I).EQ.TEMP(J)) GO TO 14
 13      CONTINUE
 14      IV(I)=J
         TEMP(J)=0.
 12   CONTINUE
      RETURN
      END
```

```
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C *                                                                 *
C *   PROGRAM REGRES CALCUALTES A GREGRESSION LINE FOR A            *
C *   NUMBER-N OF INDEPENDENT VARIABLES; THREE INITIAL              *
C *   PARAMETERS ARE REQUIRED:                                      *
C *       M -  NO. OF OBSERVATIONS                                  *
C *       N -  NO. OF INDEPENDENT VARIABLES                         *
C *     INT -  INDICATOR OF INTERCEPT INCLUSION                     *
C *            0 IF NOT INTERCEPT                                   *
C *            1 IF INCLUDED                                        *
C *     INT MUST BE GIVEN INTERACTIVELY WHEN USING PROCEDURE        *
C *     SRWEMREP TO CALL PROGRAM REP18 FOR EXECUTION                *
C *            PROGRAM REP18 FOR EXECUTION                          *
C *                                                                 *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
      PROGRAM REGRES
      DOUBLE PRECISION XX,X,Y,B,XY,EY,RES
      DIMENSION XX(120,120),XT(120,120),X(120,120),Y(120),
     +         B(120),XY(120)
      DIMENSION EY(120),RES(120)
      REAL MRES
      READ 999,INT
 999  FORMAT(I1)
      READ(1,100) M,N
 100  FORMAT(2I3)
      J1=1
      DO 11 I=1,M
         B(I)=0.
         Y(I)=0.
         XY(I)=0.
         EY(I)=0.
         RES(I)=0.
C
         DO 12 J=1,N
            XX(I,J)=0.
            X(I,J)=0.
            XT(I,J)=0.
 12      CONTINUE
C
 11   CONTINUE
      MRES=0.
      SRES=0.
      SSRES=0.
      IF(INT.EQ.0) GOTO 20
      DO 1 I=1,M
         X(I,1)=1.
 1    CONTINUE
      J1=2
      N=N+1
 20   CONTINUE
      DO 2 I=1,M
         READ(1,150) Y(I),(X(I,J),J=J1,N)
 150     FORMAT(5F14.2)
```

```
  2     CONTINUE
C
C            TRANSPOSING OF MATRIX
C
        DO 3 J=1,M
           DO 4 I=1,N
              XT(I,J)=X(J,I)
  4        CONTINUE
  3     CONTINUE
C
C            INDEPENDENT VARIABLE VALUES MATRIX MULTIPLICATION
C
        WRITE(2,181)
 181    FORMAT(/,5X,'ELEMENTS X*X(TRANS) MATRIX',/)
        DO 5 J=1,N
           DO 6 K=1,N
              DO 7 I=1,M
                 XX(K,J)=XT(K,I)*X(I,J)+XX(K,J)
  7           CONTINUE
  6        CONTINUE
  5     CONTINUE
        DO 111 I=1,N
           WRITE(2,188) (XX(I,J),J=1,N)
 111    CONTINUE
        WRITE(2,182)
 182    FORMAT(/,5X,'ELEMENTS VECTOR Y(DEP.VAR.)*X(INDEP.VAR.)',/)
        DO 8 J=1,N
           DO 9 I=1,M
              XY(J) = Y(I)*X(I,J)+XY(J)
  9        CONTINUE
  8     CONTINUE
           WRITE(2,188) (XY(J),J=1,N)
C
        D=0.
        CALL MATINV(XX,N,D)
        WRITE(2,183)
 183    FORMAT(/,5X,'ELEMENTS INVERSE MATRIX X*X(TRANS)',/)
        DO 222 I=1,N
           WRITE(2,188) (XX(I,J),J=1,N)
 188       FORMAT(4F20.8)
 222    CONTINUE
C
C              COEFFICIENTS B= XY* XX(INV.)
C
        WRITE(2,184)
 184    FORMAT(/,5X,'MODEL COEFFICIENTS',/)
        DO 13 J=1,N
           DO 14 I=1,N
              B(J)=XY(I)*XX(I,J)+B(J)
 14        CONTINUE
 13      CONTINUE
        WRITE(2,180) (B(J),J=1,N)
 180    FORMAT(4F15.8)
C
C            COMPUTATION OF RESIDUALS
C
        WRITE(2,185)
 185    FORMAT(/,5X,'RESIDUALS: SS, MEAN, VARIANCE, STANDARD DEV.',/)
        DO 15 I=1,M
```

```
         DO 16 J=1,N
            EY(I)=B(J)*X(I,J)+EY(I)
 16      CONTINUE
         RES(I)=Y(I)-EY(I)
         SRES=SRES+RES(I)
         SSRES=SSRES+RES(I)*RES(I)
 15   CONTINUE
      MRES=SRES/M
      VAR=SSRES/(32.-N)
      STD=SQRT(VAR)
      WRITE(2,180) SSRES,MRES,VAR,STD
      WRITE(2,186)
 186  FORMAT(/,5X,'ESTIMATED OBS., OBS., RESIDUAL',/)
      DO 17 I=1,M
         WRITE(2,180) EY(I),Y(I),RES(I)
 190     FORMAT(3(2X,F8.4))
 17   CONTINUE
      WRITE(2,187)
 187  FORMAT(///,100('-'))
      STOP
      END
C
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C *                                                                    *
C *   SUBROUTINE MATINV(ARRAY,NORDER,DET)                              *
C *                                                                    *
C *   PURPOSE - INVERT A SYMMETRIC MATRIX AND CALCULATE ITS            *
C *             DETERMINANT                                            *
C *   PARAMETERS -                                                     *
C *      ARRAY - INPUT MATRIX WHICH IS REPLACED BY ITS INVERSE         *
C *      NORDER - DEGREE OF MATRIX (ORDER OF DETERMINANT,              *
C *               NO. OF COLUMNS OF ROWS)                              *
C *      DET - DETERMINANT OF INPUT MATRIX                             *
C *                                                                    *
C * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * * *
C
C
C
      SUBROUTINE MATINV(ARRAY,NORDER,DET)
      DOUBLE PRECISION ARRAY,AMAX,SAVE
      DIMENSION ARRAY(120,120), IK(10),JK(10)
 10   DET=1.
 11   DO 100 K=1,NORDER
C
C         FIND LARGEST ELEMENT ARRAY(I,J) IN REST OF MATRIX
C
      AMAX=0.
 21   DO 30 I=K,NORDER
      DO 30 J=K,NORDER
 23      IF(DABS(AMAX) - DABS(ARRAY(I,J)))24,24,30
 24   AMAX = ARRAY(I,J)
      IK(K) = I
      JK(K) = J
 30   CONTINUE
C
C           INTERCHANGE ROWS AND COLUMNS TO PUT AMAX IN ARRAY(K,K)
C
 31   IF(AMAX) 41,32,41
 32   DET = 0.
```

```
      GOTO 140
   41 I = IK(K)
      IF(I-K) 21,51,43
   43 DO 50 J=1,NORDER
         SAVE=ARRAY(K,J)
         ARRAY(K,J)=ARRAY(I,J)
         ARRAY(I,J)=-SAVE
   50 CONTINUE
   51    J=JK(K)
         IF(J-K) 21,61,53
   53    DO 60 I=1,NORDER
         SAVE = ARRAY(I,K)
         ARRAY(I,K)=ARRAY(I,J)
         ARRAY(I,J)=-SAVE
   60    CONTINUE
C
C             ACCUMULATE ELEMENTS OF INVERSE MATRIX
C
   61 DO 70 I=1,NORDER
         IF(I-K) 63,70,63
   63    ARRAY(I,K) = -ARRAY(I,K)/AMAX
   70 CONTINUE
   71 DO 80 I=1,NORDER
      DO 80 J=1,NORDER
         IF(I-K) 74,80,74
   74    IF(J-K) 75,80,75
   75    ARRAY(I,J)=ARRAY(I,J) + ARRAY(I,K)*ARRAY(K,J)
   80 CONTINUE
   81 DO 90 J=1,NORDER
         IF(J-K) 83,90,83
   83    ARRAY(K,J) = ARRAY(K,J)/AMAX
   90 CONTINUE
      ARRAY(K,K) =1./AMAX
  100 DET = DET * AMAX
C
C            RESTORE ORDERING OF MATRIX
C
  101 DO 130 L=1,NORDER
         K=NORDER-L+1
         J=IK(K)
         IF(J-K) 111,111,105
  105    DO 110 I=1,NORDER
            SAVE=ARRAY(I,K)
            ARRAY(I,K)=-ARRAY(I,J)
            ARRAY(I,J)=SAVE
  110    CONTINUE
  111       I=JK(K)
            IF(I-K) 130,130,113
  113    DO 120 J=1,NORDER
            SAVE = ARRAY(K,J)
            ARRAY(K,J)=-ARRAY(I,J)
            ARRAY(I,J)=SAVE
  120    CONTINUE
  130 CONTINUE
  140 RETURN
      END
```

CHAPTER 6: REFINERY RENT MODELS AND THE ROTTERDAM SPOT MARKET

The purpose of this Chapter is to apply the second kind of Refinery Rent models estimated in Chapter 5 to the product prices time series found in the Rotterdam spot market, in order to determine the models which represent adequately the refinery rents behavioural pattern in North West Europe through time.

6.1 PETROLEUM STATISTICS AVAILABILITY

For the sake of estimating refinery rents from market prices and thus being able to verify the Refinery Rent models, several petroleum statistics are required, it is intended to give an overview of the availability of such information.

First, for refinery rents estimation purposes (i.e., application of RR models to real data) the two models below will give an indication of the kind of data required:

- A regression model resulting from the 2^5 7-area WEM Factorial Design, which explains the response in 98% sum of squares,

  $$\rho_{CC}(B) = 900.62 - 17.51ALP - .315MGD + .034ALP.MGD - 1.955CCC + .0385ALP.CCC - .0008MGD.CCC - .00007ALP.MGD.CCC - 4.02SHC + .008CCC.SHC$$

- A linear regression model in terms of a reduced number of independent variables based on the marginal rents responses from the 2^5 7-area WEM FD responses,

  $$\rho_{CC}(B) = -3.28 + .55pms_p - .72hfo_p.$$

Both models permit the analyst to estimate rent values provided the underlying data (independent variables) are readily available, though assessing on the adequacy of data becomes a difficult matter because there is usually discrepancy between published data of different sources. The use of particular data may lead to biased information thus biased estimates. The choice of a data upon another is then at times subjective reflecting only the present needs of the analysis.

The study time horizon covers the 12-year period 1972-1983 for which reasonable oil statistics have been systematically reported by several sources though in many cases not on a fully comparable basis. The required time series to place in the models above are attached in Appendix D. Accordingly:

1. Time series of prices, namely Arabian Light price and oil refined product prices (motor spirit, gas oil, fuel oils and light products) at the Rotterdam spot market are published by *Oil Platt's Price Report*, *OPEC* and *Europ Oil*, among others. They agree well generally and in any case better than other oil satistics series.

2. The refinery unit capacities data show the major discrepancies between sources since no common way of measuring them seems to exist. They are reported as the processing charge capacity available either at the beginning of a year or at the end of a year, or simply as the capacity for the year, mostly in barrels per calendar day (total input divided by 365) and seldom in barrels per stream day (the maximum the refinery processing unit would be able to process during the effective available time of operation).

   Oil & Gas Journal, *OPEC*, and *BP statistical review of world energy* report data on refining capacities; the latter only on crude distillation. The three sources have been used in the present study. Because the figures are in barrels per calendar day (i.e., they include refinery unit iddle periods for turnarounds and downtime), some kind of uncertainty arises when comparing small capacity changes between two successive years for variations in capacity may be brought about by various means, namely, by a net increase in already existing capacity, by instalments of complete new units and/or by changes in severity of operation which in turn produce changes in capacity utilization.

   If a variation were small it could be assumed there was not addition to capacity *but* that the equipment was run at a different utilization rate, still the figures would not represent actual available capacities. Similarly, if a variation represented a net addition, it would be nearly impossible to point it out since effects of increasing capacity utilization and/or net expansion are not singled out. If all figures were reported in barrels per stream day, or the availability factors of refinery units known, the effective capacity would directly follow; and it is precisely this information what is seldom found in the data.

From 1982 onwards *O&GJ* reports on closures of inactive refineries making possible the assessment on absolute actual decreases in capacity (this happens to be in hydroskimming refineries, i.e., in crude and reforming capacities), yet not on absolute actual increases.

Summarizing this point, the fact that the percentages of refinery units capacity utilization are seldom published and if so, referring only to crude distillation units, makes extremely hard to assess on real figures of existing refinery processing units capacities (other than crude distillation), and hence on net expansions.

3. Worldwide motor gasoline demand figures are published as consumption figures rather[121] and also differ among statistical sources. OECD publishes quarterly oil statistics while BP publishes annual data. Monthly data are systematically reported by CPDP in its *Bulletin Mensuel* for most of the OECD countries (some missing, namely, Finland, Portugal, Switzerland and Greece; Finland and Switzerland counting for the particular North West Europe region though not at a great level); OPEC also reports monthly demand figures. All three sources have been used to produce the figures reported here.

4. Worldwide tanker fleet availability is found in *BP statistical review of world energy* and *Lloyds Shipping Economist*. The latter reporting also on tanker fleet effective demand.

With time series 1. to 4. refinery rents can be estimated from the empirical models. In Chapter 5, RR from the first kind of RR model (i.e., those derived from the 2^5 7-area WEM FD) were already estimated using the series of data 1. - 4. presented in Table 5-6. In following sections the estimated rents from the second kind of RR model, namely, those in term of product prices derived by Linear Regresssion Analysis, are estimated.

[121] Actual demand figures may well be lower than consumption figures because of stock changes, etc. For this work consumption figures were however used as they are widely available, and do not cause generally great differences in results.

Second, for refinery rent models validation purposes, it is needed series of refinery processing units rents (refinery technology rents), information which is not found in commonly available oil statistics and possibly not available at all for the researcher.[122]

The oil companies' annual reports fail to report the kind of data required. They present refinery's throughput data, and gross and net income figures which together with input and product prices may provide the basis for a refinery rent estimate. However, these data are for the whole of the company group and not for the group's refinery specific in the area concerned, on top of being generally manipulated and adequately presented for tax purposes.

Due to this shortcoming, validation can not be carried out to all its extent, hence that proxies become necessary: next section deals with the problem of finding a proxy for refinery rents.

[122] The author has gone through many oil companies and other petroleum publications available as well as addressed to several petroleum institutions as the Institute of Petroleum (London), Petroleos de Venezuela (UK) S.A. (London), Institut Francais du Petrole (Paris); to general energy research centres, Nuclear Research Centre (Juelich, Germany); and libraries, Patent Office Library (London), University of London Library, Queen Mary College Library (London), and others, without succeeding in finding the required information.

6.2 A PROXY OF REFINERY RENTS

The proxies for models' validation are based on two different pieces of information. For a paticular year:

- the refinery processing units' capacity changes; and,

- the unit investment costs of new plant options.

When comparing the rents estimated values with actual capacity changes for the period under consideration, one would expect high level of rents to correspond to increases in capacity (due consideration to time-lags in construction) either due to enlargements or to increases in capacity utilization. Conversely, low or zero rents would correspond to no expansion or unexpected shutdowns or even closures of inactive units.

In the long run, refinery rents are an indication of the unit investment cost a refiner should consider to pay for having additional capacity installed. Because such investments should produce additional profit or make the refiner's market competitiveness stronger, its value must be greater than the *actual cost of expansion*. Thus it is assumed a refiner should have been accruing a sustained level of *high* rents for a period more than the short run before making an investment decision on new capacity.

In other words, the refiner would consider to invest when the rent were *consistently higher* than or *equal* to the *average annual capital charge* on new investment (or annualized investment cost), this is the actual cost of expansion. The average annual capital charge per metric tonne of refining capacity comprises:

- a 15% discounted cash flow return on invested capital in new refinery technology, plus,

- a fixed cost element including maintenance, labour, overhead and insurance costs.

The magnitude of the fixed cost is not known with precision, however, approximations can be obtained: fixed costs are estimated to be 10% of the unit investment cost of the technology to be expanded or newly built. The average annual capital charge value resulting from the sum of the above terms is about 25% of the unit investment cost (total capital investment devided by capacity) per metric tonne on input feed.[123]

The condition for investment on a particular refinery technology is that the associated marginal rent satisfies the relation,

$$\text{marginal rent} - .25 \text{ unit investment cost} \geq 0.$$

This condition is used in deriving the proxies of refinery rents.

As mentioned earlier, net enlargements to existing units are difficult to identify from data on capacity changes. Thus for a *particular refinery processing unit* in a *particular year*, the following is assumed,

- if a substantial increase in capacity occurred, *then* a *real increase* in capacity has also occurred; and,

- if comparing the estimated level of rents for that year, or for a previous one (to account for time-lags in construction), with that year's *average annual capital charge*, the estimated rent is greater than the latter, *then* a *real expansion* occurred and the estimated rent is a good approximation to the actual level of rent.

With the former the existence of a relationship between refinery rents, refinery unit investment costs (refinery technology costs) and refinery unit capacity changes is assumed. The 'shape' of the relationship is unknown, but it is believed they move on the same direction; it is also sustained that such a relating mechanism is not straightforward, assuming however its existence helps in approximating a figure (refinery rent) which as yet and as far as the author's knowledge goes, has not been reported and makes up an important profit indicator in the short run and a potential figure for investment decisions in the long run.

Support to the latter has been already found in O'Carroll,[124] where exercises with the same 7-area WEM covering 1964-1971 market conditions were undertaken,

---

[123] This figure seems to be a reasonable approximation to the average annual capital charge as the 7-area and 22-area WEM data base imply.

[124] O'Carroll, F.M., 'Price Determination and Economic Mechanisms in the European Oil Industry 1964-71', Ph.D thesis, op.cit., pp.120-122.

> The values given for local reforming and cracking [marginal values] represent the immediate financial incentive to increase capacity for these processes, and we should therefore expect high marginal values to be associated with subsequent increases in capacity as a result of plan expansion and building. [...]. The existence of a relationship between marginal value and investment is, on the whole, supported.

Finally the refinery rent proxy is considered:

The refinery rents *proxies* are *the average annual capital charges* based on *the unit investment costs of refinery technology through time*, and, *the associated capacity expansion*.

Thus to estimate the average annual capital charges the time series of refinery unit investment costs are required. It should be noted that these figures are also imperfect. There is a wide spectrum of processes available for a single refinery processing unit as well as licensors for commission on new construction. The offers vary between licensors and between countries in and outside Europe.

The intention is to verify for North West Europe (i.e., area B of the 7-area WEM). The region includes Scandinavia, United Kingdom, Ireland, Netherlands, Belgium, North of France, FR Germany, Switzerland, and Austria.

In order to obtain *reliable* figures for the three technologies, namely, catalytic reforming, catalytic cracking and hydrocracking, the following is required:

1. Domestic (country) investment costs of the technology concerned, for the years 1972-1983, assuming most of the equipment is built in Europe.

2. Exchange rates 1972-1983 country currency against he USA Dollar to unify currencies (and make figures comparable to the monetary unit of the model).

3. Average per year on all country investment costs will give the desired estimates of actual technology investment costs for North West Europe.

Following the former outline the time series (in \$/mt/Year) investment costs 1972-1983 were intended to fill in. A good approximation to the domestic unit investment cost (step 1. above) should follow by knowing every intermediate cost component of the particular technology for every refinery within the country, and for every country within the area. This entails indeed a great deal of work which includes access to the oil industry files, and to the different manufacturers in the market. It would certainly require another thesis to make the evaluation of oil refinery technologies available in North West Europe.[125]

The alternative followed was to obtain the actual, aggregate investment cost figure. In so doing, once again a confrontation with lack of published information arose; the available, if at all, comes from casual publications or direct institutional communications

With no access to oil companies information, the researcher is constrained to whatever he can get or estimate to carry on analyses.

Great deal of information regarding technological and chemical aspects of refining, cost estimating techniques, and others, is actually published for the USA refinery sector.[126] Unfortunately, such a publication is not available for North West Europe. There do exist series of price indices on investment goods and others, but not actual investment costs.

The information gathered is summarized in Table 6-1. With such a reduced set of data it is not much what one can do. The most reliable data are the three last for the FR Germany for they are based on oil companies' information and used for an ongoing work at a german research centre.[125]

In view of this restriction, the series of investment costs 1972-1983 for the technologies above were estimated on the basis of these few data by following some conventions:

1. Use of the price index for technology construction when available, or the price index for investment goods instead, to estimate the series of investment costs 1972-1983.

---

[125] A research to be reported in a doctoral thesis is currently being undertaken at the Programme Group on Systems Research and Technological Development (STE) of the KFA, Juelich, FR Germany, on the oil refinery technology evalutaion of the FR Germany.

[126] See Nelson, W.L., *Guide to Refinery Operating Costs*, op.cit.

For the FR Germany the price index for technology construction was applied to the 1984 base data; this index is periodically published in *Chemische Industrie*.

For Netherlands and the UK the country price index for investment goods was applied to the 1980 cracking and hydrocracking data respectively.

In this way, series of investment costs were obtained in relation to either 1980 or 1984 as the base year:

a. A reforming time series based on 1984 data, FRG, DM currency.

b. Two cracking time series, one based on 1984 data, FRG, DM currency; and another one based on 1980 data, NETH, GL currency.

c. Two hydrocracking time series, one based on 1984 data, FRG, DM currency; and another one based on 1980 data, UK, STL currency.

2. Use of GDP/GNP country deflators to transform the investment costs 1972-1983 above into 'nominal' 1972-1983 figures to which the respective exchange rates can be applied.

With these two conventions point 1., (page 289), is fulfilled. Points 2. and 3. follow readily. The resulting investment costs in \$/mt/Year (nominal) are given in Table 6-2.

Table 6-1. Domestic investment costs for catalytic reforming, catalytic cracking and hydrocracking in some european countries.

Year	Reforming	Cracking	Hydrocracking	$/mt/Year REF CC HYD
1980	87. DM FRG	213. DM FRG	268. DM FRG	48. 117. 147.
		276. GL NETH	46. STL UK	139. 107.
1982		250. DM FRG	350. DM FRG	105. 144.
1984	123. DM FRG	260. DM FRG	295. DM FRG	43. 91. 104.

Sources: 1980 figures from IEA, Etsap project, STE, KFA, Juelich, FR Germany; 1982 figures from Esso Magazin, 1/82; 1984 figures from STE, direct communications. Exchange rates applied from Table D-9, appendix D.

Table 6-2. 1972-1984 North West Europe estimated catalytic reforming, catalytic cracking, and hydrocracking investment costs.

Year	REF	Cracking		Hydrocracking		Average $/mt/Y		
	FRG	FRG	NETH	FRG	UK	REF	CC	HYD
1972	12.2	25.9	26.0	29.4	35.0	12.2	26.0	32.2
1973	16.3	34.6	36.0	39.1	41.0	16.3	35.3	40.0
1974	19.7	41.6	45.0	47.0	47.0	19.7	43.0	47.0
1975	23.5	49.5	55.0	56.0	56.0	23.5	52.3	56.0
1976	24.7	52.3	58.0	59.0	50.0	24.7	55.2	55.0
1977	29.0	61.6	83.0	70.0	56.0	29.0	72.3	63.0
1978	36.8	78.0	105.0	88.0	69.0	36.8	91.5	79.0
1979	44.4	93.7	124.0	106.0	85.0	44.4	109.0	96.0
1980	50.4	106.5	139.0	121.0	107.0	50.4	122.8	114.0
1981	44.7	94.7	131.0	107.3	95.0	44.7	113.0	101.0
1982	46.4	98.0	123.0	111.0	91.0	46.4	110.5	101.0
1983	46.6	98.6	120.0	111.6	82.0	46.6	109.0	97.0
1984	43.2	91.0	112.0	103.7	78.0	43.2	102.0	91.0

Source: GDP/GNP deflators and exchange rates, 1972-1984 found in Table D-9, Appendix D, were applied to the figures of Table 6-1.

The highlighted figures of Table 6-2 are the time series used for calculating the average annual capital charges, i.e., ***the final proxies for validation***, estimated as the 25% of the unit investment costs. These series are presented in Table 6-3 and depicted in Figure 30 next page.

Table 6-3. 1972-1984 North West Europe estimated average annual capital charges of refinery technology.

Year	Catalytic Reforming	Catalytic Cracking	Hydrocracking
1972	3.05	6.50	8.05
1973	4.08	8.83	10.00
1974	4.93	10.75	11.75
1975	5.88	13.08	14.00
1976	6.18	13.80	13.75
1977	7.25	18.08	15.75
1978	9.20	22.88	19.75
1979	11.10	27.25	24.00
1980	12.60	30.70	28.50
1981	11.18	28.25	25.25
1982	11.60	27.63	25.25
1983	11.65	27.25	24.25
1984	10.80	25.50	22.75

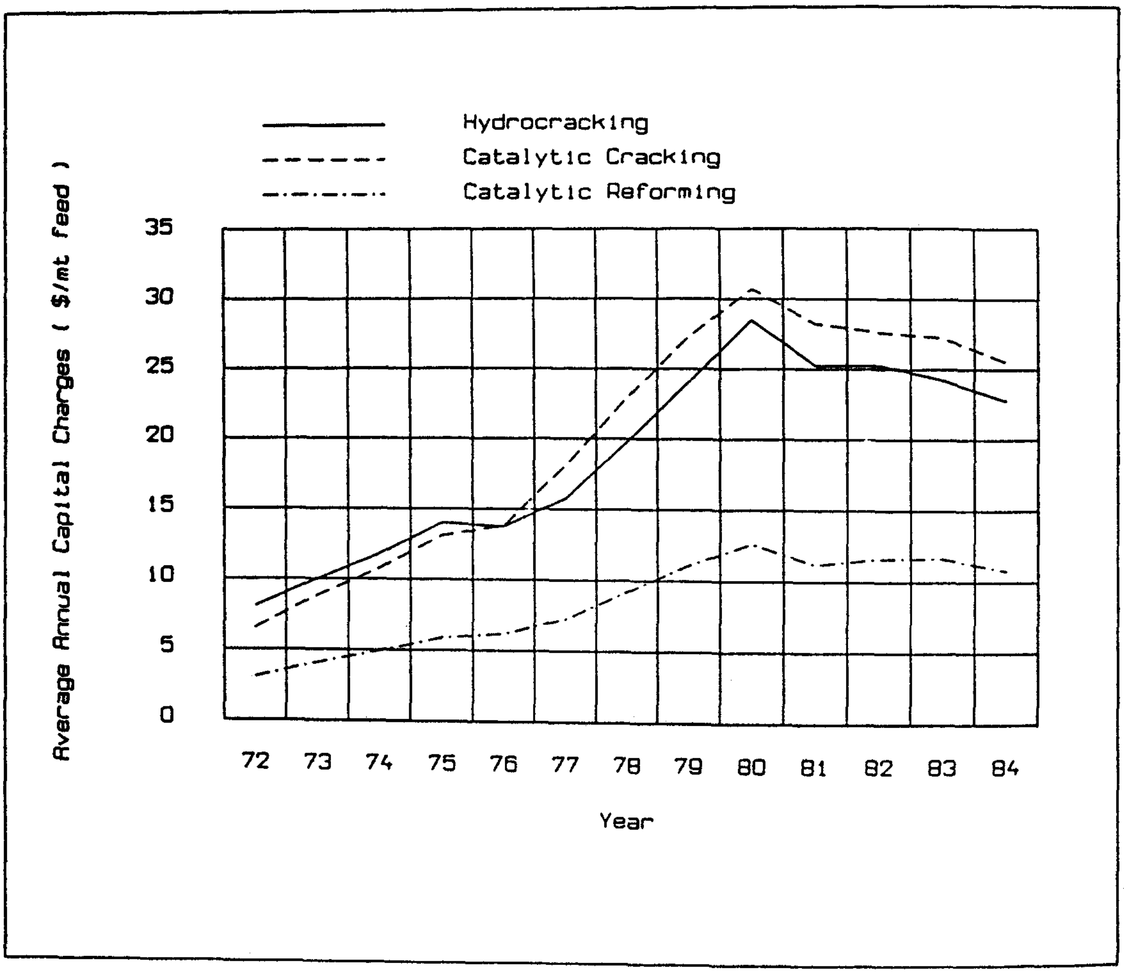


Figure 30. 1972-1984 North West Europe estimated average annual capital charges of refinery technology.

6.3 1972 - 1983: A HISTORICAL BACKGROUND

By looking at historical trends in product demands and prices, and at refinery units capacity changes, additional understanding and assumptions concerning historical developments of levels of rents can be drawn.

Referring to Figure 28 on page 215 (prices) and Figure 29 on page 216 (demands), and Figure 31 and Figure 32 on page 296 (capacities), it is possible to see that the changes in refinery processing units capacities, specially catalytic cracking and hydrocracking's, are related to the development of product prices through time. This has an interesting implication: since rents in the short run are price determined, and the level of rents indicates future possible investments, higher prices determine higher rents hence greater incentive to expand reflected in the increasing capacity. This supports our previous assumption that a relationship exists between rents and investment.

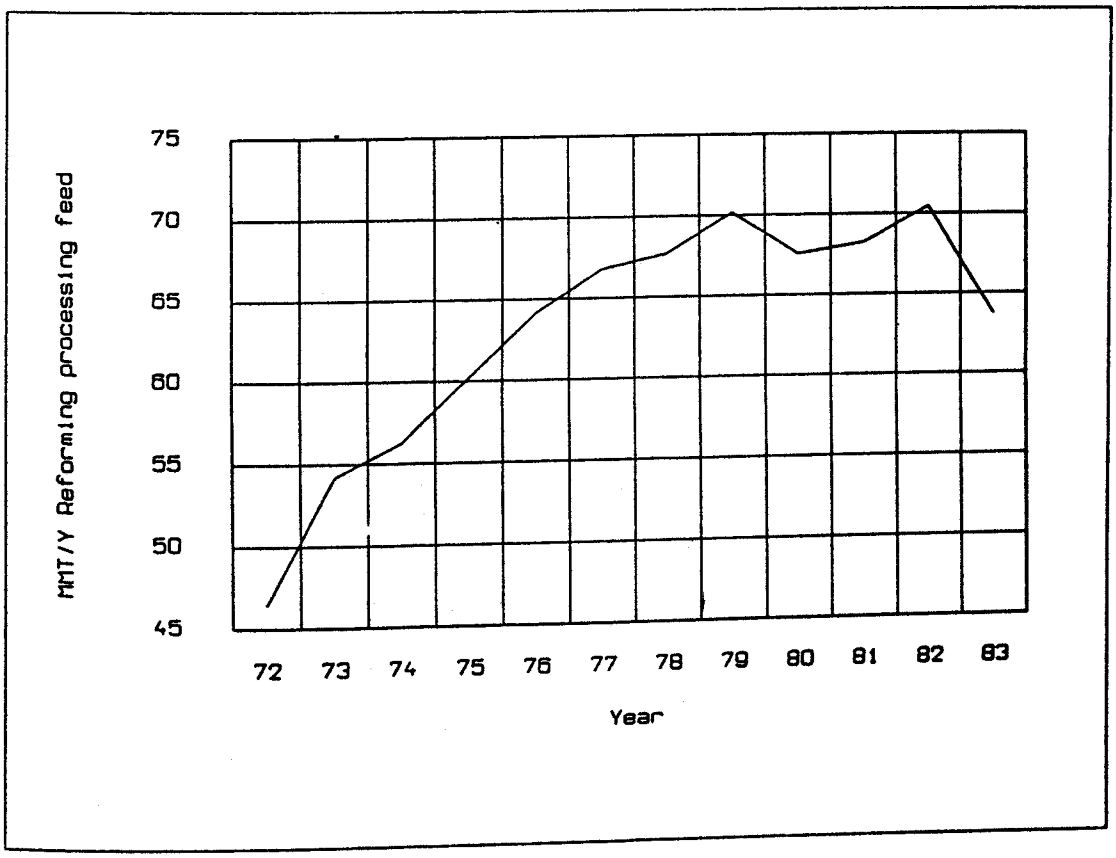


Figure 31. 1972-1983 North West Europe catalytic reforming capacity.

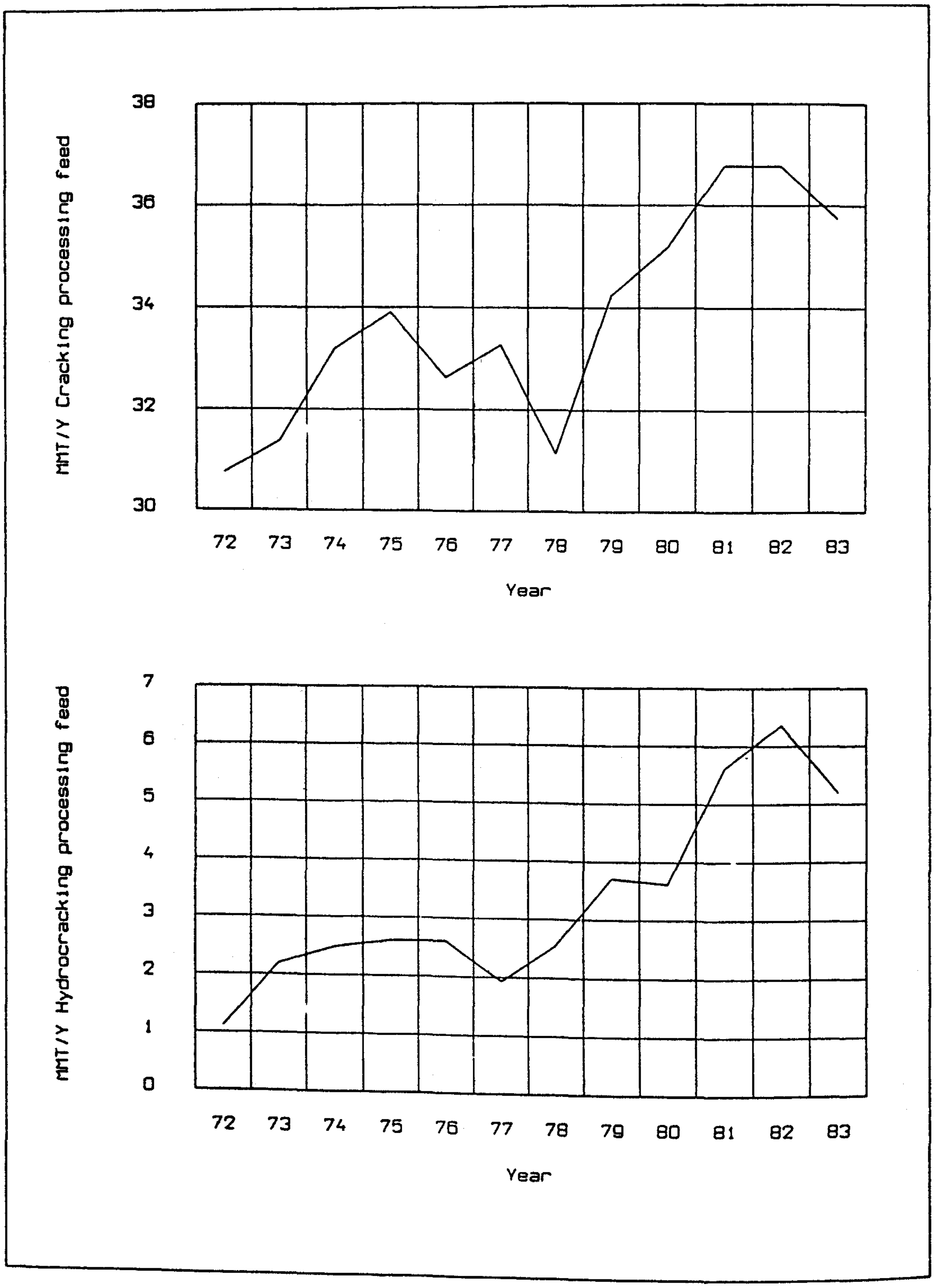


Figure 32. 1972-1983 North West Europe catalytic cracking and hydrocracking capacities.

From both previous figures an illustrative table with the yearly changes is drawn. This is Table 6-4 below. Signs '+' and '-' represent respectively increases and decreases in magnitudes.

Table 6-4. 1972-1983 North West Europe, product demands, refinery capacities, product prices: changes related to previous year.

	Product demands			Refinery units			Crude and product prices			
	mgd	dfo	hfo	REF	CC	HYC	AL	PMS	DFO	HFO
1972	+	+	+	+	+	+				
1973	+	+	+	+	+	+	+	+	+	+
1974	-	-	-	+	+	+	+	+	+	+
1975	+	+	-	+	+	+	+	-	+	+
1976	+	+	+	+	-	=	+	+	+	+
1977	+	+	-	+	+	-	+	-	+	+
1978	+	+	+	+	-	+	+	+	+	-
1979	+	+	+	+	+	+	+	+	+	+
1980	+	-	-	-	+	-	-	+	-	+
1981	-	-	-	+	+	+	-	+	-	+
1982	+	-	-	+	-	+	-	-	-	-
1983	+	+	-	-	+	-	-	-	-	-
1984				-	+	+				

A historical account is following:

1973 Increases in main oil product demands together with increases in hydroskimming and cracking (catalytic and hydro) capacities; increases in crude and product prices. Cracking up, interpreted as change in capacity utilization, the input feed, gas oil and/or heavy fuel oil directed to satisfy final demand. High level of rents are expected by the end of 1973 beginning of 1974 since a major oil price increase occurred in autumn 1973.

1974 Product demands go down while prices continue to rise as well as hydroskimming and cracking capacities. The fact capacities increase even though product demands go down, could be a clear indication of a net expansion of capacity occurring in 1974 as a result of investments made in previous years, some two years before to account for construction lags.

1975 Demands for motor spirit and gas oil increase while the demand for heavy fuel oil drops. Catalytic cracking and hydroskimming capacities increase which could be a reflection of 1973-1974 investments. For reforming could mean an increase in utilization to account for motor gasoline increases in demand. Rents are expected either to fall or remain constant in spite of higher usages due to price decreases.

1976 The demands for the three products increase accompanied by decreases in cracking capacity usage and increases in prices. The level of rents might well show an increase driven by the price increase of products. The fact that demand increases do not reflect an increase in cracking capacity utilization could be explained by the use of light crudes and straight-run distillates to meet the chemistry of demand hence that hydroskimming refineries could have still been producing positive rents which eventually might have led to investment for expansion.

1977 Motor gasoline and gas oil demands increase; heavy fuel oil demand drops. This makes heavy fuel oil to be used as cracking feed to produce the catalytic cracked spirit and gas oil required for final use, hence that hydrocracking capacity usage drops too. However, even if catalytic cracking increases, the drop in premium motor spirit price may cause a decline in both catalytic cracking and reforming rents.

1978 Increases in the three slates of demand: now heavy fuel oil is directed to final demand and to hydrocracking as decreases in catalytic cracking capacity occur. Hydrocracking is the unit satisfying the increases in gas oil and motor spirit demands. Rents could eventually rise as heavy fuel oil is dropping in price, however, this effect may be neutralized by the less use of cracking.

1979 A new crude price increase occurs at the end of 1979 with a consequent product price increase at Rotterdam. Also increases in product demands occur. Increases in catalytic cracking and reforming capacities are expected to rise the level of rents.

1980 Increases in motor spirit demand are satisfied by cracking both gas oil and heavy fuel oil since their own demands decrease. The increase in premium motor spirit price and the drops in crude and heavy fuel oil prices may produce a rent in the catalytic cracking unit.

1981 Chemistry of demand and refinery units capacity utilizations show the same situation as in 1974. An increase in previous year's prices generates decreases in product demands and additionally in product prices. Increases in capacities could have been brought about by previous investment decisions (for example in 1979-1980) as was the case in 1974. Low but positive levels of cracking rents expected.

1982 Falling gas oil and heavy fuel oil demands and catalytic cracking capacity utilization. Motor spirit demand increases; the now excess middle and heavy distillates are hydrocracked to satisfy together with reformates the increase in motor spirit demand. Rents are expected to remain constant or lower.

1983 The motor spirit and gas oil demand increases bring about catalytic cracking capacity utilization. The catalytic reforming capacity goes down due to absolute closures of inactive units in Western and in particular, in North West Europe. The product price profile deteriorates as a reflection of the March 1983 OPEC agreement to cut price down to U$29. per barrel of Arabian Light. Rents expected to lower.

In accordance to this historical background a graph of expected total level of rents 1972-1983 is depicted in Figure 33 on page 300. This general trend may be expected for complex refineries since for hydroskimming ones a record of closures as from 1980 onwards is registered meaning the catalytic reforming units have not been producing rents for investment during the early eighties.

It is worth to point out this account is only a guide of (some) happenings. Other factors likely to affect rents responses are not considered above, namely,

• at any one year, particular weather conditions (other than normal seasonal changes) may affect the chemistry of demand hence the product price profile hence the level of rents;

• inventories, specially of crude oil;

• other refinery processing units as residue desulphurization, coking and alkylation play an important role in the whole of the refinery and certainly interact with existing units;

• the substitution process of oil refined products for alternatives which is taking place; and,

• social/political events may cause changes in the chemistry of supply/demand that in turn may have serious implications in market prices, therefore in refinery operations and rents (a recent example is the U.K. coal strike which had clear effect on heavy fuel oil prices and rents, a matter of discussion of Chapter 7).

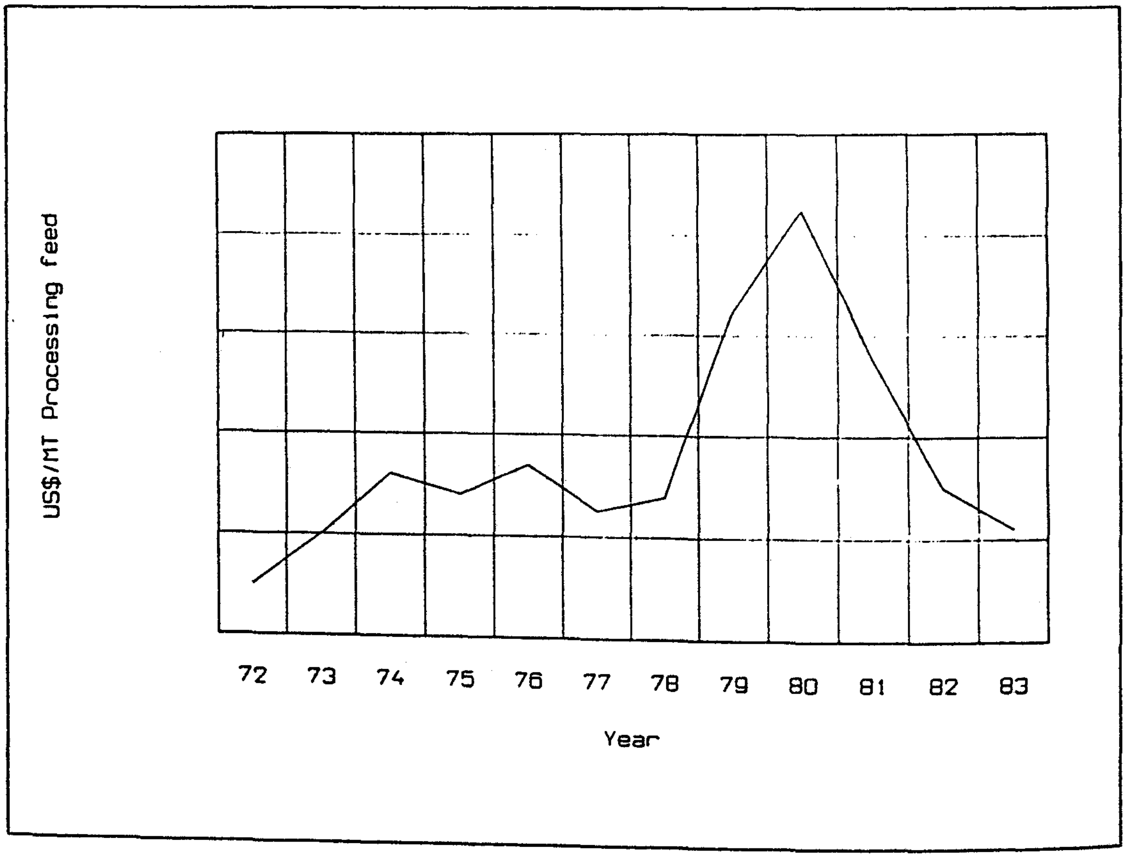


Figure 33. 1972-1983 North West Europe expected refinery rents general trend.

6.4 FITTING THE SECOND KIND OF REFINERY RENT MODEL: Application to North West Europe

For every refinery processing unit twelve models are used to estimate refinery rents from Rotterdam spot prices. The models are derived from parameters of tables 5-7 to 5-10, Chapter 5.

Further, graphs showing the estimated refinery rents at Rotterdam by using each of the models are depicted. The product prices time series used are the 1972-1983 Rotterdam spot prices of Table D-10, Appendix D. The sequence of graphs is as follows:

- Figure 34 on page 302 to Figure 37 on page 305 for catalytic cracking rent in section 6.4.1;
- Figure 38 on page 306 to Figure 41 on page 309 for catalytic reforming rent in section 6.4.2;
- Figure 42 on page 310 to Figure 45 on page 313 for hydrocracking rent in section 6.4.3.

It is noted that each figure depicts the trends in refinery rents given by three different approximation models. Each group of three models, in turn, represents a given oil system condition, namely, that condition implied by the experiment setting up. In this way, when talking for instance about models '7-area WEM Parametric dfo' the refinery rent models are assumed to be derived when the system is continuously increased in the demand for gas oil, always in the North West Europe region.

The acceptance of one or another model may lead to the conclusion that the oil market in time was behaving around the conditions experimentally set up to derive the model.

6.4.1 Catalytic Cracking

Models based on the 7-area WEM Factorial Design responses:

1 $\rho_{CC}(B) = -3.28 + .55p_{pms} - .72p_{hfo}$

2 $\rho_{CC}(B) = -1.27 + .38p_{pms} + .5p_{dfo} - 1.07p_{hfo}$

3 $\rho_{CC}(B) = 2.23 + 1.36p_{dfo} - 1.55p_{hfo}$

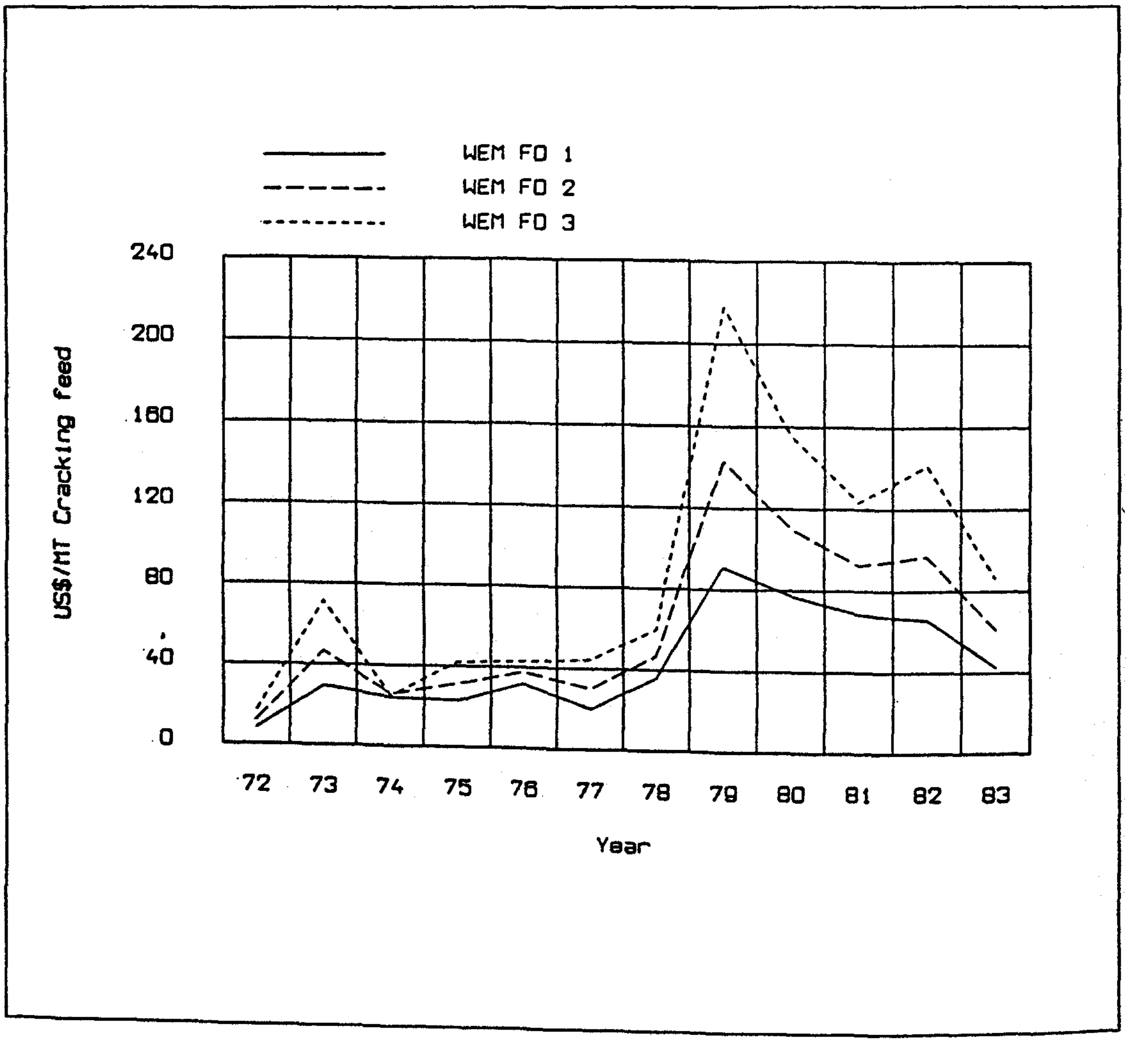


Figure 34. 1972-1983 catalytic cracking estimated rent: second kind of Refinery Rent models, 7-area WEM FD responses.

Models based on responses from the 7-area WEM Gas Oil demand parametric cases:

1 $\rho_{CC}(B) = 1261.7 - 6.95p_{pms} + 4.73p_{hfo}$

2 $\rho_{CC}(B) = -235.86 + 1.71p_{pms} + .25p_{dfo} - 1.70p_{hfo}$

3 $\rho_{CC}(B) = 44.71 + .21p_{dfo} - .38p_{hfo}$

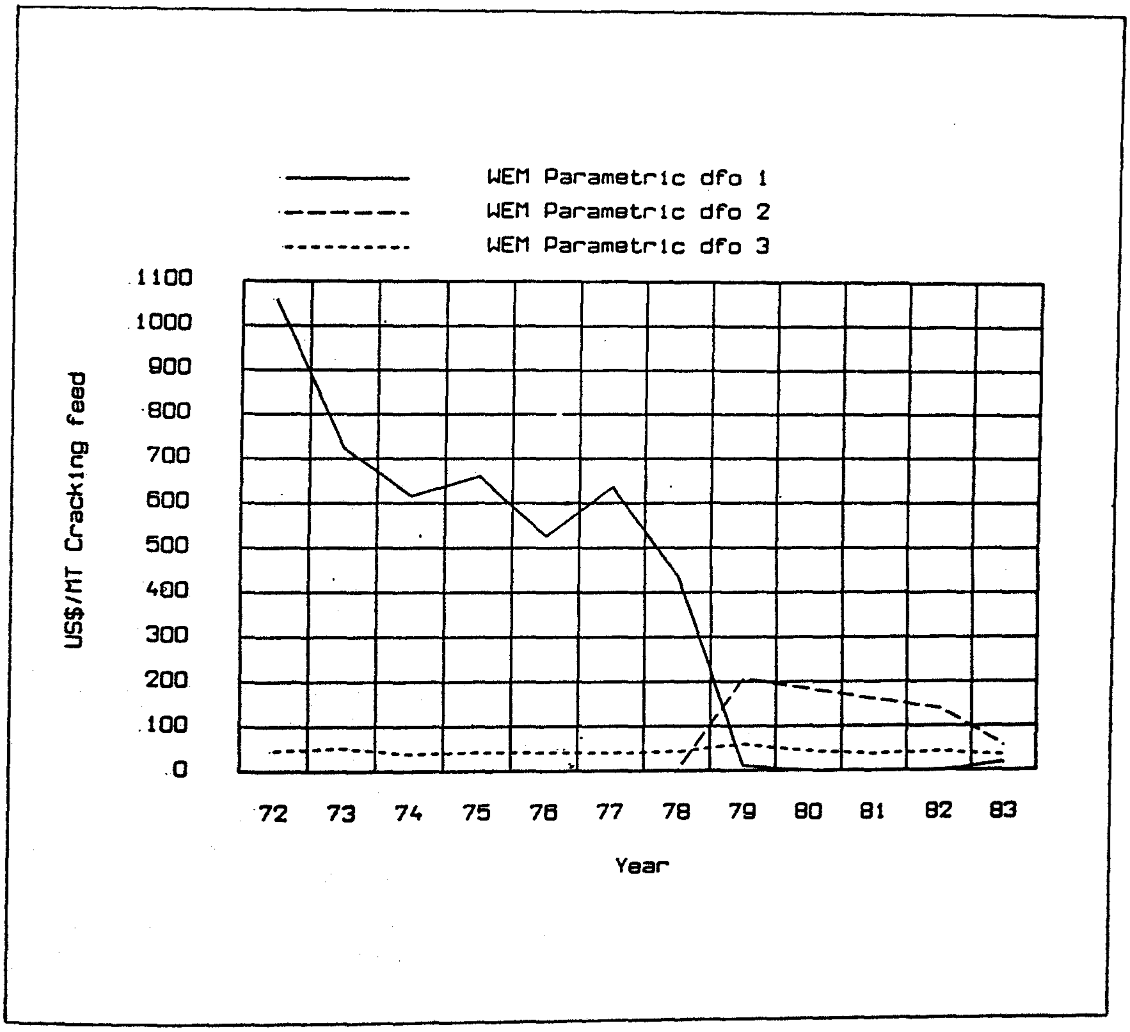


Figure 35. 1972-1983 catalytic cracking estimated rent: second kind of RR models, 7-area WEM DFO demand change responses.

Models based on responses from the 7-area WEM Premium Motor Spirit parametric cases:

1 $\rho_{CC}(B) = 84.20 + .38p_{pms} - .84p_{hfo}$

2 $\rho_{CC}(B) = 51. + .36p_{pms} + .11p_{dfo} - .85p_{hfo}$

3 $\rho_{CC}(B) = 1728. - 5.81p_{dfo} - 1.14p_{hfo}$

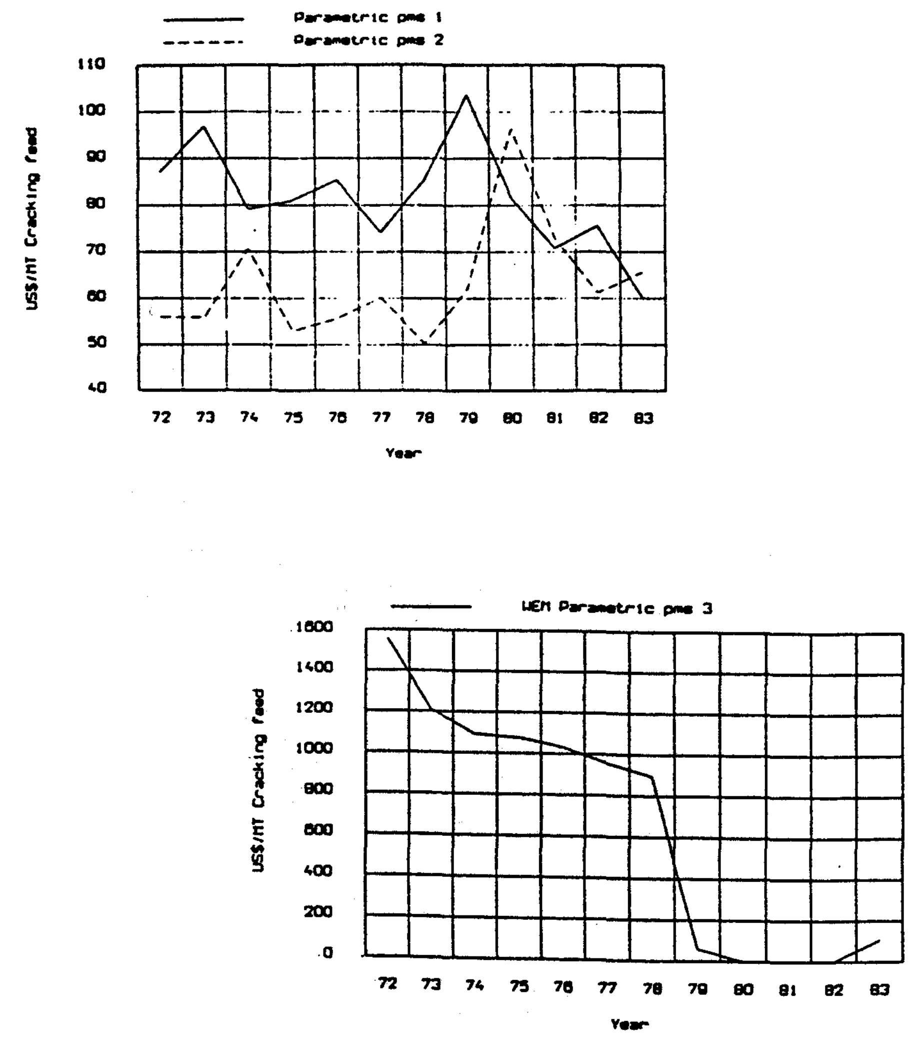


Figure 36. 1972-1983 catalytic cracking estimated rent: second kind of RR models, 7-area WEM PMS demand change responses.

Models based on responses from all 7-area WEM experimental cases, namely Factorial Design and Parametric Programming:

1 $\rho_{CC}(B) = 2.60 + .57p_{pms} - .81p_{hfo}$

2 $\rho_{CC}(B) = -.5 + .40p_{pms} + .45p_{dfo} - 1.06p_{hfo}$

3 $\rho_{CC}(B) = -1. + .99p_{dfo} - 1.07p_{hfo}$

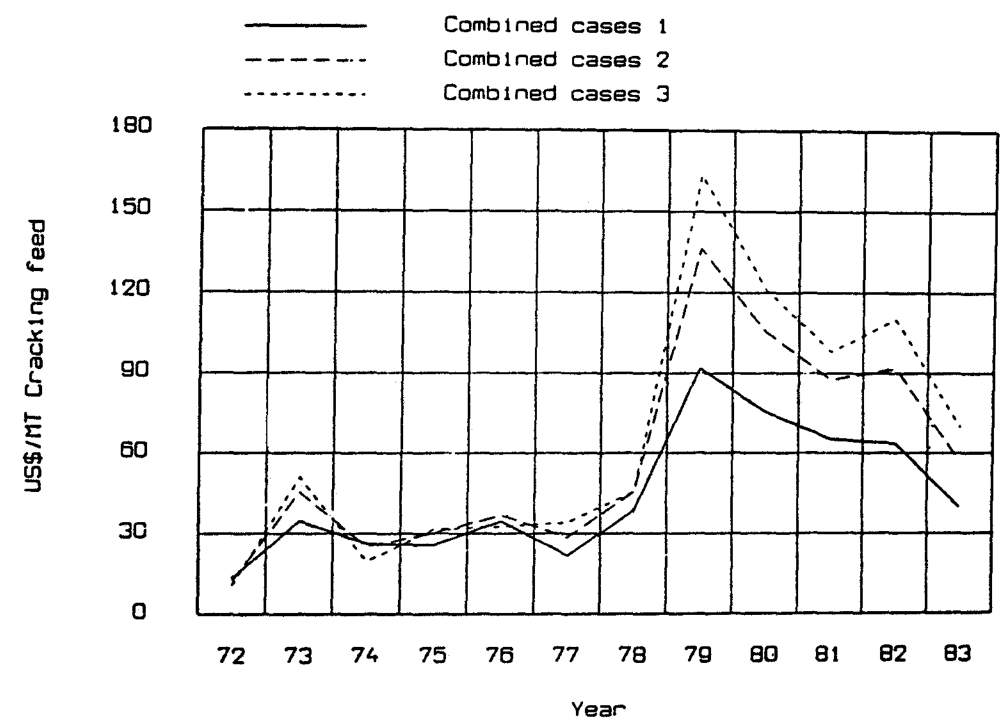


Figure 37. 1972-1983 catalytic cracking estimated rent: second kind of RR models, 7-area WEM all combined responses.

6.4.2 Catalytic Reforming

Models based on the 7-area WEM Factorial Design responses:

1 $\rho_{REF}(B) = -2.17 + .89p_{pms} + .0133p_{ker} - 1.p_{srb}$

2 $\rho_{REF}(B) = 1.65 + .61p_{pms} - .89p_{hfo}$

3 $\rho_{REF}(B) = -2.32 + .89p_{pms} + .031p_{hfo} - 1.014p_{srb}$

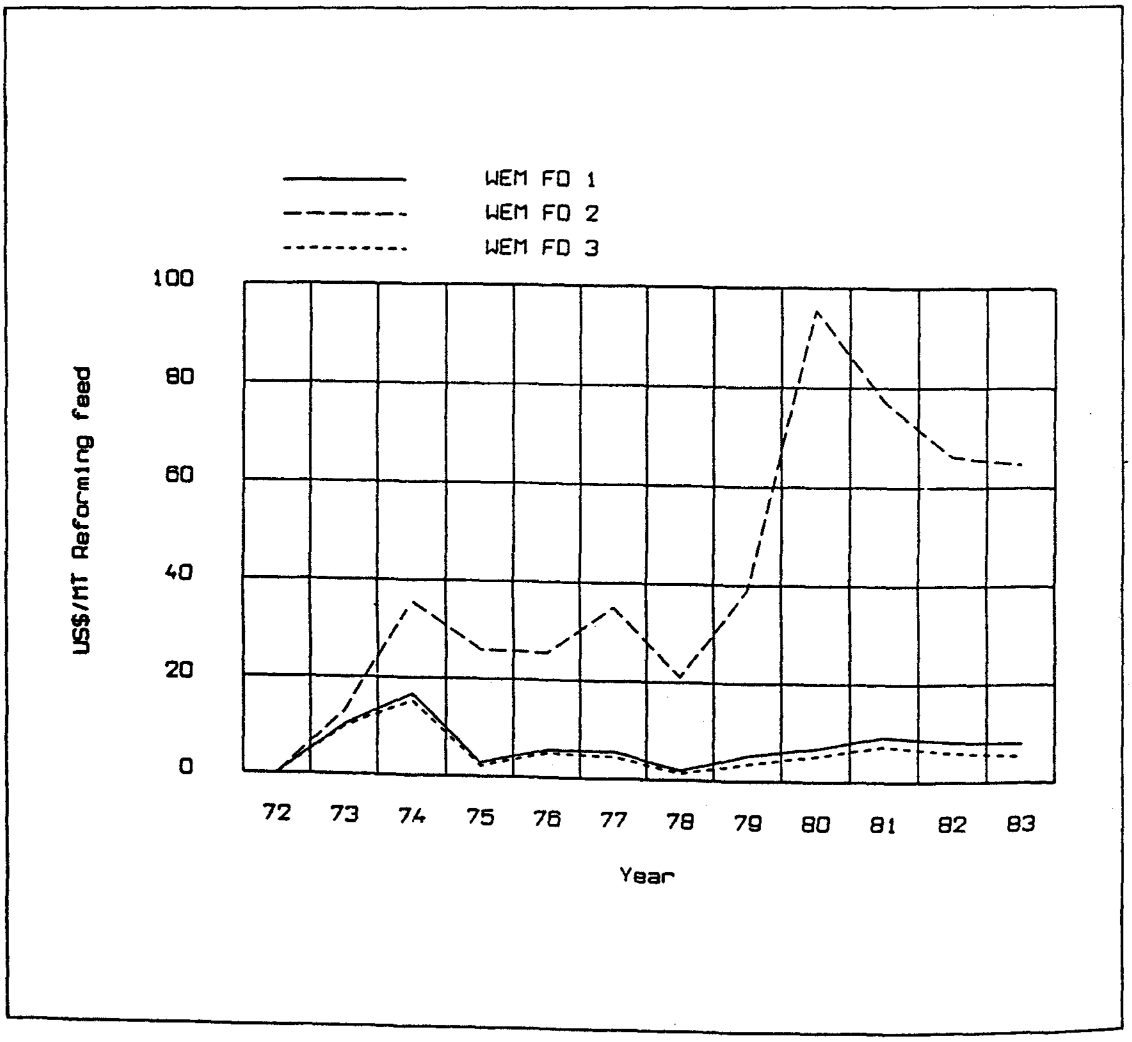


Figure 38. 1972-1983 catalytic reforming estimated rent: second kind of Refinery Rent models, 7-area WEM FD responses.

Models based on responses from the 7-area WEM Gas Oil demand parametric cases:

1 $\rho_{REF}(B) = -416.5 + 1.6p_{pms} - .50p_{ker} - .75p_{srb}$

2 $\rho_{REF}(B) = 7927.4 - 51.2p_{pms} + 40.8p_{hfo}$

3 $\rho_{REF}(B) = 141.5 + .9p_{pms} - .54p_{hfo} - 1.06p_{srb}$

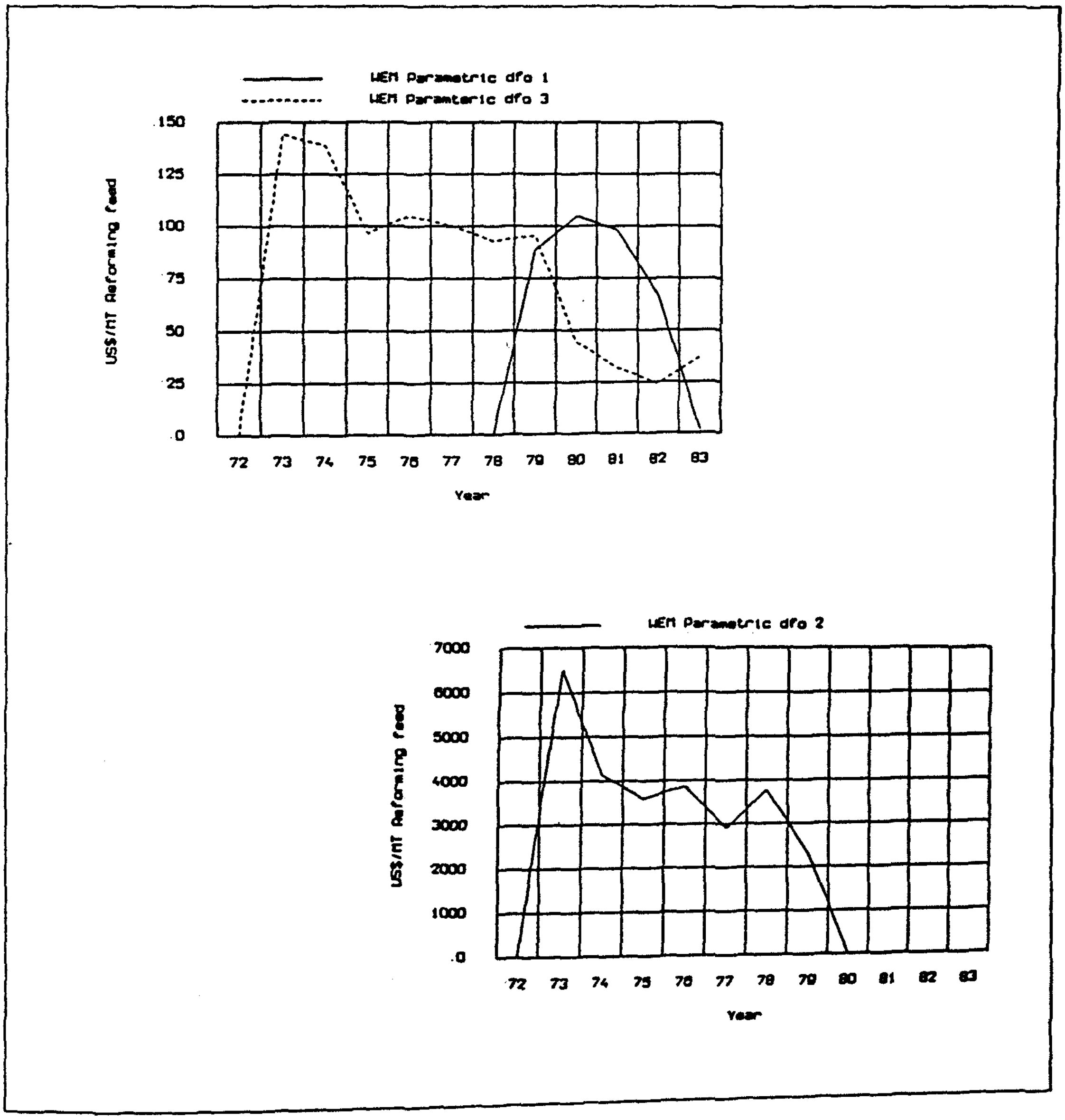


Figure 39. 1972-1983 catalytic reforming estimated rent: second kind of RR models, 7-area WEM DFO demand change responses.

Models based on responses from the 7-area WEM Premium Motor Spirit parametric cases:

1 $\rho_{REF}(B) = 108.4 + .9p_{pms} - .15p_{ker} - 1.27p_{srb}$

2 $\rho_{REF}(B) = -717. + .9p_{pms} + 2.11p_{hfo}$

3 $\rho_{REF}(B) = -3.88 + .91p_{pms} + .20p_{hfo} - 1.17p_{srb}$

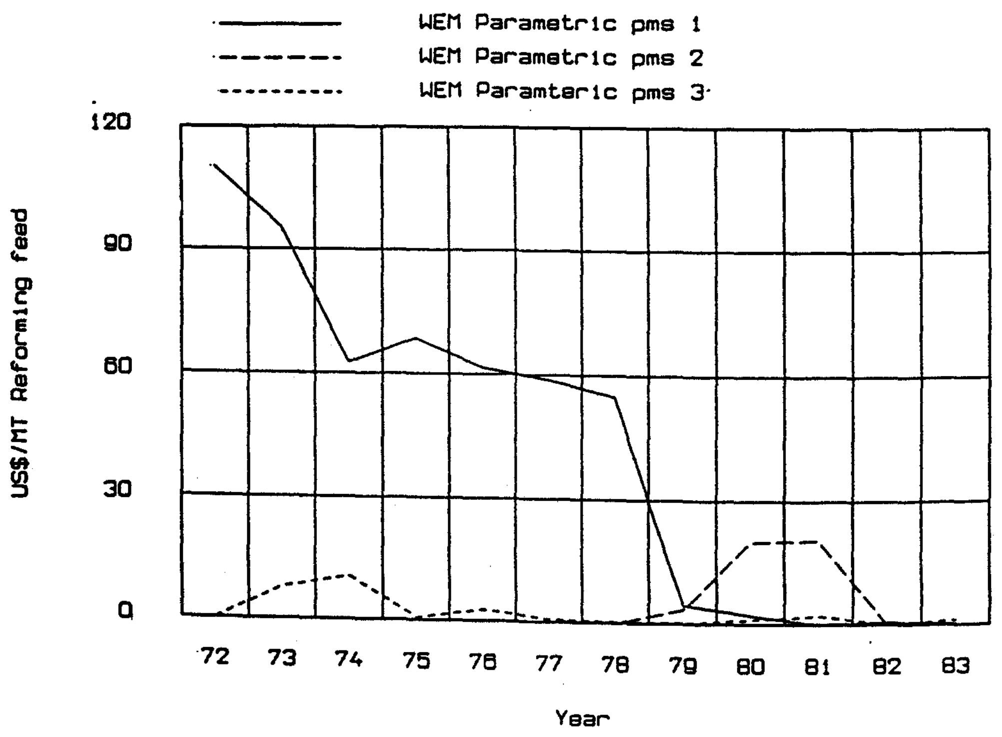


Figure 40. 1972-1983 catalytic reforming estimated rent: second kind of RR models, 7-area WEM PMS demand change responses.

Models based on responses from all 7-area WEM experimental cases, namely Factorial Design and Parametric Programming:

1 $\rho_{REF}(B) = -3.48 + .9p_{pms} + .08p_{ker} - 1.06p_{srb}$

2 $\rho_{REF}(B) = - 21.4 + .5p_{pms} - .44p_{hfo}$

3 $\rho_{REF}(B) = - 2.75 + .91p_{pms} + .08p_{hfo} - 1.06p_{srb}$

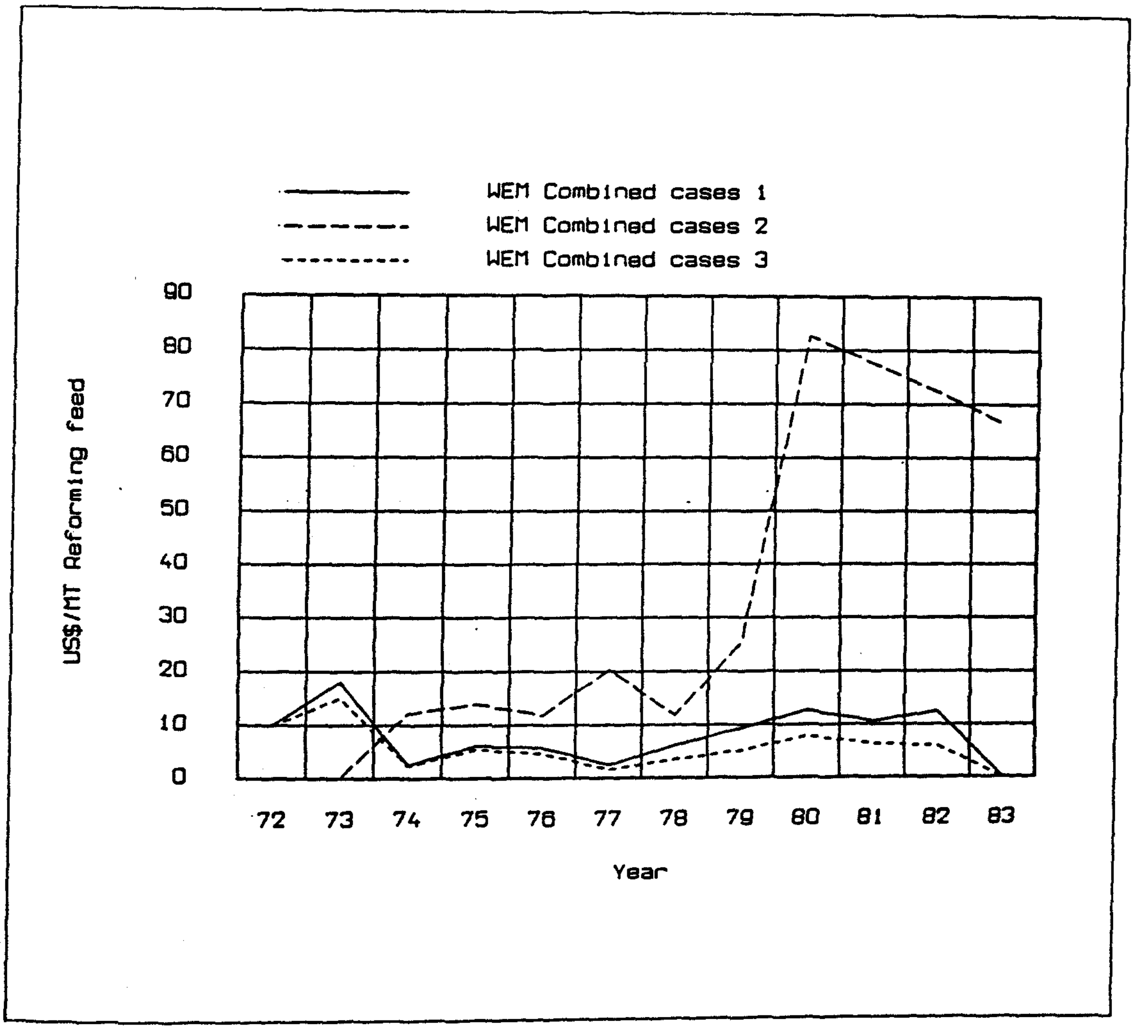


Figure 41. 1972-1983 catalytic reforming estimated rent: second kind of Refinery Rent models, 7-area WEM all combined responses.

6.4.3 Hydrocracking

Models based on the 7-area WEM Factorial Design responses:

1 $\rho_{HYC}(B) = -1.27 + .15p_{pms} - .29p_{dfo} + .13p_{ker}$

2 $\rho_{HYC}(B) = 3.32 + .12p_{pms} - .58p_{dfo} + .39p_{naph}$

3 $\rho_{HYC}(B) = 3.48 + .12p_{pms} - .50p_{dfo} - .11p_{ker} + .42p_{naph}$

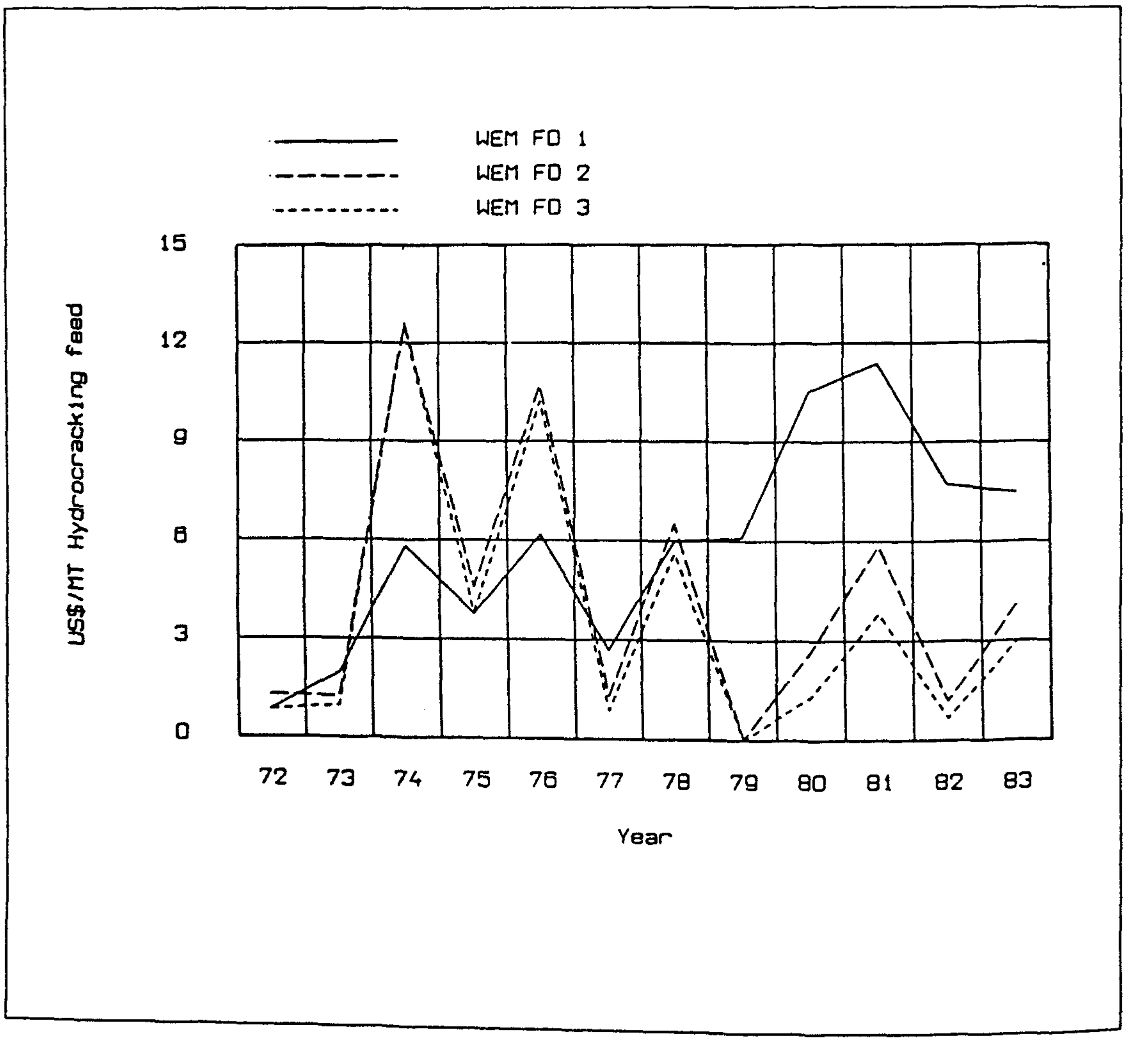


Figure 42. 1972-1983 hydrocracking estimated rent: second kind of Refinery Rent models, 7-area WEM FD responses.

Models based on responses from the 7-area WEM Gas Oil demand parametric cases:

1 $\rho_{HYC}(B) = 839. - 1.16p_{pms} - 2.4p_{dfo} + .57p_{ker}$

2 $\rho_{HYC}(B) = 311.8 - .32p_{pms} - 1.27p_{dfo} + .42p_{naph}$

3 $\rho_{HYC}(B) = 332. - .33p_{pms} - 1.40p_{dfo} + .08p_{ker} + .4p_{naph}$

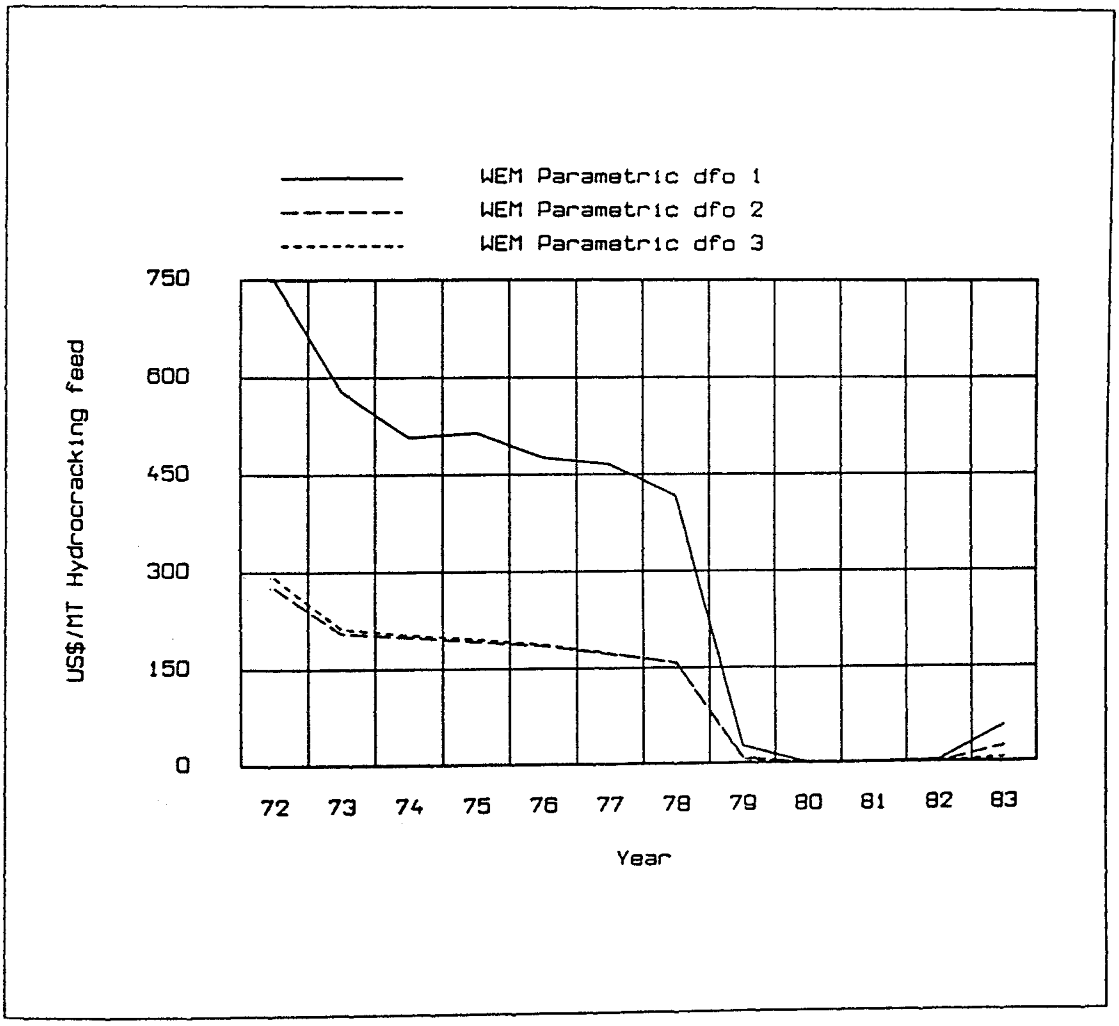


Figure 43. 1972-1983 hydrocracking estimated rent: second kind of Refinery Rent models, 7-area WEM DFO demand change responses.

Models based on responses from the 7-area WEM Premium Motor Spirit parametric cases:

1 $\rho_{HYC}(B) = 389. + .24p_{pms} + 79.77p_{dfo} - 80.77p_{ker}$

2 $\rho_{HYC}(B) = 73.91 + .18p_{pms} - 1.11p_{dfo} + .56p_{naph}$

3 $\rho_{HYC}(B) = 48.8 + .18p_{pms} - 8.p_{dfo} + 6.88p_{ker} + .58p_{naph}$

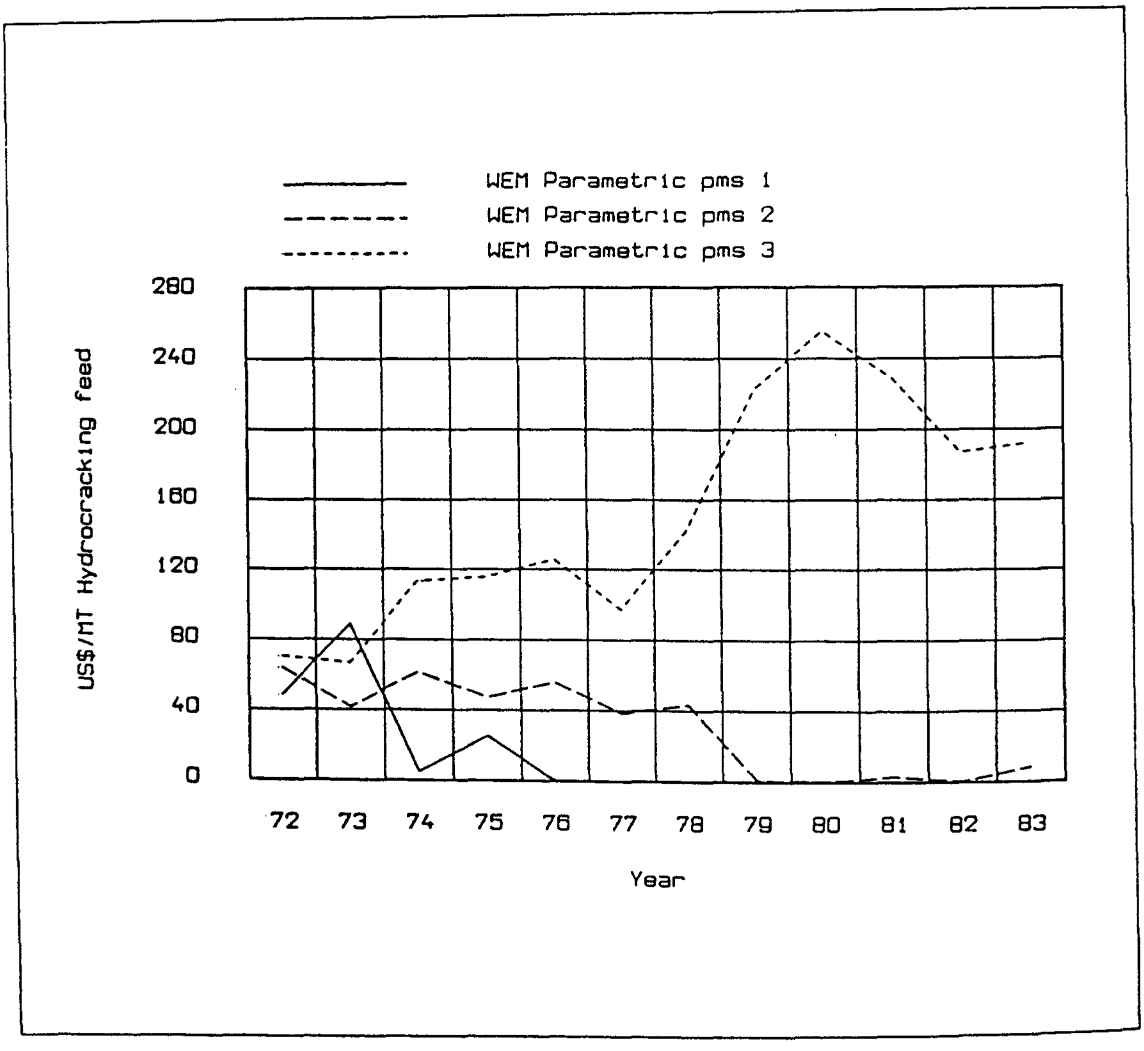


Figure 44. 1972-1983 hydrocraking estimated rent: second kind of Refinery Rent models, 7-area WEM PMS demand change responses.

Models based on responses from all 7-area WEM experimental cases, namely Factorial Design and Parametric Programming:

1 $\rho_{HYC}(B) = 5.42 + .21p_{pms} - 12.60p_{dfo} + 12.16p_{ker}$

2 $\rho_{HYC}(B) = .23 + .20p_{pms} - .77p_{dfo} + .49p_{naph}$

3 $\rho_{HYC}(B) = 2.60 + .16p_{pms} - .29p_{dfo} - .56p_{ker} + .61p_{naph}$

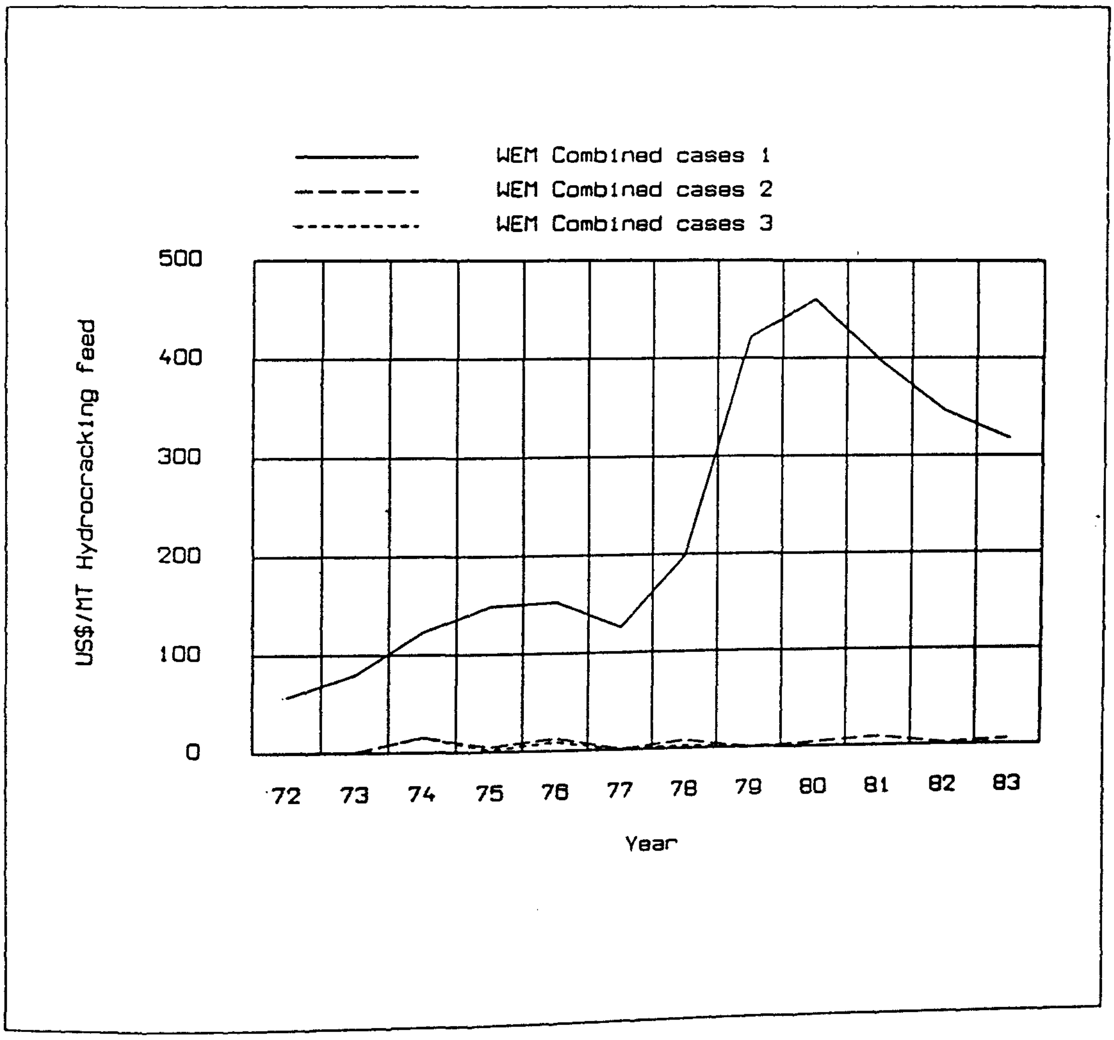


Figure 45. 1972-1983 hydrocracking estimated rent: second kind of Refinery Rent models, 7-area WEM all combined responses.

6.5 VERIFICATION OF THE REFINERY RENT MODELS

In despite of the fact there are not available series of refinery processing units rents against which the estimates could be verified, in this section it is intended to give an 'ad hoc' validation on the available historical data.

The validation procedure was already outlined in section 6.2 of this Chapter. For a particular model validation the steps to follow are summarized below:

1. Figure 33 on page 300 presents a hypothetical refinery rent trend for North West Europe based on the historical background of section 6.3.

2. Substantial increases in refinery processing unit capacities are considered *real increases* in time and not as increases due to higher utilization of equipment. These are the peaks in capacities in Figure 31 on page 295 and Figure 32 on page 296.

3. For a real increase in time a previous investment had to be made. Previous levels of rents, given by the particular model under validation, are compared with the corresponding average annual capital charge estimated in section 6.2, and summarized in Table 6-2.

4. If the comparison in 3. agrees, and additionally a general trend given by 1. is maintained, it is concluded the model is representative of the level of rents of the particular refinery processing unit.

Verifications are made separately for every refining technology, accordingly, catalytic cracking (section 6.5.1), catalytic reforming (section 6.5.2) and hydrocracking (section 6.5.3). Section 6.6 concludes Chapter 6.

6.5.1 Catalytic Cracking

Observing the development of catalytic cracking capacity, Figure 32 on page 296, three real increases or net expansions seemed to have occurred. These are, around 1974-75, in 1979 and around 1981-82; the latter means that investment should have been made in the years 1972-73, 1976-77, and 1979-80. For those investments to occur, the levels of catalytic cracking rents should have been at least equal to the values given by the estimated cracking average annual capital charges in North West Europe, Table 6-3:

1972	6.50	US$ per mt feed
1973	8.83	"
1976	13.80	"
1977	18.08	"
1979	27.25	"
1980	30.70	"

Next these values and the shape of catalytic cracking capacity in time (Figure 32 on page 296) are compared to the values given by the whole set of estimated catalytic cracking rent models:

1. The values and shapes of estimated rents using the first kind of RR model (i.e., those in terms of factorial effects, 2^5 7-area WEM FD) on cracking do not reproduce the annual cracking charges for the period 1974-1978, and some of the models do not even reproduce for the remaining years, therefore these models can be disregarded as possible approximations to catalytic cracking rents.

2. From the second kind of RR model, the following ones can be used to produce reasonably good approximations to the level of cracking rents:

   a. The three models based on '7-area WEM FD' responses, Figure 34 on page 302,

      1 $\rho_{CC}(B) = -3.28 + .55p_{pms} - .72p_{hfo}$

      2 $\rho_{CC}(B) = -1.27 + .38p_{pms} + .5p_{dfo} - 1.07p_{hfo}$

      3 $\rho_{CC}(B) = 2.23 + 1.36p_{dfo} - 1.55p_{hfo}$

b. One of the models based on '7-area WEM Parametric dfo' responses, Figure 35 on page 303,

$$3 \quad \rho_{CC}(B) = 44.71 + .21p_{dfo} - .38p_{hfo}$$

When gas oil demand is the factor exogenously changed, is the price of gas oil the main determinant of rent (as opposite to the case below), hence that models 1 and 2 which include the premium motor spirit price do not give good approximations.

c. Two of the models based on '7-area WEM Parametric pms' responses, Figure 36 on page 304,

$$1 \quad \rho_{CC}(B) = 84.20 + .38p_{pms} - .84p_{hfo}$$

$$2 \quad \rho_{CC}(B) = 51. + .36p_{pms} + .11p_{dfo} - .85p_{hfo}$$

The fact that model 3 which does not include the price of premium motor spirit is not a good approximation indicates that for increasing premium motor spirit demand, is its price the most relevant determinant of cracking rent.

d. The three models based on '7-area WEM combined cases', Figure 37 on page 305,

$$1 \quad \rho_{CC}(B) = 2.60 + .57p_{pms} - .81p_{hfo}$$

$$2 \quad \rho_{CC}(B) = -.5 + .40p_{pms} + .45p_{dfo} - 1.06p_{hfo}$$

$$3 \quad \rho_{CC}(B) = -1. + .99p_{dfo} - 1.07p_{hfo}$$

These three models which combine almost all range of possibilities in exogenous market conditions, produce very good estimates indeed.

The trends produced by all models above are generally comparable to the expected historical ones (Figure 33 on page 300); besides, the estimated rent values at 1973 and 1979 they generate are greater than the corresponding average annual capital charges.

As a summary graph, Figure 46 depicts two of the catalytic cracking estimated models, namely, model 2 of the '7-area WEM combined cases', and model 1 of the '7-area WEM FD' cases. The cracking capacity expansion and the average annual capital charges for cracking are also shown.

It is worth noting that the yearly levels of rents produced by the models are all higher than or equal to the corresponding average annual capital charges, except in 1984, year in which cracking rents are reported to have decreased drastically. Also the expansion during the eighties is noticeable, particularly in 1983-1984.

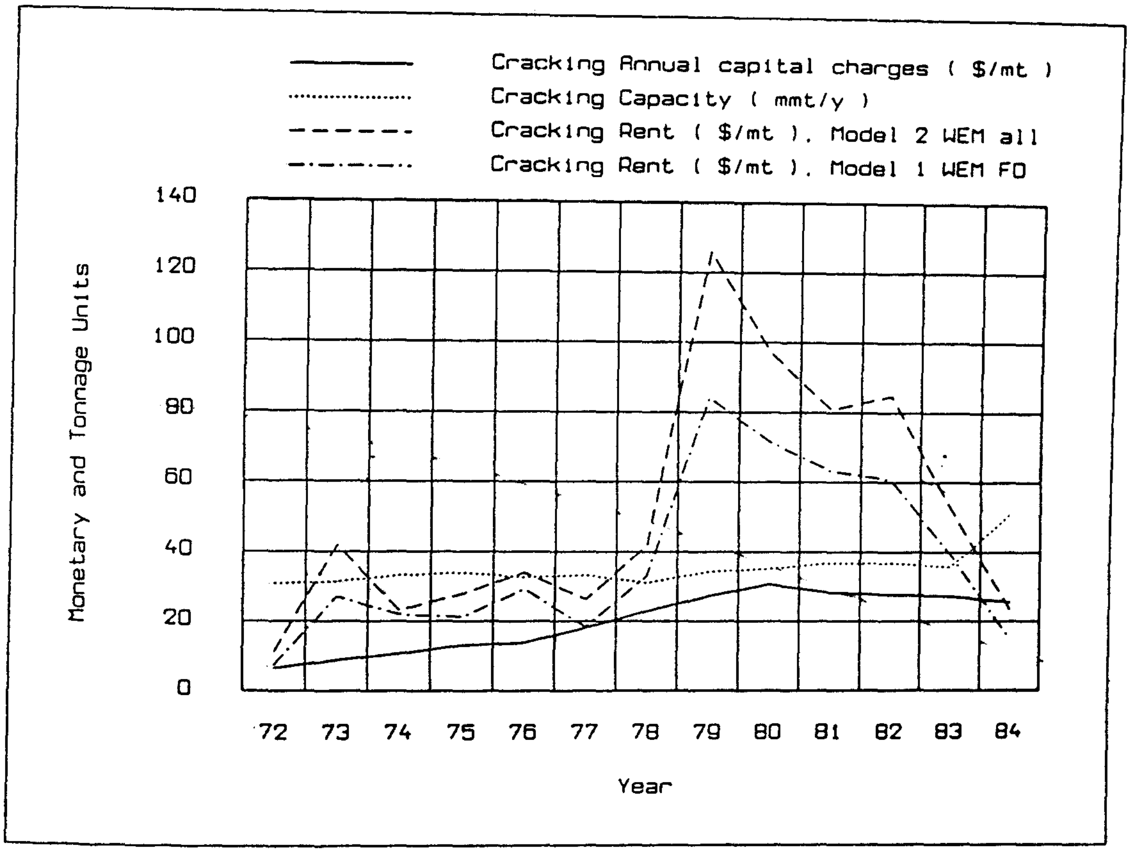


Figure 46. 1972-1984 North West Europe cracking rent, capacity and average annual capital charges.

6.5.2 Catalytic Reforming

The trend in catalytic reforming capacity from Figure 31 on page 295 shows that there has been a progressive investment in hydroskimming capacity until around 1977-1978 for last increase occurred in 1979. However from 1980 onwards hydroskimming refineries have been progressively closing down excess capacity in Western Europe. The slight improvement in 1982 is a reflect then of an increase in capacity utilization and not of net investment. Indeed, by the beginning of 1983 catalytic reforming capacity had been already reduced by 6.5 mmt/y.[127]

Assuming this progressive expansion, rents greater than the reforming average annual capital charges are expected. This is to say greater than those of Table 6-3 for reforming:

1972	3.05	US$ per mt feed
1973	4.08	"
1974	4.93	"
1975	5.88	"
1976	6.18	"
1977	7.25	"
1978	9.20	"

Comparing these values and the shape of the catalytic reforming capacity in time (Figure 31 on page 295) with the set of values from the estimated reforming models, the following results:

1. The 90% and 95% SS models of the first kind of RR model (Figure 26 on page 202) could be used to estimate reforming rents in spite of accounting for zero rent in 1972-1973, the time at which crude oil was relatively cheap as compared to product prices and the refiner was indeed making a positive rent. The other two models (98% and 99% SS) produce excessively high rents, far beyond the estimated annual capital charges, and knowing that rents in hydroskimming deteriorated already in the early 80's, those values could not in principle have happened in North West Europe.

---

[127] The fact that Table 6-4 shows an increase in capacity in 1982 is due to the way data were compiled. They represent capacities at the beginning of each year thus they are in fact previous year's effective capacity. The closures during 1982 are then accounted in 1983.

2. From the second kind of RR model there is only one model producing relatively good estimates, that is, model 1 of the group '7-area WEM Parametric pms', Figure 40 on page 308,

$$1 \qquad p_{REF}(B) = 108.4 + .9p_{pms} - .15p_{ker} - 1.27p_{srb}$$

This model reproduces the kind of expected catalytic reforming rents for until 1978 the level of rents are greater than corresponding annual capital charges in reforming to decrease sharply after 1979 and become zero from 1981 onwards.

The fact that the model corresponds to experiments under increasing premium motor spirit demand may reflect the use of reforming in North West Europe to meet increases in motor gasoline demand (additionally to the octane upgrading function). With two exceptions (1974 and 1981) motor gasoline demand increased with respect to previous year's in the period 1972-1983 (see Table 6-3). The tendency however seems to be changing now when is cracking the one taking most of the motor spirit production.

The second model is found most suitable for reforming rent estimation, though perhaps some others from the estimated second kind of RR model on reforming could also be used.

Price exchange ratios premium to kerosine and premium to naphtha corresponding to the '7-area WEM Parametric pms' set of experiments are as follows:

Experiment	p_{pms}/p_{ker}		p_{pms}/p_{naph}	
	minimum	average	minimum	average
WEM pms	1.25	1.29	1.23	1.28

This gives necessary conditions for a positive reforming rent to occur of $p_{pms}/_{ker} \geq 1.25$ and $p_{pms}/p_{srb} \geq 1.23$. These ratios assure a refiner a high reforming rent, experimentally higher than US$ 39./mt on processing feed in average.

At Rotterdam, smaller ratios are accounted for (see Table D-13, Appendix D), but it seems that the smaller the ratios, i.e., the less premium motor spirit produced per metric tonne of naphtha or kerosine, the lower the catalytic reforming rent. The estimated model agrees then with values deteriorating from 1979 and market price exchange ratios declining.

From Table D-13 the Rotterdam price exchange ratios 1978-83 are drawn and the model estimated rents shown:

Year	p_{pms}/p_{naph}	p_{pms}/p_{ker}	US$/mt
1978	1.16	1.16	54.80
1979	1.12	0.99	4.58
1980	1.14	1.06	2.08
1981	1.13	1.10	0.00
1982	1.15	1.06	0.70
1983	1.08	1.07	0.11

Figure 47 on page 321 shows the models' estimated rents, the annual capital charges and the reforming capacity in time. It is seeing that in spite of reforming rents being continuously diminishing specially from 1979 onwards, the reforming capacity was increasing until about 1982 when closures then took place. This may be explained by the higher rents in cracking which accounted for the reforming losses, and wrong expectations of an increasing demand for motor spirits that would eventually require more reforming capacity.

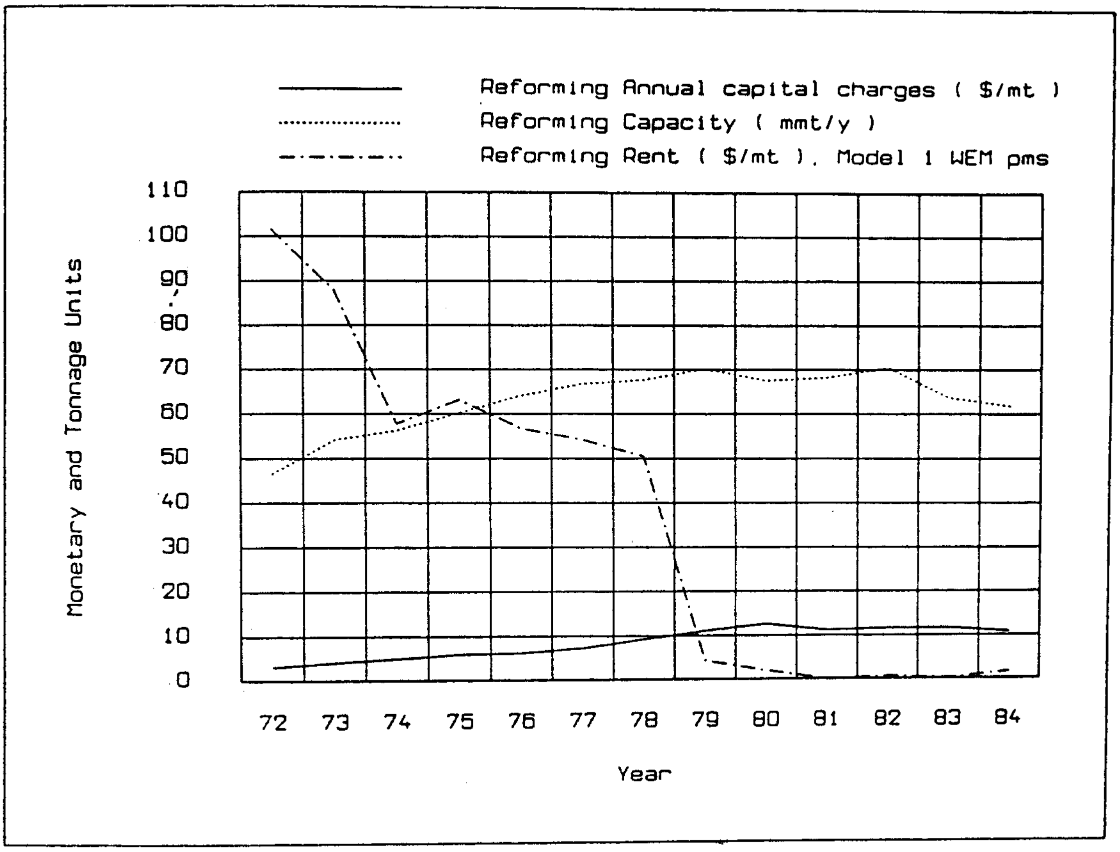


Figure 47. 1972-1984 North West Europe reforming rent, capacity and average annual capital charges.

6.5.3 Hydrocracking

The development of hydrocracking capacity from Figure 32 on page 296 is rather stable during the years 1973-76 with slight increases which are interpreted as increases in utilization of capacity due to motor spirit demand increases. Three hydrocracking capacity expansions, namely, in 1973, in 1978-79 and in 1981-82 are noticed, meaning investments might have been made in 1971-72, 1975-76 and 1979-80 respectively. For this to occur expected levels of rents should have been higher than the hydrocracking average annual capital charges during the corresponding years as presented in Table 6-3:

1972	8.05	US$ per mt feed
1975	14.00	"
1976	13.75	"
1979	24.00	"
1980	28.50	"

The comparison of these values and the trend in hydroskimming capacity during 1972-1983, Figure 32 on page 296, shows that:

1. The 95% SS model of the first kind of RR model produces values of rents higher than the above hydrocracking average annual capital charges, however it generates zero rents for the years 1979-80, an thus it would be discarded as an appropriate approximation model to hydrocracking rent.

2. The second kind of RR model suitable for hydrocracking rent estimation are as follows:

   a. One of the models based on the '7-area WEM Parametric pms' responses, Figure 44 on page 312,

   3 $\rho_{HYC}(B) = 48.8 + .18p_{pms} - 8.p_{dfo} + 6.88p_{ker} + .58p_{naph}$

   b. One of the models based on the '7-area WEM Combined cases' responses, Figure 45 on page 313,

   1 $\rho_{HYC}(B) = 5.42 + .21p_{pms} - 12.60p_{dfo} + 12.16p_{ker}$

The two models of the second kind of RR model produce levels of rents which agree well with expected rents behavioural patterns, their levels at the above investment dates are greater than the estimated average annual capital charges.

Figure 48 depicts both models generated levels of rents, average annual capital charges, and capacity development during 1972-1984: hydrocracking rents cover the average annual capital charges for each year, and capacity expansion keeps a close relation to rents in time.

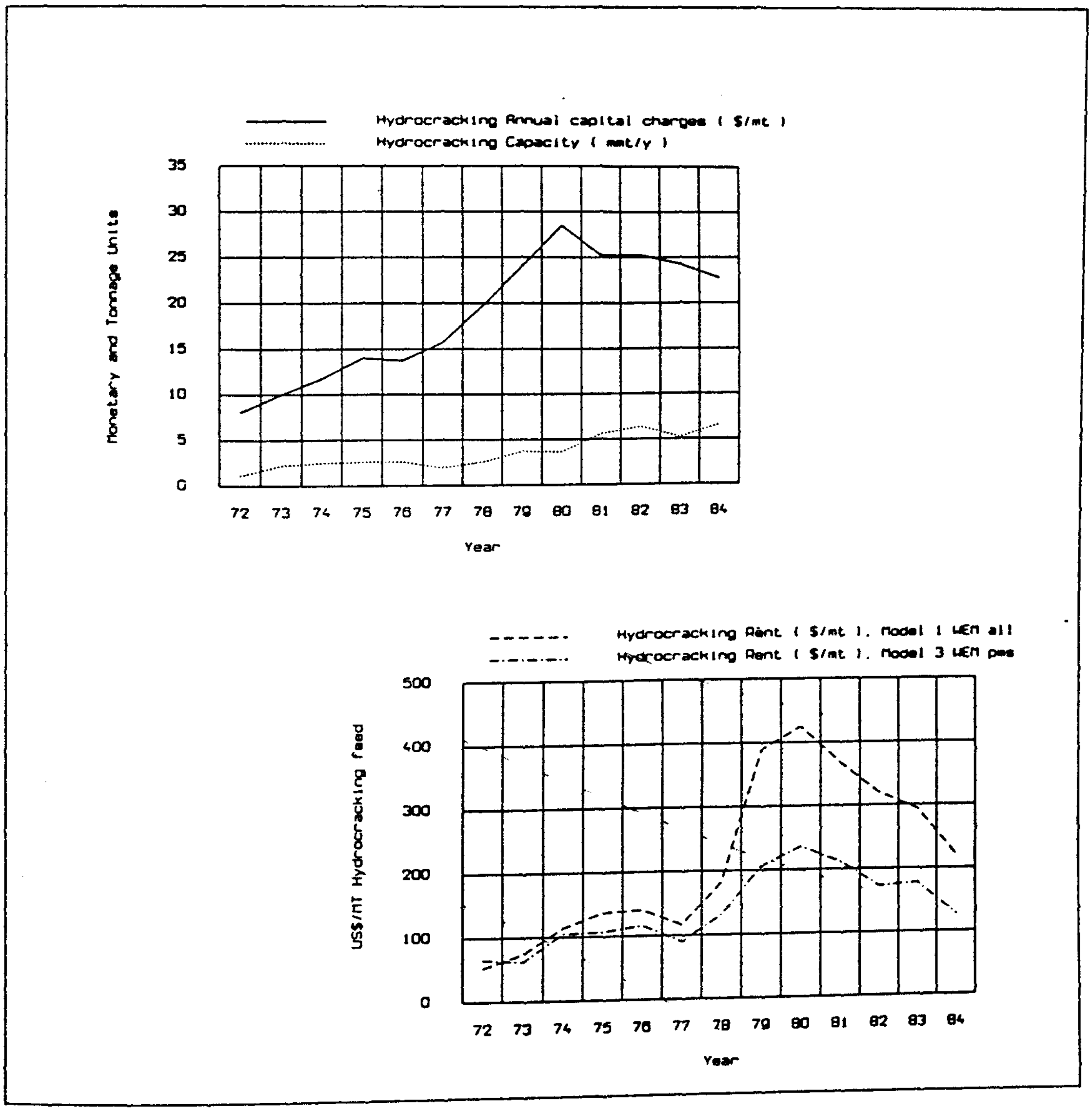


Figure 48. 1972-1984 North West Europe hydrocracking rent, capacity and average annual capital charges.

6.6 CONCLUSIONS

Chapter 6 concludes the experimental work for which a theoretical background was presented in Chapter 5. With all the experimental work done, that includes gathering statistics, estimating data, approximating functions, etc., the options open in the attempt to find refinery rents from simple models have been exhausted.

The models derived from the 2^5 7-area WEM Factorial Design did not bring about the kind of response expected. Instead simple regression models in terms of few variables (product prices) have proved to produce reasonably good estimates with the advantage of being easier to manage.

By constructing proxies of refinery rents the selection of the *best* of all rent approximations found for the technologies of catalytic reforming, catalytic cracking and hydrocracking in North West Europe was made possible. Further research should be similarly conducted for the other areas of the 7-area WEM.

Unidimensionality proved to be valid in North West Europe in the case of refinery rents. The assertion relies on the way the models were built. Arabian Light was ever *the* fixed price, the marker, only changes in the chemistry of demand, and not in the price structure were considered in order to get the different parametric cases, hence that all crude and product prices were aligned with that of the marker. Moreover, since the models need the Rotterdam spot prices, and these are certainly related to the Arabian Light spot price because previous work confirmed that,[128] and refinery rents in the short run are price determined, there exists the link between rents and spot prices.

More refined studies may be plausible to make on the availability of more perfect statistics than those the author has at present.

[128] O'Carroll, F.M., 'Price Determination and Economic Mechanisms in the European Oil Industry 1964-71', Ph.D thesis, op.cit.

CHAPTER 7: CRACKING RENTS UNDER CHANGING MARKER PRICE

In this chapter it is the intention to investigate the conditions for a change of the marker energy source which would make the system dependent on a second energy marker price, how this two-tier price structure affects catalytic cracking rents, and how Unidimensionality is regained.

7.1 INTRODUCTION

The First Principle of Energy Substitution establishes the Unidimensionality in energy pricing in the short run. This principle has been particularly confirmed for the case of product prices in North West Europe and (in Chapter 6 of this work) for refinery rents in the same area. Given the energy area interconnections, Unidimensionlity is eventually valid for the remaining areas with due consideration of freight rates and transport costs that account for geographical differences.

In the short run Arabian Light is the marker: in the long run it will lose its position when its reserves are exhausted. The aim is to show as a conclusion to the present work how Unidimensionality still holds when there is a change in the *marker energy source*. Such a change may occur in the presence of two or multiple tier price structures that develop when either two crudes or a crude and any other source of energy act as markers simultaneously. When a two-tier price structure develops, the energy price relations are no longer in *equilibrium*. Some prices may align with one price marker, others with the next one. *Two-tier price structures may be temporal*, but if no adjustment of the present marker takes place in due course, a *new marker will appear*. Under present conditions were Saudi Arabia not to adjust its price[129] production would gradually cease. Because this last position is not likely to happen since the crude producer *needs* the oil revenues, *he* will ultimately lower his price.

The change in marker price is a long run issue to which the Third Principle of Energy Substitution applies. ·A particular case that con-

---

[129] Actual spot Arabian Light price is about 27. - 28. $/bl, official of 29. $/bl. The development of this chapter will show that the marker price should be adjusted to about 20. $/bl if Saudi Arabia is to maintain leadership, producing product spot prices in competition with alternative energy substitutes of oil products.

cerns us here is the *price relationship heavy fuel oil/coal*, and hence the *price link crude/coal*, coal as the second energy marker, and the conditions for equilibrium from the point of view of the refiner.

7.2 CRACKING RENTS AND THE CRUDE/COAL PRICE LINK

Coal/nuclear is assumed here to become the new *marker energy source*. There is evidence that this process has been taking place in the market, the reasons are given below and conform with the basis for studying the economic relation crude/cracking rent/coal:[130]

1. Arabian Light is the marker, the refined product prices are aligned with its price, in particular the prices of heavy fuel oil and motor spirit, the least and the most valuable ones respectively at the market place.

2. Cracking is the dominant factor in the refinery. It produces rent by converting the distillate fuel oil into white (motor spirit, kerosine, diesel) products. How high the rent is, depends on the price of crude relative to the prices of white products, and to the crude runs in distillation, for constant white product demand.

   Worldwide cracking rent has been rising steadily during the early eighties, a period during which the Arabian Light price has been relatively high, and Saudi production has dropped by some 233. mmt from 1980 to 1983 (see Table 7-1 at the end of the Chapter).

3. Coal substitutes for heavy fuel oil, the price of heavy fuel oil and the total cost of coal are linked. In equilibrium both are competitive. However, this relation may produce a disequilibrium in the system which in turn *reverts to the price of the Arabian Light crude, the marker.*

   Worldwide, the years 1980-1984 of high cracking rent were accompanied by decreases in oil consumption of about 287. mmt, and increases in the use of other alternative energy sources, namely, coal (91. mmt increase), hydro (47.5 mmt increase) and nuclear (101.6 mmt increase), see Table 7-1. Since coal/uranium attack the heavy fraction of the barrel, substitution of heavy fuel oil by coal/uranium took place: heavy fuel oil consumption decreased by 206.7 mmt in the same period.

In North West Europe the situation was slightly different, whereas oil and in particular, heavy fuel oil consumptions did decrease during the

[130] The reader can refer to Chapter 4, where the case presented here was introduced and developed up to some length using the Single Refinery model (sections 4.2.9 and 4.2.10).

eighties, and hydro and natural gas consumption remained fairly constant, the increase in coal consumption was about 6. mmt in 1979 to 1980 to remain then rather constant in the period 1980-1983. Nuclear increased notably, about 39.5 mmt o.e. from 1980 to 1983, and heavy fuel oil consumption dropped by a similar amount, 38.7 mmt during the same period.

The increase in nuclear production/consumption in North West Europe appears to have replaced most of the fuel oil drop, however this necessitates further investigation since coal was also replaced by nuclear. A disaggregation by sector of end use of heavy fuel oil, coal and nuclear in recent years would be necessary to assess the actual drop in coal consumption due to coal/nuclear substitution, and the actual increase in coal consumption due to heavy fuel oil/coal substitution in order to draw any reasonable conclusion on the heavy fuel oil substitution process. We will concentrate here on the heavy fuel oil/coal substitution only on the grounds that relative to oil, coal and nuclear substitute the same part of the barrel, hence for the sake of the analysis both are equally competitive, though in fact the threshold prices heavy fuel oil/coal and heavy fuel oil/nuclear differ. The development followed can be done in the same way with nuclear.

The general refiner's action is to buy crude and sell finished products so as to maximize his revenue through refining, in such a way as to produce the *minimum amount of fuel oil* and the *maximum of white products* compatible with his infrastructure and the market demand.

Since motor spirit consumption was remarkably constant, and heavy fuel oil consumption dropped sharply, then motor spirit demand was satisfied with more cracking at the expense of less crude consumption and less heavy fuel oil in the market. The refiner was making less than *minimum* heavy fuel oil. It is then suggested that there existed a demand for heavy fuel oil at the refinery (for cracking) which was not satisfied by additional crude runs but by direct heavy fuel oil purchases. It paid to buy heavy fuel oil for cracking rather than crude. Cracking rents increased at the point of expansion, and there was room for alternative fuel oil substitutes to penetrate. It is intended to look closer at this mechanism.

The crude/cracking rent/coal interaction is approached in following sections: in section 7.2.1 from a theoretical viewpoint, in section 7.2.2 graphically, and in sections 7.2.3 and 7.2.4 by looking at the market place.

7.2.1 A Theoretical Approximation

We can formulate a simple three-equation model reproducing the economic relationship crude/cracking rent/coal:

$$P_c = \alpha P_{wp} + \beta P_{hfo} \qquad (7.1)$$

$$P_{wp} = \gamma P_{hfo} + \rho_{cc} \qquad (7.2)$$

$$P_{hfo} \leq 1.5\ C_c \qquad (7.3)$$

where,

P_c is the price of crude,

P_{hfo} is the price of heavy fuel oil,

P_{wp} is the price of white products, i.e., motor spirit and/or kerosine, gas oil; and,

ρ_{cc} is the catalytic cracking rent in \$/mt of white product produced, and,

C_c is the total cost of coal in \$/mt, and 1.5 is the conversion factor coal/fuel oil equivalent in mt.[131]

Equations (7.1) to (7.3) form a theoretical (mental) model aiming to explain the behaviour of rents when two prices are simultaneously fixed (a two-tier price structure), namely, P_c and C_c.

Equation (7.1) reflects the price of crude netbacked from the yields in distillation.[132] Variable costs (negligible) are omitted for sim-

---

[131] Hereinafter we will talk indifferently about *price* or *cost* of coal since coal price resembles more the total cost of production, transportation and handling, whereas generally the price of crude does not keep any relation to its low production cost.

[132] We let aside the criticisms posed to the netbacking method (as for example in Poleo, V., 'Interactive Netbacking System', (London: Petroleos de Venezuela (UK) SA, 1982)), unpublished reference, which go beyond the purpose here, and attach to the common prac-

plicity, and no rent whatsoever arises in crude distillation (surplus worldwide).

The estimate of equation (7.1) is based on the Arabian Light/Heavy Fuel Oil parametric cases (put forward in section 7.2.2 next) taking as price the Arabian Light in North West Europe CIF-Rotterdam, the resulting coefficients are, $\alpha = .4$, and $\beta = .52$:

$$P_c = .4P_{wp} + .52P_{hfo}.$$

Equation (7.2) represents the economic link between fuel oil and white products when cracking. From Chapter 6 we have some good approximations to this equation, where γ results as 1.31 as follows:

Model 1 of the 'WEM FD' group:

$$\rho_{CC}(B) = -3.28 + .55P_{wp} - .72P_{hfo}$$

rearranging terms,

$$P_{wp} = (\ \rho_{CC}(B) + 3.28 + .72P_{hfo}\ )1/.55$$

$$P_{wp} = \rho_{cc} + 1.31p_{hfo}$$

where the term ρ_{cc} is equal to $(\rho_{CC}(B)+3.28)/.55$.

The refiner then converts about about 1.31 tonnes of fuel oil into 1 tonne of white products (motor spirit) to produce a cracking rent depending on input and output prices. In equilibrium this rent is equal to the *cracking fully built-up cost* or *cracking average annual capital charge*.

Equation (7.3) is the energy price relation coal/heavy fuel oil when both are competitive. If binding, it represents the *maximum* price heavy fuel oil can attain to keep equilibrium.

---

tice of netbacking among refiners to assess their refiners' margin from spot product prices.

Now let us take the system (7.1) to (7.3): there are five unknowns and three equations, namely two degrees of freedom in fixing prices in order to have a unique solution.

The following situations arise:

1. $P_{hfo} \leq 1.5C_c$. Suppose $P_{hfo} < 1.5C_c$, in this case equation (7.3) is putting no pressure on the price of heavy fuel oil. The latter is competitive with coal and will be marginally burned/demanded in the market.

   On the other hand, *if*

   $P_{wp} = 1.31P_{hfo}$ + built-up cost

   where built-up cost = ρ_{cc} is in equilibrium, ρ_{cc} becomes a known variable, and we have:

   $P_c = \alpha P_{wp} + \beta P_{hfo}$

   $P_{wp} = 1.31P_{hfo}$ + built-up cost

   P_{hfo} and P_{wp} will depend on the price of crude, the marker.

   Suppose now, $P_{hfo} = 1.5C_c$, and rents equal to equilibrium rents, in this case the price of crude will not go up more than the limit the relationship heavy fuel oil price/coal cost in equilibrium imposes to it. This is to say,

   $P_c = \alpha P_{wp} + \beta 1.5C_c$, at maximum,

   $P_{wp} = 1.3(1.5C_c)$ + built-up cost.

   The second equation gives the price of white products compatible with the coal cost and from here, the first equation gives *a price of crude in line with the cost of coal*. The cost of coal is the *marker price* now, if equilibrium is to be kept.

2. $P_{hfo} > 1.5C_c$. This situation is a disequilibrium position. Heavy fuel oil is not competitive with coal and will not be marginally burned, rather cracked at the refinery.

   From equations (7.1) and (7.2):

   $$\alpha P_{wp} + \beta P_{hfo} > \alpha P_{wp} + \beta 1.5C_c \qquad (7.4)$$

$$\gamma P_{hfo} + \rho_{cc} > \gamma 1.5 C_c + \text{built-up cost} \qquad (7.5)$$

Relation (7.5) says that P_{wp} is higher now than at equilibrium, hence in equation (7.4) the price of crude on distillation is higher than it would be otherwise, had the heavy fuel oil price been in line with the cost of coal.

The solution to (7.4) and (7.5) would therefore be:

- ρ_{cc} is kept equal to built-up cost, in this case, no more heavy fuel oil is made, nor it is cracked, crude runs will depend on the price of crude.
- ρ_{cc} is allowed to increase beyond built-up cost, less heavy fuel oil is produced and more cracked.

Combining equations (7.1) to (7.3) we have,

$$P_c = (\alpha \gamma 1.5 + \beta 1.5) C_c + \alpha \rho_{cc}$$

and from here the price link coal/crude is expressed in terms of the cracking rent and some conversion factors as in equation (7.6) below:

$$P_c/C_c = 1.5 (\alpha \gamma + \beta) + \alpha \rho_{cc}/C_c \qquad (7.6)$$

In equation (7.6) when fixing the Arabian Light price it is noted:

- the lower the coal cost relative to Arabian Light price is, the higher the rent is, or in other words, the higher the price exchange ratio P_c/C_c for fixed P_c, the higher the cracking rent is.
- If ρ_{cc} = equilibrium rent, then expression (7.6) gives the combination P_c/C_c in line with that rent, in this way C_c, the cost of coal, can control the price of crude.

It should be noted that the above discussion assumes a (fairly) constant demand for white products, otherwise the demand for crude or heavy fuel oil and the rents would be additionally affected by the increases/decreases in white product demand.

7.2.2 A Graphical Approximation

Although equations (7.1) - (7.3) help in understanding the two-tier price mechanism, they are of course a simplified representation of the whole system. Between these three there exist some hundred more relating the price of crude to the other crude prices, the latter to the other oil products and rents, and the heavy fuel oil to coal per area. The 7-area WEM takes account of all relations. In order to make heavy fuel oil competitive, its price must be bounded by the total cost of coal, and this in turn must agree with the solution of the remaining equations.

To investigate the effects of fixing the price exchange ratio P_c/C_c on cracking rents and on the level of heavy fuel oil/coal substitution, the 7-area WEM is set up as follows:

- The Arabian Light price FOB Jetty is varied between the extreme cases of 10. and 300. \$/mt (respectively 1.4 and 41. \$/bl).
- Coal cost is varied between the ranges 0. to 225. \$/mt f.o.e., CIF-area.
- Chemistry of demand remains constant throughout.

This setting up produces many run cases and solutions per area. As general findings it can be pointed out that:

- For fixed P_c, worldwide the higher the C_c is, the lower the cracking rent, the lower the amount of heavy fuel oil replaced by coal and the higher the Saudi Arabian (Arabian Light) production.
- For fixed C_c, worldwide the higher the P_c is, the higher the cracking rent, the higher the amount of heavy fuel oil replaced thus the less heavy fuel oil make, the more cracked, and the lower the Saudi Arabian Production.

It was precisely the second position that developed after the 1979 oil price increase and produced the overexpansion of cracking capacity worldwide.

Figure 49 on page 334 shows in the form of a contour line diagram, the cracking rents varying from 10. to 450. \$/mt, as solution to equations (7.1) to (7.3) for North West Europe. The results depicted there are more accurate but not inconsistent for most practical applications with the oversimplified three-equation model.

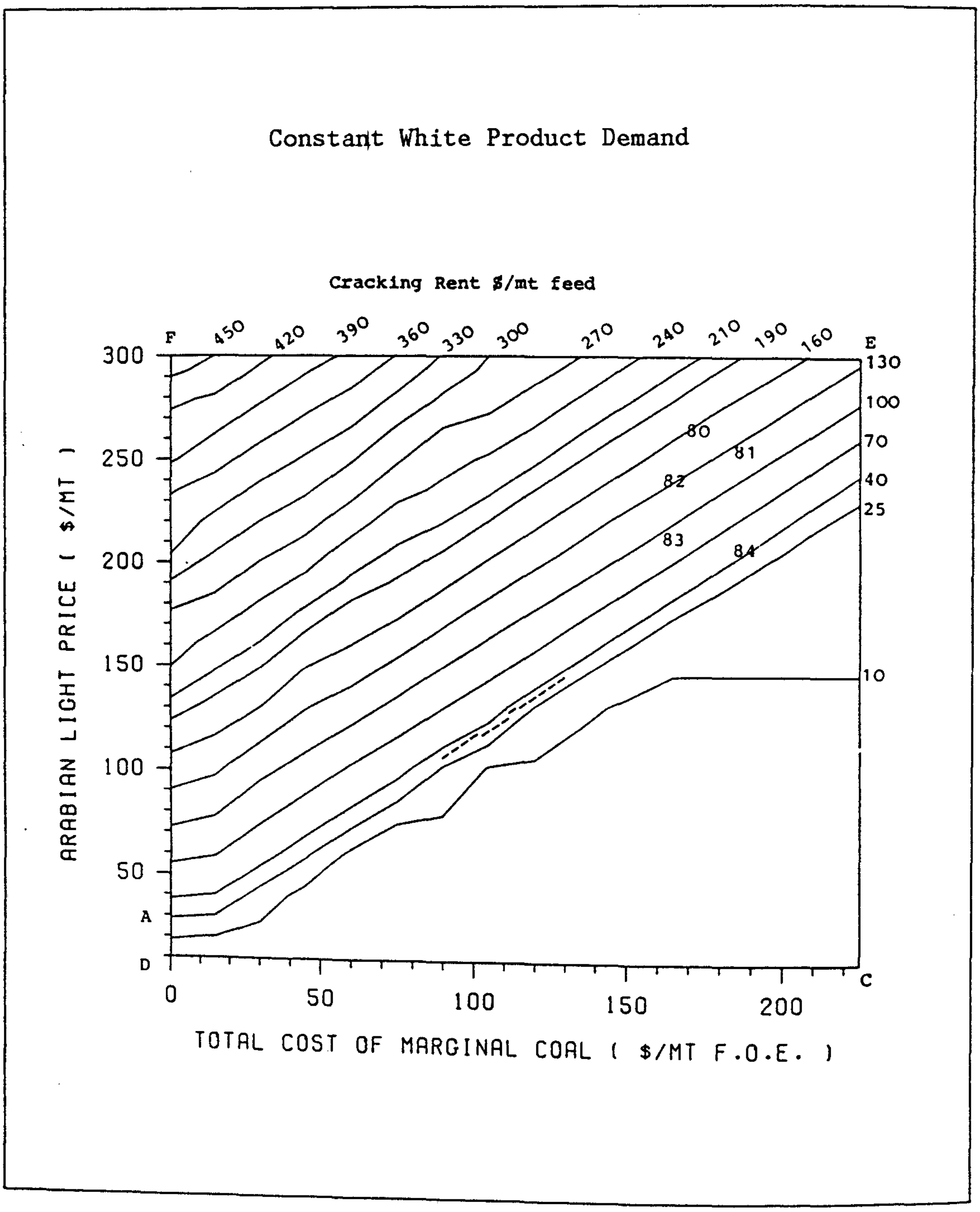


Figure 49. North West Europe cracking rent isolines $/mt, for given Arabian Light and Total Cost of Marginal Coal ($/mt f.o.e.).

It is worth noting:

- Even at very low price combinations of (0.,0.) and (10.,0.), a minimum of cracking rent arises (some 4. \$/mt). This is explained by the inclusion of quality specification constraints in products (effect of quality imputed costs).

- As expected, the higher the ratio P_c/C_c, the higher the cracking rent is. High ratios implying high Arabian Light prices relative to the heavy fuel oil/coal price.

The following regions are distinguished in the graph:

Region ABCD: rent is less than equilibrium rent (i.e., less than about 25. \$/mt on feed, see Table 6-3, page 293), crackers will not be built; more heavy fuel oil is made and sold in the market at a competitive price with the cost of alternative energy sources. If rent drops to less than 10. \$/mt, say near zero-rent level, crackers will eventually be shut down, heavy fuel oil replaces coal in old installations.

This region includes the equilibrium rent contour line AB.

Region ABEF: rent is higher than equilibrium rent, it pays to build crackers and hence less heavy fuel oil will be produced, the gap is filled by coal/uranium which as we have seen is taking place. Increasing refinery profits.

7.2.3 1980-1984 Cracking Rents out of line with Equilibrium Rents

Equilibrium rents in the short run mean a level equal to the long run equilibrium rent, which in annual terms is the average annual capital charge. Thus a refiner is perceiving an equilibrium rent at any one year when the latter is *equal* to the estimated average annual capital charge.

In order to show the disequilibrium rents in North West Europe for recent years, the results of experiments are contrasted against reality.

Unfortunately, data is lacking on coal costs before 1980. For the years 1980 to 1984, data from Table 7-4 (end of this Chapter) is used: the annual capital charges, the Arabian Light and heavy fuel oil prices at Rotterdam, in order to find the corresponding cracking points in the graph of contour lines.

First, at Rotterdam crude and heavy fuel oil nominal prices for the period, the associated cracking rents lie on the contour lines as follows:

Year	Arabian Light spot $/mt	price $/bbl	HFO Price $/mt	Cracking Rents contour lines $/mt feed	Annual Capital Charge
1980	268.92	(36.4)	170.25	160.00	30.70
1981	254.30	(34.5)	183.87	130.00	28.25
1982	236.77	(32.0)	164.42	150.00	27.63
1983	213.71	(28.9)	163.43	100.00	27.25
1984	207.82	(28.2)	178.00	40.00	25.50

The cracking rent values are indicated in the graph of Figure 49 on page 334 with the numbers of the year they correspond. They are in line with the nominal prices of Arabian Light and heavy fuel oil at the same years. It should be noted at those prices the *cracking rents* were *considerably higher* than the corresponding average annual capital charges for cracking which explains the expansion occurring in 1983-1984 as a result of investment in new capacity in the years 1979 to 1982.

Second, the high cracking rents are also in line with heavy fuel oil prices breaking the upper limit imposed by coal cost, this is to say heavy fuel oil was not competitive with coal as an end use energy source and the surplus was cracked.

Taking the coal prices CIF-Rotterdam, it is realized that according to equation (7.3), the heavy fuel oil price did not satisfy the condition,

$$P_{hfo} \leq 1.5C_c$$

Indeed, the following summary shows that if heavy fuel oil price had been in line with coal cost, then Arabian Light prices would have not been as high as they nominally were in 1980-1984. To have equilibrium rents at the heavy fuel oil prices in line with actual coal costs (the maximum price heavy fuel oil can reach in equilibrium, that is column 'Coal Price $/mt f.o.e.' of the table below), the corresponding Arabian Light prices should be as indicated in the table.

Year	HFO Price $/mt	Coal price $/mt	Coal price $/mt f.o.e.	Annual Capital Charges	Equilibrium Arabian Light $/mt	$/bbl
1980	170.25	63.00	95.00	30.70	110.0	15.0
1981	183.87	87.00	131.00	28.25	145.0	20.0
1982	164.42	85.00	128.00	27.63	143.0	20.0
1983	163.43	73.00	110.00	27.25	128.0	18.0
1984	178.00	60.00	90.00	25.50	100.0	14.0

Third, to restore equilibrium either heavy fuel oil price or coal cost must adjust. Because in the price relationship heavy fuel oil/coal coal is the bound for heavy fuel oil price and not the other way around, it is more likely that equilibrium is reached when heavy fuel oil price drops, unless unexpectedly there is a cut in the supply of coal and heavy fuel oil rises due to the sudden increase in demand. A clear example was the UK coal strike (March 1984 - Feb 1985) on which comment is made later.

On the other hand, since the yields in distillation must netback the price of crude, the price of heavy fuel oil can not be lowered at will but must be in line with the price of crude and, in turn, with the price of motor spirit. If the price of crude *does not* drop, the refiner sooner or later will inevitably demand less of it, and the producer will see crude liftings down, hence revenue. A disequilibrium will exist and cracking rents will increase drastically because fuel oil will be higher priced than coal therefore cracked.

The producer to keep the marker position will have to lower crude price to an equilibrium level with coal, and this will give in turn an equilibrium rent for the refiner, or possibly a surplus of cracking capacity, all under conditions of reasonably constant demands of motor spirit and middle distillates except for normal seasonal variations.

The theoretical 1980 to 1984 Arabian Light prices in equilibrium with coal cost and rent presented in the table above, are represented by the hatched segment of the line in Figure 49 on page 334. They imply prices of Arabian Light between 100. and 145. $/mt (14. to 20. $/bbl) to maintain equilibrium.

Now let us see how much the crude price would have to drop to maintain at present price leadership and revenue, assuming the 1985 cracking average capital charge is about 28. $/mt on feed (average of 1980-1984), with a coal cost of 80. $/mt (120. $/mt f.o.e.) 1985 dollars. The 120. $/mt f.o.e is an estimated figure. As Deam points out,[133]

> South African coal landed in Rotterdam adjusted for handling costs, user cost, stack cleaning, and thermal efficiency is about $ 120/tonne f.o.e.

The price of Arabian Light will have to drop to 145. $/mt (20. $/bl). Below that the refiner will clearly have less than equilibrium rent, he will be at a loss, and the crude producer (SA) will lose production, area ABCD of graph; above that he will benefit by cracking heavy fuel oil rather than selling it, and SA will secure revenues and market share, area ABEF of graph.

[133] Deam, R.J., 'Coal cost caps Crude Price', (Queen Mary College, University of London, September 1985), p.2.

7.2.4 Cracking Rents and the UK Coal Strike

The effect of the UK coal strike on the (spot) price structure during the period March 1984-March 1985 is noticeable, particularly for heavy fuel oil and premium motor spirit. This was also reflected in the associated mechanism linking crude/cracking rent/coal put forward above.

Figure 50 on page 340 (top) depicts the monthly nominal prices at Rotterdam for premium motor spirit, heavy fuel oil and Arabian Light during January 1983 and August 1985. It is noticed that 1983 prices are well in line with Arabian Light's (and of course, as we saw earlier, the heavy fuel oil price was well beyond the threshold given by the cost of coal).

The December 83-February 1984 increase/decrease in heavy fuel oil and premium motor spirit respectively can be attributed to normal seasonal variations. However the high heavy fuel oil prices during March 1984-February 1985 are a reflection of the increase in heavy fuel oil demand due to the coal strike. Heavy fuel oil price averaged 180. \$/mt at Rotterdam, with a peak of 189. \$/mt in February 1985, a hard winter in Europe. An opposite effect occurred in the premium motor spirit price which dropped considerably during the period, to start to recover just after the coal strike ended, March 1985. During the same period Arabian Light price remained rather constant: about 207. \$/mt (28. \$/bl).

This heavy fuel oil/premium motor spirit price disparity due to the strike acted as the control instrument in bringing cracking rents down to *quasi-equilibrium*. With the aid of the contour line cracking rents diagram we depict in the same Figure 50 (bottom) the monthly rents during 1984-1985. It is seen they remained fairly constant after the intital drop in March 1984 (beginning of the strike) to reach *equilibrium* rents, 30. \$/mt, around January-February 1985. In fact we do not have the exact cracking capital charges for 1985 but assuming there were not dramatic changes, 30. \$/mt is within the range of annual capital charges during 1980-1984, and fairly approximate to the average of the period 1980-1984, 28. \$/mt on feed.

As soon as the strike ended cracking became attractive again and rents went up, less heavy fuel oil make in the market. Prices of heavy fuel oil decreased as coal entered again into competition, however they have still passed the threshold of cost coal in \$/mt f.o.e. (at least until August 1985, the last figure available).

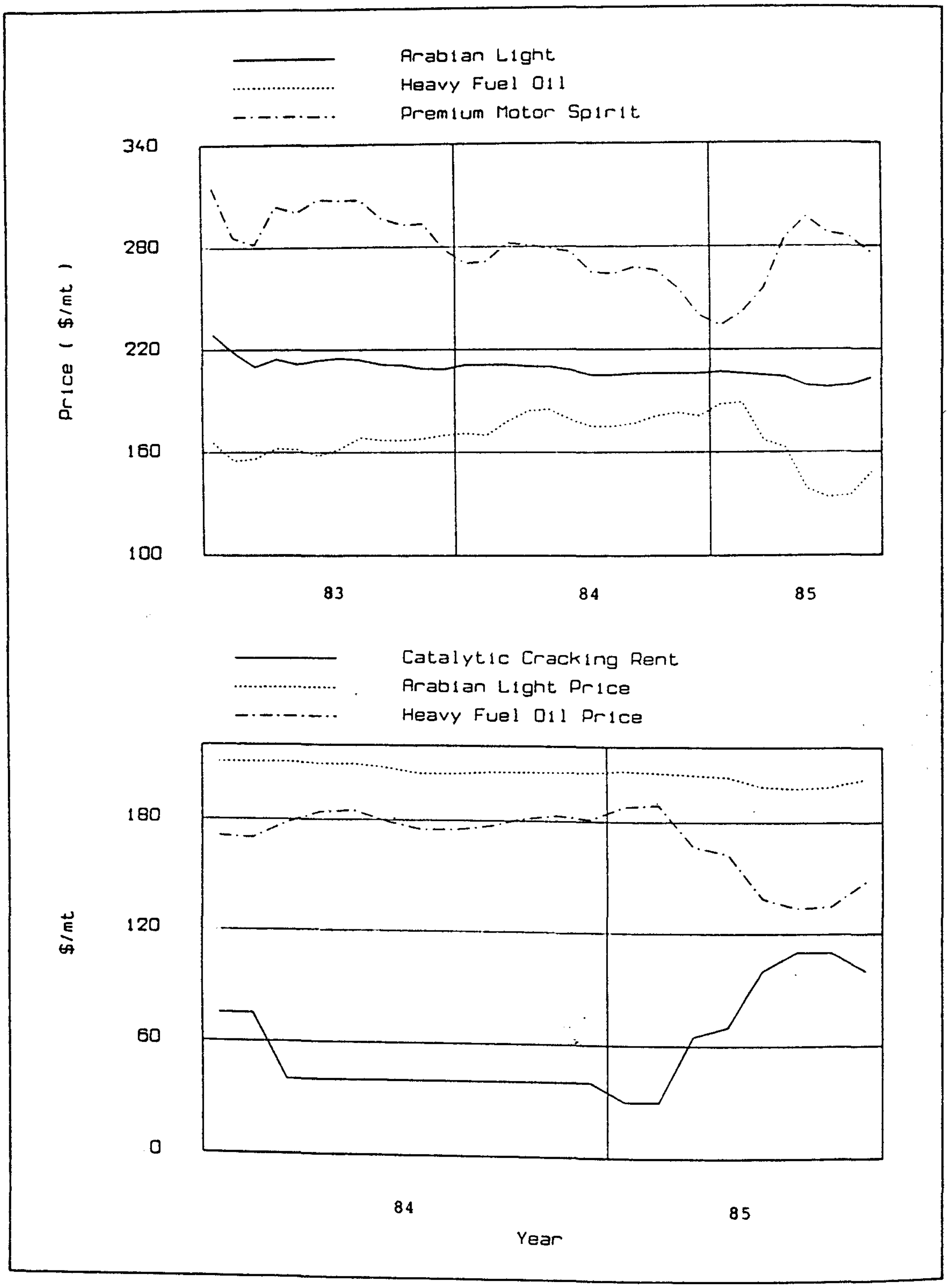


Figure 50. 1983-1985 Rotterdam Arabian Light, premium motor spirit and heavy fuel oil prices; 1984-1985 cracking rents in NWE.

For the heavy fuel oil price to come in line with coal cost, some of the following actions should eventually happen:

- Arabian Light price drops to between 100. to 145. \$/mt (14. to 20. \$/bl), as dicussed in last section.

- Arabian Light price remains constant and heavy fuel oil price increases to reach equilibrium rents: this is not likely to occur unless another case of restricted coal supply appears and heavy fuel oil make is above minimum to meet demand.

- A deficit of coal fired plants exists, which does not seem to be the case in North West Europe (now and/or in the near future).

- Weather conditions: a hard winter would increase the total competitive heavy fuel oil/coal demand, causing the heavy fuel oil price to rise, more heavy fuel oil to be made and rents to decline.

We believe that it is by means of the first action that the crude price leadership and market share is to be regained/maintained.

To conclude the point, before the coal strike the price link Arabian Light/Heavy fuel oil was dominating the cracking rent trend thus expansion occurred as a result of high cracking rents creating a hole for coal to penetrate, substituting for heavy fuel oil.

During the strike there developed a shortage in the supply/demand of coal and hence the price relationship coal/heavy fuel oil was the factor forcing rents to drop to equilibrium without the Arabian Light price being lowered. Heavy fuel oil became competitive with coal because the latter's price increased considerably due to scarcity, less heavy fuel oil was cracked, reflected in the loss of profitability of the refiners during the coal strike, i.e., the cost of the strike was partially carried by the refinery industry.

Such a situation is however a special case and does not reflect the behaviour of the market under normal conditions of supply/demand.

7.3 COAL AS A MARKER ENERGY SOURCE

Crude is the short run energy marker because the production of alternative energy substitutes is fixed in the short run; there are lag times in technological development and implementation. The long run however sets the pace for the development and use of alternatives.

It has been shown in previous sections how the energy trend is gradually given prominence to substitutes for a specific part of the composite crude barrel, that is, the bottom heavier part. It has been also shown that substitution of coal/uranium for heavy fuel oil does not bring about a stable energy market when prices for crude and finished products are likely to remain high, namely when the price of heavy fuel oil passes the threshold of coal cost.

Moreover, that is not the best position for the crude producer: the history of Arabian Light price and Saudi production (see Figure 51 on page 343) shows that in the period 1973-1980 Saudi ever benefited from increasing crude prices as revenues were thereby also increased and production swinging according to demand. After the price increase at the end of 1979 the situation changed, the price of crude became too high, and further decreases of prices were not accompanied by lifting production but the opposite, to decreased prices we find decreased production and thus revenue.

The average Saudi production for the period 1973-1981 was 9. mb/d, the price of crude in nominal terms before 1979 was within the range imposed by the coal cost, i.e., 13. $/bl in 1978, maximum for the period. In 1979 the price of crude became nearly threefold higher (about 30. $/bl., 226. $/mt in average), yet Saudi acted as the swinger during 1979-1981, but from 1982 onwards it began to lose market share as the use of alternatives took place. The lost production is about 4. mb/d (200. mmt/a) due to the coal substitution: the price of Arabian Light became higher than 20. $/bl in 1979 and remained so.

Similar arguments apply to the case of the premium part of the barrel and the methanol substitution for gasoline by processing natural gas. The threshold price there is even lower: results from recent studies show that 12. $/bl can hold Saudi Arabian price and production for a period of 40 years. And then methanol from coal could hold it for another 40 years.[134]

---

[134] See Deam, R.J., and Giesecke, C., 'The price of crude can be controlled by the cost of methanol', (Juelich, FRG: Programme Group on Systems Research and Technological Development, April 1983); and, Giesecke, C., 'World computer model of oil markets: OPEC

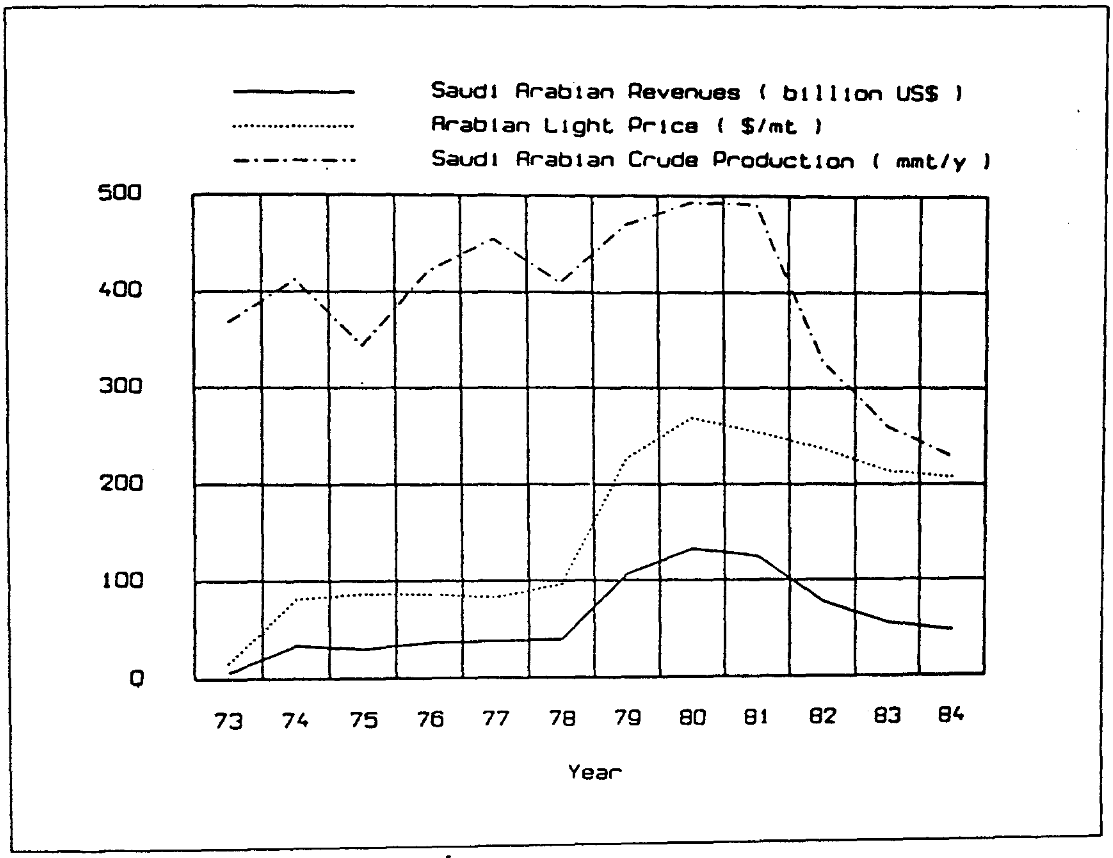


Figure 51. 1973-1984 Saudi Arabian crude price, production and revenues.

So in the long run, the Third Principle of Energy Substitution establishes,

> There is an optimum price for Saudi crude which maximises its asset value in the long term and which minimises the long term cost of all energy to consumers.

It is sustained then that for Saudi Arabia to hold the position of *energy price setter* and maximise revenue, the price of crude should drop (now) and remain in line with the costs of alternative energy sources and production increase accordingly (to keep revenues) until

---

pricing strategy model in the short and long term', Ph.D thesis, op.cit.

eventually reaching maximum: "Saudi's potential maximum supply is considered to be in excess of 30. mbd."[135]

At that point the next marginal source will take the leadership in price setting and marginal production. But before this happens, there will be a transition period in which even if coal (uranium, natural gas, etc.) takes the price leadership, Saudi Arabian crude continues to be the marginal source. Its resources could well last for nearly another century, considering that Saudi estimated reserves are around 180. billion barrels, and its present rate of production is about 4.-5. million barrels day. Obviously if production *has* to go at maximum, the transition period will be shortened.

If Arabian Light price is not adjusted, coal will take sooner the leadership and then determine the prices of all energy carriers in international trade thereby changing the structural relationships between energy forms.

And then the process starts again: coal withholds the marker position until production is dwindled and then the next marginal source will become the marker. Unidimensionality will hold with a different set of technical and economic relationships. The three Principles will need to be altered in line with the current marker source.

---

[135] Deam, R.J., 'Coal cost caps Crude Price', op.cit., p.18.

7.4 CONCLUSIONS

The equilibrium in the energy system persists in time provided Unidimensionality in pricing holds. In the long run there exist for the crude producer and the refiner price *floors* that bring about a balance in production and pricing.

Price floors for the producer are set by the costs of producing alternative energy carriers, in due course, the new price markers. It has been shown that coal can set an upper limit to the current nominal FOB price of crude of about 20. $/bl (145. $/mt), higher crude prices will open the room for coal to take over the role of marker energy source. This price and lower (in real terms) assure the producer long run sustained level of revenues and equilibrium rents for the refiner.

When the price floor is passed, the producer sees its market share shrinking and the refiner rents much above equilibrium thus producing overexpansion of capacity. This short run effect may appear beneficial for the refiner but it is in the long run interest of both parties to keep prices within the equilibrium limits.

Table 7-1. 1973-1983 Worldwide energy consumption by type.[a)]

Year	Saudi Prod. mmt/y	Oil mmt/y	HFO mmt/y	Coal	Hydro	Nat. Gas	Nuclear
				–	mmt/y oil equivalent		–
1973	367.9	2330.1	733.3	799.2	283.1	818.8	45.4
1974	412.4	2248.8	696.0	801.4	296.2	823.6	56.6
1975	343.9	2182.9	644.0	784.7	305.3	789.4	79.7
1976	421.6	2327.3	691.3	829.5	305.9	819.4	95.6
1977	455.0	2391.9	698.6	842.3	315.8	821.4	118.4
1978	409.8	2462.0	698.0	839.3	339.7	845.3	134.9
1979	469.9	2487.8	706.5	909.1	343.6	892.5	135.8
1980	493.0	2356.2	655.8	951.2	346.4	893.1	148.1
1981	491.3	2252.5	593.3	977.6	351.3	887.2	174.0
1982	327.9	2174.9	539.0	983.2	368.5	857.3	189.8
1983	260.0	2141.0	498.0	1003.4	387.6	835.3	211.0
1984	228.6	2069.0	499.8	1042.1	393.9	882.7	249.7

Source: *BP statistical review of world energy*, June 1984, June 85; *Petroleum Economist*, October 1985.

a) excluding Centrally Planned Economies.

Table 7-2. 1973-1983 North West Europe energy consumption by type.

Year	Oil Prod. mmt/y	Motor Gasol mmt/y	Heavy f.oil mmt/y	Coal	Hydro	Nat. Gas	Nuclear
				– mmt/y	oil	equivalent	–
1973	748.9	54.16	115.4	253.6	91.9	129.9	16.6
1974	699.3	51.64	104.4	247.8	96.4	147.2	18.6
1975	664.4	58.92	101.0	234.7	97.1	153.4	25.3
1976	710.3	60.83	104.2	248.3	91.7	163.6	29.2
1977	697.3	63.51	97.9	246.6	110.1	169.9	35.6
1978	715.0	66.81	99.8	246.5	104.3	174.6	36.1
1979	732.4	67.32	101.0	259.8	107.8	187.6	40.0
1980	680.1	67.74	85.6	265.5	103.1	184.4	46.3
1981	632.6	65.34	68.4	262.4	105.6	182.6	63.2
1982	604.3	66.32	60.3	265.7	106.3	177.1	70.5
1983	585.7	68.60	46.9	262.4	110.7	183.5	79.5

Source: *BP statistical review of world energy*, June 1984, and Table D-6, Appendix D.

Table 7-3. 1980-1983 Coal import prices CIF Rotterdam.

Year	Steam $/mt	F.O.E. $/mt	Coking $/mt	F.O.E. $/mt
1980	58.2	87.0	67.7	101.6
1981	69.6	104.4	76.8	115.2
1982	68.0	102.0	77.0	115.5
1983	58.0	87.0	66.0	99.0
1984	60.0	90.0		

Source: IEA, *Energy Prices and Taxes*, 2nd. quarter 1984.

Prices are CIF Rotterdam and average of 7 EEC countries. The difference in value between 1982 and 1983 is a reflect of higher exchange rates US$/country currency for the coal importers, and not of an increase in the coal producers' price.

Table 7-4. 1979-1984 North West Europe selected statistics.

Year	Crack a.c.c. $/mt	Crack Cap. mmt/y	Arabian Light spot price $/mt	($/bbl)	Heavy f.oil $/mt	Coal Import Price CIF $/mt
1979	27.25	34.24	226.75	(30.7)	133.62	
1980	30.70	35.17	268.92	(36.4)	170.25	58.0 - 67.7
1981	28.25	36.77	254.30	(34.5)	183.87	69.6 - 104.4
1982	27.63	34.93	236.77	(32.0)	164.42	68.0 - 102.0
1983	27.25	35.74	213.71	(28.9)	163.43	58.0 - 87.0
1984	25.50	40.90	207.82	(28.2)	178.00	60.0 -

Source: column 1 from Table 6-3, Chapter 6; column 2 from Table D-3, Appendix D; columns 3 and 4 from Table D-8, Appendix D, and *OPEC bulletin*, XVI, 7, (September 1985). 1985; column 5 from Table 7-3 above.

CONCLUSIONS AND RESEARCH GUIDELINES

The main objective of this thesis has been to study the determinates of the oil refinery rents and to formulate simple models whereby the refiner could find the short term rentability/profitability of his refinery.

The work required both a study of theoretical aspects to arrive at the understanding and analytical formulation of the oil refinery rent (Chapter 3), and a study of empirical aspects in order to create the models within the theoretical framework. The latter study was carried out by means of two Linear Programming models, the Single Refinery Model and the 7-area World Energy Model.

While it was not the purpose to pursue a demonstration of the three *Principles of Energy Substitution* (they are formulated and demonstrated elsewhere), the two main modelling hypotheses, the First Principle of Energy Substitution, Unidimensionality in energy pricing, and the existence of a competitive equilibrium market for oil being realized in the oil spot markets, were on the whole supported once more.

There are not two refineries that produce the same level of rents unless their technological infrastructures are identical and satisfy the same market. Whereas the methodologies applied in experimentation, namely, Parametric Programming and Factorial Experimental Design, and the individual experimental hypotheses are valid, the results are largely dependent on the type of refinery, the chemistry of supply/demand (market related) and thus on geographical location. It has been shown that hydroskimming refineries in North West Europe produce low or no rent, and further, they need to be in surplus at some point to satisfy increases in motor spirit demand when seasonality requires it (Chapter 4). While statistics evidence the fact by reporting closures of inactive units, it has been now empirically demonstrated. Contrariwise, complex refineries produce almost always rents.

The 2^5 Factorial Design applied to the 7-area WEM (Chapter 5) produced 32 responses for each marginal value attached to prices of crudes and products, to refinery and shipowner's rents. The analysis of rents was not straightforward: the first kind of Refinery Rent models derived for catalytic reforming, catalytic cracking, hydrocracking and alkylation in North West Europe from the 32 responses and in terms of main factors and interactions, are highly interactive, too lengthy, and uneasy to handle for practical applications. The fact that the

Factorial Design was originally planned to study the formation of crude and product prices, and not to derive the refinery rent models may have had an implication on the inadequacy of the models found. Indeed, as already mentioned in section 5.7, an alternative set of factors would be necessary to design a FD for the sake of modelling refinery rents. Such a set should of course include the Arabian Light price, and factors such as the catalytic reforming and cracking capacities, the demands for motor spirit and middle distillate, and a more severe perturbation in the chemistry of supply. Yet, the models resulting from this new Factorial Design could be equally impractical but could produce better approximations. The methodology is however applicable and useful albeit the associated computational constraints of time, programming efforts and checking feasibilities.

The second methodological approach applied with the 7-area WEM, Parametric Programming, produced more succesful results. The two experimental variables or parameters were the demands for middle distillate and motor spirit in North West Europe, and were considered one at a time. Both sets of parametric experiments produced about 43 responses. This plus the 32 responses from the 2^5 FD make up 75 responses for model estimation. The second kind of Refinery Rent models for each technology (catalytic reforming, catalytic cracking and hydrocracking) comprised the following set of models in terms of product prices:

- Models based on the 2^5 FD responses.
- Models based on the parametric cases of middle distillate demand changes.
- Models based on the parametric cases of motor spirit demand changes.
- Models based on all above responses.

Because the independent variables chosen were prices of premium motor spirit, gas oil, naphtha and jetkero suitably combined to the refinery unit being modelled, the resulting models were significantly simpler and more practical than the first kind of Refinery Rent models. At most they comprised three independent variables (Chapter 6) in contrast to the first kind, with a minimum of seven (Chapter 5).

The verification of the models against reality was troublesome. To start with there is no possibility of a 100% validation since the refinery rents are not published statistical information. On the other hand, our models require some average figure associated with a specific place. The North West Europe area has Rotterdam as spot market and the hypotheses of the 7-area WEM need relate all trade operations to the spots. The proxy of refinery rent to be used should represent the level of rent generated at an average refinery in the area associated with Rotterdam. The proxies used were based on refinery unit investment costs for the technologies of reforming, cracking and hydrocracking. Ideally one should have these investment costs on a fully comparable basis for each country within the area in consideration. As for North West Europe, information was gathered from the Federal Republic of Germany (FRG), U.K. and The Netherlands though the information was not uniform, this is to say, for the FRG the three technologies' investments costs were obtained, whereas for U.K. and The Netherlands one or another was missing. For other countries they were simply not comparable or not available. The intention was to verify for the period 1972-1983, and here appeared the second problem: the data were related to one year only (either 1980, 1982 or 1984, Table 6-1) and costed in local currencies. Hence technological construction indices and deflator series were needed to create the investment costs for the 12-year period in each country, and then to use associated country exchange rates against the dollar and average. Finally the 1972-1983 investment costs (Table 6-2) were used to obtain the proxy of refinery rents as the 25% of the investment cost. This figure was estimated to be the average annual capital charge of investing in refining technology, and is in agreement with usual refining cost estimation practices. Thus the average annual capital charges were the proxies (Table 6-3) together with the refinery units' capacities through time.

Refinery capacities data were taken from *Oil & Gas Journal* and OPEC direct communications. The remaining necessary time series of product prices at Rotterdam are readily available from various published sources, namely *Platt's Oil Report*, *OPEC bulletin*, *Petroleum Argus*, and others.

The comparisons of the estimated 1972-1983 refinery rents with the refinery rent proxies produced significant agreement in many cases. Moreover, to levels of rents greater than the corresponding proxy value, a subsequent increase in capacity was associated. Some discrepancies arose in relation to timing which could be explained by several factors, namely, the time-lags in refinery construction, the imperfection of statistics and/or the suitability of the kind of model proposed.

There remains still a problem. In some cases like that of catalytic cracking, not only one but several models seem to agree well by reproducing the rent's behavioural pattern during 1972-1983, and producing rents greater than equilibrium (average annual capital charges). The choice of the model is not clear but is associated with market conditions during the time, i.e., with supply and demand patterns. For instance, a model of catalytic cracking rent estimated on the responses from the gas oil demand parametric cases gives different level of rents (though the same general trend) to the same model estimated on the basis of the motor spirit demand parametric cases. On the whole, when rents are estimated from Rotterdam product prices, the North West European refinery system is more sensitive to changes in premium motor spirit demand, if taken a factor alone, and to combined changes of motor spirit/gas oil demands, Arabian Light price, cracking and shipping capácities, as the verification in section 6.5 showed. The latter does not contradict the fact that the first kind of Refinery Rent model does not reproduce responses or behavioural patterns. Those models have a positive finding, that of confirming the complex interactions in the oil system and thus the dependency of rents on many distinct factors. However, the study shows that the models to be chosen are not those in terms of all factors but on spot product prices.

Although the models produce reasonably good approximations to actual level of rents, they are far from perfect. Limitations are imposed, **first**, by the availability and quality of the data hence the need for a comprehensive data base (implying additional effort in gathering, estimating, etc.); **second**, by the particular type of refinery modelled in the 7-area WEM. Whereas the model includes nine technologies, each with a different number of processes, it is noted that technologies in the oil refining sector have lags in development and implementation. In the period 1972-1983 many new processes have been incorporated in the existing and/or newly built plants. This is a potential area of further research. It is a sensible task to update the 7-area WEM technological matrix, by changing already modelled processes and incorporating new ones. Even though, it could well happen that the models found above remained valid since many of the model's technological coefficients are highly insensitive to changes. Moreover, there is a technological sub-matrix in every LP problem that can be perturbed in the range $(-\infty,+\infty)$ without causing any change whatsoever in the LP solution, this is the sub-matrix intersecting the unused activities with ineffective factors. The sub-matrix was identified for the Single Refinery model (section 2.5.5). Attempts were made to find it for the 7-area WEM, however the exercise is not yet finished and therefore results are not sufficient to be reported or to draw any conclusion here. The completion of this case would be relevant in setting priorities as to where the effort in changing and/or approximating technological coefficients should last be concentrated. For the refiner it would also be of importance to know which refining processes are never economic to use and which factors are never restrictive.

As regards the predictive nature of the estimated Refinery Rent models, any quantitative estimation could only come about if some approximations of future spot prices were available. The 7-area WEM can be suitably used to produce product prices some three month ahead of those ecountered in the market place, this is to say, the prices produced by the model appear with a lag of about three months time in Rotterdam. This is reported elsewhere and represents the main confirmation of Unidimensionality at Rotterdam.

The present work should be considered complete when Refinery Rent models for the areas of South Europe and North Africa, Middle East and rest of Africa, Japan and Australasia, North America, and Caribbean and South America (the other six areas of the model), were also approximated, even though the lack of necessary data from some of those areas is greater than for North West Europe.

A further important contribution would be the incorporation of the other energy carriers with associated supply/demand and processing areas, and transportation links. In this way the quality specifications of the oil product prices that represent requirements of the market mainly imposed by local environmental protections, will decide when and where a particular oil product is likely to be replaced by a substitute energy form. In this respect the attempt was made though in a very preliminary basis, of including coal as a second energy carrier (Chapters 4 and 7). The main intention there was to explain the recent oil market disequilibrium caused by excessive cracking expansion and eventually by a high price of the energy marker, Arabian Light: the price of crude in line with actual coal prices resulted in the range 15. to 20. $/bl (compared to the actual spot of about 28. $/bl, and official of 29. $/bl). Yet this exercise was done on basically economic grounds: the increasing importance that environmental aspects are having and the pressure public opinion exerts on the energy issue create another relevant frame of study.

By incorporating all other forms of energy, the structure of the model will produce the short run *energy market equilibrium* given the price of *the one* energy source. If the price of the energy marker remains a political issue, and dictates all other energy prices in international trade in the short run, then Unidimensionality will be ever valid, in the long run the prices of other forms of energy will ultimately act as feedback mechanism putting upper limits to the *price* politically set. The competitive nature of the energy world would be better understood and assessed through this implementation.

Within the static framework of the 7-area WEM, various study cases can arise:

- The Third Principle of Energy Substitution, which establishes the existence of an optimum price of the marker energy form that maximizes the long run revenue of its producer and minimizes the consumer's costs, creates the space for investigating the entrance of other sources as markers, namely, of heavier crudes, tar sands, oil shales, as was done with coal, and natural gas in another thesis.[136]

- The model as it stands provides an instrument of investigating the determinates of freight rates and shipowner's rents in the short run in the same context as the refinery rents were studied in this work.

Options of including long run investment decisions have been already added to the model. By creating processes representing expansion of capacity at a cost equal to equilibrium rent, i.e., to the corresponding average annual capital charge, the structure of the market will determine where and when in terms of prices, the building of additional capacity should occur. *When* that happens in *time*, is however not directly given by the model. This would be part of extending the model to be dynamic, which would benefit the analyst by providing an overview of the future energy market and the time path of interfuel substitution, but it would make the analysis lose its more fine quantitative and interpretive condition given by the static model. Besides, predictions on prices of primary energy carriers would be necessary as well as options in the model to substitute one marker by another in a dynamic way given the series of marker prices, which being a political matter have to be exogenously fixed.

To conclude, there is room for a variety of applications and theoretical studies to be undertaken within the energy system represented in a mathematical framework (e.g., the 7-area World Energy Model) where mathematicians, economists, operational researchers, chemical and petroleum engineers, can participate to elucidate questions that at present and in the future are of concern for the energy system as well as important for a worldwide economic order to prevail.

---

[136] Giesecke, C., 'World computer model of oil markets: OPEC pricing strategy model in the short and long term', Ph.D thesis, op.cit.

APPENDIX A. GENERAL ECONOMIC CONSIDERATIONS

We set out here some economic definitions which are used throughout the present study. Other relevant concepts are put forward as required by the various sections.

A *commodity* is a consumable good produced by a firm and for which a particular exchange market exists. Terms as *product* and *outputs* are interchangeably used. The concept is applied to the oil refinery output as the oil commodities of final demand -the oil refined products.

A *factor of production* (alternatively, mean of production, input factor, productive unit, productive resource) is any service entering in the production of a (oil) commodity.

A *single-product firm* is an entity producing only one commodity. A *multi-product multi-process firm* is that firm producing more than a single commodity by means of various production processes. As a result, a number of products may be jointly produced by a single process. The theory of the firm has been developed on the basis of a single product firm but extensions to a multi-product firm can be followed. The oil refinery is a multi-product multi-process firm.

A *production function* $f(x)$ in a profit maximizing firm, is any transformation or technological process such that for a given set of inputs $x=(x_1,x_2,...,x_n)$ an output y is produced in such a way as to minimize the use of resources (factors of production) while maximizing profit and output for the firm.

A production function is said to have *constant*, *decreasing* or *increasing returns to scale*, if an increase in input factor x by a constant factor α respectively causes,

(i) a proportional increase in output $f(x)$, $f(\alpha x) = \alpha f(x)$
(ii) a smaller proportional increase in $f(x)$, $f(\alpha x) < \alpha f(x)$
(iii) a greater proportional increase in $f(x)$, $f(\alpha x) > \alpha f(x)$.

And a *homogeneous production function of degree s*, is that production function for which an increase by a proportion α^s in the input factors can be expressed as $\alpha^s f(x) = f(\alpha^s x)$.

Whenever $s = 1$, $s > 1$ or $s < 1$, $f(x)$ exhibits respectively, constant, increasing or decreasing returns to scale.

The linear production functions attached to a LP problem are homogeneous production functions of degree 1, thus exhibiting constant returns to scale. This directly follows:

let $f(x) = y = \Sigma a_{ij} x_j$, i=1,m, j=1,n, be a LP production function and s a scalar, then because of linearity we have,

$$f(sx) = \Sigma\, b_{ij} s x_j = s\Sigma\, b_{ij} x_j = sf(x)$$

which is the definition of constant returns to scale above.

A *cost function C(x)* is the mean of describing the economic possibilities of a firm when producing and output *y*. Traditionally total costs of a firm are divided into total *fixed* costs and total *variable* costs.

Fixed costs, also *overheads*, are costs which in the short run do not vary with output becoming however variable in the long run. Fixed costs include,

(a) salaries and other expenses of administrative staff,
(b) salaries of temporary staff (contracts on a fixed term basis),
(c) standard depreciation allowances (wear and tear of machinery),
(d) expenses for maintenance of buildings and machinery,
(e) expenses for maintenance of land on which the plant is built,
(f) an additional normal profit may be included in the fixed costs.

Variable costs, also *operating*, *prime*, or *direct* costs are costs which vary directly with output. Variable costs include,

(a) labour cost,
(b) raw material costs, and,
(c) running expenses of machinery: fuel and power costs.

Marginal analysis is an economic (neo-classical) analytical tool whereby the optimal operating conditions of a profit-maximizing firm and the best individual action of the utility-maximizing consumer can be determined when equilibrium market positions prevail.

The followers of the neo-classical theory (marginal analysis concepts of productiviy and utility) have emphasized that market prices for factors of production and commodities are associated to their scarcity. And scarcity refers to limiting positions, e.g, when total availability of certain production factor or commodity is reaching zero level. At this point, a unit not yet produced or consumed is the marginal unit, i.e., that unit causing marginal costs to equate with marginal revenue and market price.

For a profit-maximizing firm, profit is given by revenues minus costs, i.e., $P = R - C$, and the maximum occurs at the point where the derivative of P with respect to output is zero.

$$dP/dy = dR/dy - dC/dy = 0, \qquad dR/dy = dC/dy$$

and,
$$dP/dy = \lim_{\Delta y \to 0} \frac{R(y+\Delta y) - R(y)}{\Delta y}$$

$$dC/dy = \lim_{\Delta y \to 0} \frac{C(y+\Delta y) - C(y)}{\Delta y}$$

In marginal analysis the terms $\Delta_{\to}0$ refer to the point of the last unit produced or consumed, i.e., to the unit *at the margin* of operation and also to the point at which marginal conditions are met.

Marginal productivity (alternatively, marginal physical product, marginal return) of the production function f(x), with respect to an input factor-i, is the change in output per unit change in the input factor-i. It is the derivative of the production function f(x) with respect to the input factor-i.

$$MP(x) = df(x)/dx = \begin{bmatrix} MP_1(x) \\ MP_2(x) \\ \cdots \\ MP_n(x) \end{bmatrix} = \begin{bmatrix} \partial f(x)/\partial x_1 \\ \partial f(x)/\partial x_2 \\ \cdots \\ \partial f(x)/\partial x_n \end{bmatrix}$$

The *average productivity* or average return of f(x) with respect to input factor-i, is the total output y produced divided by the total input factor-i required to produce that output, $AVP_i(x) = y/x_i = f(x)/x_i$.

Euler's Theorem. The Euler's theorem states that, if a function $f(x)$ has constant returns to scale, i.e., it is an homogeneous production function of degree 1, then total output equals the sum of the marginal products of the individual input factors. Moreover, the value of output will be totally exhausted if each factor is paid the value of its marginal product.

Letting the following variables be:

y total output production,

x_i amount of input factor-i,

$\partial y/\partial x_i = a_i$ the marginal product of y with respect to input factor-i,

p price of unit of oputput-y,

$p_i = p\partial y/\partial x_i$ value of marginal product-i,

then,

$$y = f(x) = \Sigma a_i x_i = \Sigma (\partial y/\partial x_i) x_i,$$

and,

$$py = \Sigma p(\partial y/\partial x_i) x_i = \Sigma p_i x_i.$$

APPENDIX B. SOME ALGEBRAIC NOTES ON THE SR MODEL

B.1 Algebraic formulation of the catalytic cracking rent

Sub-System S.3 on page 100, Chapter 3, in relation to the catalytic cracking unit (equally applicable to any other refinery processing unit) is the system of linear equations (dual relationships) attached to the catalytic cracking processes. Next, a detailed formulation is presented by giving all technological and economic components interacting at the individual refinery processing unit. The following variables are defined:

x_j, $j=1,k$, the activity level of catalytic cracking process-j;

p_i, $i=1,n$, the price of input-i to the production process-j (for example prices of waxy fuel oil, long gas oil, refinery fuel, i.e., intermediate products entering the process),

ρ_{cc} the unitary (marginal) catalytic cracking rent,

c_j, $j=1,k$, the cost per unit of catalytic cracking feed processed by catalytic cracking process-j

p_o, $o=1,m$, the price of catalytic cracking output-o (i.e., prices of catalytic cracked gas oil and spirits, gas and fuel oils),

p_q^o, $q=1,t$, $o=1,m$, the imputed costs to quality specification-q on output product-o

a_{ij}, $i=1,n$, $j=1,k$, the input-i required per unit of activity level processed by catalytic cracking process-j,

a_{oj}, $o=1,m$, $j=1,k$, the output of product-o produced per unit of activity level processed by catalytic cracking process-j,

a_{qj}^o, $o=1,n'$, $q=1,t'$, $j=1,k'$, the contribution to quality specification-q on output product-o by the unit activity level processed by production process-j, where, $k' \le k$, $t' \le t$, $n' \le n$; and,

a_{ccj}, $j=1,k$, the per unit requirement of catalytic cracking capacity when production process-j is employed.

The general economic equation for catalytic cracking process-j, reads as:

$$\left(\sum_{i=1}^{n} a_{ij} x_j p_i\right) + a_{ccj} x_j \rho_{cc} + c_j x_{chj} =$$

$$\left(\sum_{o=1}^{m} a_{oj} x_j p_o\right) - \left(\sum_{o=1}^{k'} \sum_{q=1}^{l'} a^o_{qj} x_j p^o_q\right), \quad \text{for } j=1,k.$$

The LP solution gives the vector of values $p=(p_i, p_o, p^o_q)$, the unitary rent ρ_{cc} and the activity level x_j associated to catalytic cracking process-j. The values, except those associated to (original) balance equations, namely, p_i and p_o, might eventually be zero.

For any x_j, j=1,k, being an activity at zero level, the corresponding input/output price equation does not exist. Thus the unitary rent ρ_{cc} will be uniformly associated to every unit of catalytic cracking capacity used by the catalytic cracking active processes.

The total catalytic cracking rent is then,

$$\sum_{j=1}^{J} a_{ccj} x_j \rho_{cc} = \Lambda_{cc} \rho_{cc} = r$$

where, J is the set of catalytic cracking processes in use, $J \leq k$, and Λ_{cc} is the total catalytic cracking capacity available.

If the foregoing procedure is carried out for every refinery processing unit whose capacity is fully utilized, the sum of all individual rents-r is the total rent accruing to the refiner.

B.2 LP conditions for a marginal rent to be zero

Throughout the study we have assumed the refiner is seeking the best allocation of available resources, i.e., that which minimizes his overall refinery performance costs.

The LP formulation and solution assures an optimal allocation of resources; moreover it provides the dual profit maximization function which at the optimum must cover the total refining (plus distribution) costs.

Let us address to the two general LP problems,

$$\begin{array}{lll} \text{Min} & z = cx & \text{(P)} \\ \text{s.t.} & Ax \leq b & \\ & x \geq 0 & \end{array} \qquad\qquad \begin{array}{lll} \text{Max} & z' = yb & \text{(D)} \\ \text{s.t.} & yA \geq c & \\ & y \geq 0 & \end{array}$$

The LP theory demonstrates that if x^o, y^o are feasible solutions of (P) and (D) respectively, then:

$$cx^o = y^o b \quad \text{iff} \quad (c - y^o A)x^o = 0 \quad \text{and} \quad y^o(Ax^o - b) = 0.$$

From (P), $Ax^o \leq b$, pre-multiplying this expression by vector y^o,

$$y^o A x^o \leq y^o b \tag{B.1}$$

From (D), $y^o A \geq c$, post-multiplying this expression by vector x^o,

$$y^o A x^o \geq cx^o \tag{B.2}$$

From (B.1) and (B.2) it follows,

$y^o b = cx^o \leq y^o A x^o \leq y^o b = cx^o$, implying, $cx^o = y^o A x^o = y^o b$, then,

$$cx^o - y^o A x^o = 0, \quad (c - y^o A)x^o = 0,$$

and,

$$y^o A x^o - y^o b = 0, \quad y^o (A x^o - b) = 0 \qquad \text{(B.3)}$$

For expression (B.3) to hold, either $(Ax^o - b) = \Sigma\, a_{ij} x_j^o - b_i = 0$, for all i, meaning the productive resource-i is fully used, and only under these circumstances a positive dual value is generated; or $\Sigma a_{ij} x_j^o - b_i \leq 0$, i.e., there is surplus capacity of resource-i, implying $y_i^o = 0$.

For each refinery processing unit-u there exists a capacity constraint reading as:

$$\sum_{j=1}^{n} a_{uj} x_j^o \leq b_u$$

If there is surplus capacity we have,

$$\sum_{j=1}^{n} a_{uj} x_j^o - b_u < 0$$

and hence, with $y_i^o = \rho_u$,

$$\rho_u \,(\Sigma\, a_{uj}\, x_j^o - b_u) = 0 \qquad \text{iff} \qquad \rho_u = 0,$$

in this way, the condition of zero-rent under surplus capacity is satisfied by the LP formulation and solution algorithm.

APPENDIX C. SINGLE REFINERY AND 7-AREA WEM INPUT DATA

SR and 7-area WEM Variables	Code
Crude Supply Activities	
Marginal crude supply	A1AL1J
Non-marginal crude supply	A1cc1J
Transport Activities	
Crude supply by tanker to area and entrepot	Arccat, Arccet
Crude supply by pipeline to area and entrepot	Arccap, Arccep
Crude transfers entrepot to area	Aeccat
Product transfers (exports/imports inter-area)	Aa5bpt
Refining Activities	
Crude and vacuum distillation	Aa3mcc
Alkylation	Aa3Am1
Catalytic reforming	Aa3Pmm
Catalytic cracking	Aa3CHm
Hydrofining	Aa3Hmm
Hydrocracking	Aa3Ymm
Residue desulphurization	Aa3Rcc
Coking	AaeKmm
Lead Reduction (octane correction)	Aa3Tg1
Refinery Fuel Transfers	
Fuel to refinery	Aa43pR
Heavy fuel oil to bunkers	Aa5BUN
Blending Activities	
Intermediate to finished products	Aa44np
Finished products to demands	Aa45p1
Feasibility vectors	
Product disposals	Aa4ppS
Product procurements	Aa4ppB
Explicit slack on crude distillation capacity	Aa3DSt
Explicit slack on tanker availability constraints	A06Stt

where the symbols above represent: a,b areas, r region, e entrepot, t tanker size, cc crude initials, mm process operating mode, n,p,pp product initials, g gasoline (regular/premium); their values are presented in tables C-1 to C-7 following.

SR and 7-area WEM Constraints and Equations	Code
Objective Function	LR0000
Crude oil supply and transport: mass balance equations and availability constraints	
Crude oil supply at jetty	1laccJ
Crude oil supply at field	1laccF
Limit on local crude oil supply	1a****
Crude oil balance at refinery (marginal and non-marginal crude supply balances area-a)	1a1cc0
Crude oil balance at entrepot (transhipments)	1r1cc0
Port constraints	1a1Btt, 1a1Stt
Pipeline capacity	1a1Bnn
World tanker availability	106Btt
Fuel balances area-a	
Refinery fuel balance area-a	1a3RFL
Bunker fuel balance area-a	1a5BUN
Refinery processing unit capacities area-a	
Crude distillation	1a3BCD
Vacuum distillation	1a3BCV
Alkylation	1a3BCA
Catalytic reforming	1a3BCP
Catalytic cracking	1a3BCC
Hydrofining	1a3BCH
Hydrocracking	1a3BCY
Residue desulphurization	1a3BCR
Coking	1a3BCK
Refinery processing restrictions and ratios area-a	
High sulphur gas oil control	1a3BH1
High sulphur bitumen control	1a3BHV
Low sulphur bitumen control	1a3BVL
Intermediate product balances area-a	
Isobutane balance	1a4AG0
Refinery gas, LPG balance	1a4GA0
Unsaturated gases balance	1a4UG0
Straight-run gasoline balance	1a4SG0
Straight-run benzine balance	1a4SB0
Naphtha balance	1a4NA0
Kerosine balance	1a4KE0
Catalytic cracked spirits balances	1a4XS0, 1a4YS0
Steam cracked naphtha balance (non-crude supply)	1a4ZS0
Gas oil balances (middle distillates/vacuum)	1a4DF0, 1a4DH0 1a4DL0
Waxy distillates balances (low/high sulphur)	1a4TD0, 1a4WD0

Fuel oil balances (low/high/medium sulphur)	la4LFO, la4HFO la4MFO
Motor spirits balances (regular/premium)	la4RMO, la4PmO

Product quality specifications area-a

Pour point (on gas oil and fuel oils)	la4DFP, la4LFP la4HFP
Flash point (on gas oil)	la4DFF
Sulphur content (on gas oil and fuel oils)	la4DFS, la4LFS la4HFS
Viscosity (on fuel oils)	la4LFV, la4HFV
Lead content (on gasolines)	la4RMB, la4PMB
D+L @ 100°C (on gasolines)	la4RMD, la4PMD
Olephin content (on gasolines)	la4RME, la4PME
Research octane number (RON, on gasolines)	la4RMR, la4PMR
36:1 V/L temperature (on gasolines)	la4RML, la4PML

Finished products demands balances area-a

Feedstock balance	la5FSO
Kerosine balance	la5KEO
LPG, Gas balance	la5GAO
Motor spirits (regular/premium)	la5RMO, la5PMO
Gas oils balance	la5DFO
Fuel oils balances (low/medium/high sulphur)	la5LFO, la5MFO la5HFO
Bitumen balance	la5BTO
Coke balance	la5EKO

where the symbols above represent: a area, r region, e entrepot, tt tanker size, nn pipeline name, cc crude initials; and the crude oil supply restriction represented in the model is:

1M6SEA constraint on total Middle East crude transported by tanker to area M (lower bound, for a minimum local consumption).

The values of all symbols are presented in tables C-1 to C-7 following.

Table C-1. 7-area World Energy Model Geographical Areas and Crude Production Regions.[a)]

Area Code	Countries in world area
B	North West Europe: Austria, Benelux, half France, Great Britain, Iceland, Ireland, Scandinavia, Switzerland and West Germany
J	Australasia: Australia, Japan, New Zealand and South East Asia
K	Caribbean, Central and South America
M	Middle East but Israel and Syria, and Africa but Algeria, Lybia and Egypt
R	Eastern Europe
T	South Europe and Mediterranean: Algeria, Cyprus, Egypt, half France, Greece, Israel, Italy, Lybia, Portugal, Rumania, Spain, Syria, Turkey and Yugoslavia
U	North America East Coast: Canada and Central East USA
V	North America West Coast: Alaska and West USA

Region Code	Crude Production Region
1	North Sea: part of area B
2	North and West Africa: parts of areas M and T
3	Middle East: area M, and Egypt from area T
4	Far East: Indonesia and Australia, area J
5	North America, Ecuador and Venezuela: areas U, V and part of K

a) The WEM stands as a 7-area world oil model excluding Centrally Planned Economies; the additional area-R is here representing the export of some oil products of final demand from eastern to western Europe. The eastern european refinery system is not modelled at any degree of detail whatsoever. The export products are represented on Table C-3.

Table C-2. 7-area WEM crude coding and API gravities.

Crude Code	Crude Name	Origin Country/Region	WEM Area	API Gravity
EF	Ekofisk	Norway/1	B	44.0
FT	Forties	Great Britain/1	B	36.3
AH	Arabian Heavy	Saudi Arabia/3	M	28.0
AJ	Iranian Light	Iran/3	M	33.9
AL	Arabian Light	Saudi Arabia/3	M	34.2
AZ	Arzew	Algeria/3	T	44.1
BG	Brega	Lybia/3	T	40.4
BS	Basra	Iraq/3	M	34.0
EM	El Morgan	Egypt/3	T	31.7
GS	Iranian Heavy	Iran/3	M	31.0
HM	Hassi Massaoud	Algeria/3	T	44.7
KT	Kuwait Export	Kuwait/3	M	31.4
MA	Kirkuk	Iraq/3	M	36.1
MJ	Mandji	Gabon/2	T	28.9
MN	Oman	Oman/3	M	32.9
MR	Murban	Abu Dhabi, UAE/3	M	39.4
NL	Nigerian Light	Nigeria/2	T	36.1
NM	Nigerian Medium	Nigeria/2	T	25.9
QE	Dukhan	Qatar/3	M	41.4
QM	Marine	Qatar/3	M	36.6
SR	Sarir	Lybia/3	T	36.5
SY	Syria	Syria/3	T	24.8
US	Um Shaif	Abu Dhabi, UAE/3	M	37.0
ZK	Zakum	Abu Dhabi, UAE/3	M	40.1
GL	Gippsland	Australia/4	J	44.3
MS	Minas	Indonesia/4	J	34.1
BQ	Bachaquero	Vanezuela/5	K	13.5
EE	Oriente	Ecuador/5	K	30.4
FC	Oficina	Venezuela/5	K	33.3
TJ	Tia Juana Medium	Venezuela/5	K	26.3
AS	Alaska South	Alaska/5	V	33.8
BR	Bradford	USA/5	U	41.1
CA	California	USA/5	V	25.0
PB	Bantry	Canada/5	U	24.8
PL	IPPL Light SB	Canada/5	U	35.9
PR	Rainbow	Canada/5	U	38.6
PS	IPPL Mixed Sour	Canada/5	U	38.0
PW	IPPL Medium SB	Canada/5	U	38.6
XA	Arkansas Miss.	USA/5	U	32.8
XE	Texas East	USA/5	U	37.8
XG	Texas Gulf L	USA/5	U	39.0
XH	Texas Gulf H	USA/5	U	31.0
XL	Louisiana	USA/5	U	32.3
XS	Texas West Sour	USA/5	U	31.0
XW	Texas West Medium	USA/5	U	33.8

Table C-3. 7-area WEM chemistry of supply, base case 1976.[b)]

Crude	Crude Oil supply a Jetty area (mmt/y) — World Areas						
	B	J	K	M	T	U	V
EF	13.685						
FT	11.698						
AH				44.01			
AJ				168.23			
AL				264.81			
AZ					40.063		
BG					33.83		
BS				56.23			
EM					31.70		
GS				25.62			
HM					14.03		
KT				108.878			
MA				51.018			
MJ				19.52			
MN				35.655			
MR				53.300			
NL				58.56			
NM				43.47			
QE				11.083			
QM				12.035			
SR					58.023		
SY				12.520			
US				10.915			
ZK				12.553			
GL		19.618					
MS		74.853					
BQ			15.513				
EE			9.088				
FC			35.443				
TJ			68.225				
AS							10.583
BR						8.125	
CA							57.995
PB						6.535	
PL						14.668	
PR						7.105	
PS						12.613	
PW						28.433	
XA						30.000	
XE						100.085	
XG						16.303	
XH						43.960	
XL						91.813	
XS						39.245	
XW						78.825	

Table C-3. (Cont.)

	B	J	K	M	T	U	V
Indg. crude	6.49	27.03	100.858		6.79		
Total	31.87	121.50	229.400	998.40	184.44	477.71	68.578
Non crude							
GAS	3.37	4.70	.80	.26	3.14	7.06	.30
SRG	.71	.81	.0	.0	.0	19.01	.0
SCN	.45	.52	.0	.0	.0	12.04	.0

Area R total non-crude supply to Western Europe

DFO	HFO	LFO	RMS
16.00	6.00	6.00	6.00

b) The crude supply at jetty area (figures Table C-3) are the actual area productions for both local consumption and exports; the crude distribution to other areas is done by the model solution by choosing the cheapest transportation route among all the routes available. Indigenous crude is assumed to be used at the local area only. Non-crude supplies, including eastern european's (i.e., DFO, HFO, LFO and RMS from area R), are inputs to the system.

Table C-4. 7-area WEM chemistry of demand, base case 1976.

Oil product (mmt/y)	World Areas						
	B	J	K	M	T	U	V
GAS	6.46	16.21	4.57	.45	8.37	32.39	1.83
LFS	19.78	30.42	9.87	.74	16.23	24.85	2.12
PMS	42.10	17.97	9.61	8.54	29.60	73.60	11.74
RMS	14.28	28.24	27.54	3.51	12.47	210.14	32.51
KERO	13.14	40.20	10.18	8.82	12.60	40.53	12.33
DFO	123.21	62.81	30.30	9.06	96.56	163.23	13.41
LSFO	11.59	26.80	.0	.41	6.86	24.81	4.66
MSFO	33.67	44.05	.0	.0	15.25	34.73	5.25
HSFO	69.75	69.46	61.56	2.56	91.54	81.11	6.71
BIT	11.45	15.92	1.93	.86	9.57	22.48	3.23
COKE	.46	.20	.	.	.57	11.87	2.26
Totals	345.89	352.28	155.56	34.95	299.62	719.74	96.05

Table C-5. 7-area WEM total refined product demand, base case 1976.

Product	(mmt/y)	Product	(mmt/y)
GAS	70.28	LSFO	75.13
LFS	104.01	MSFO	132.95
PMS	193.16	HSFO	382.61
RMS	328.69	BIT	65.44
KERO	137.80	COKE	15.36
DFO	498.58		
Total refined products demand 2004. mmt			

Table C-6. 7-area WEM refinery processing units capacities, base case 1976.

Unit	Refining capacities (mmt of feed) World Areas						
	B	J	K	M	T	U	V
ALK	.34	.44	.82	.46	1.34	29.2	12.938
CCRAK	32.11	32.93	43.81	7.64	27.35	269.77	36.21
COKER	.495	.20	.0	.0	.573	11.873	2.263
ATDIST	565.68	503.81	395.81	182.23	536.36	742.24	121.34
HYDROF	33.42	61.72	49.76	19.04	68.74	52.89	7.190
HYCRAK	2.32	3.95	1.90	6.15	2.78	23.02	16.
PLATF	63.01	32.326	17.60	12.36	50.65	137.84	25.29
RESDES	.0	16.29	.0	1.79	.0	.66	.0
VCDIST	65.93	128.71	120.82	46.27	89.01	268.80	58.49

Table C-7. 7-area WEM fleet availability, base case 1976.

WEM Code	Ship size (dwt)	Worldwide Capacity (million dwt)
6	≤ 24.999	17.460
8	25.000 - 49.999	33.050
10	50.000 - 79.999	32.620
11	80.000 - 124.999	43.610
12	125.000 - 199.999	33.350
13	≥ 200.000	164.590
	Total	324.680

All data sources are indicated in World Energy Models Ltd., *Seven Area Short Term World Energy Model, A Technical Description* (London, 1976):

- For crude oil supply, *Petroleum Times*, and *Oil & Gas Journal*, (op.cit., pp.30-31).
- For oil product demands, OECD, *Quarterly Oil Statistics*; Bureau of Mines: *Mineral Industry Surveys USA*; CPDP, *Bulletin Mensuel*, *BP statistical review of world energy*, and company industrial information; special data arrangements are reported as, for instance, non-adjustments to stock changes, (op.cit., p.31 and ff.).
- For refining capacities, *Petroleum Times*, 23.1.76, for Canada, Bureau of Mines: *Mineral Industry Surveys of USA*, and *Oil & Gas Journal*, for the remaining areas.
- For fleet availability, *World Tanker Fleet Review*; total supply 'including both, dedicated oil tankers and combined carriers', (op.cit., p.39).

APPENDIX D. TIME SERIES DATA

Table D-1. 1969-1983 worldwide hydroskimming and catalytic cracking capacities, mmt/y average processing.

	North America	W. Europe	World
1969–CD	672.833	672.903	1976.165
CR	113.650	70.190	225.227
CC	305.640	40.046	409.503
1970–CD	699.416	737.900	2154.787
CR	118.312	73.723	238.271
CC	312.628	45.564	40.046
1971–CD	737.732	793.384	2298.835
CR	130.590	79.218	260.788
CC	306.179	47.257	427.042
1972–CD	755.521	860.046	2496.723
REF	134.727	87.799	284.736
CC	293.304	55.340	431.391
1973–CD	808.503	923.292	2688.204
REF	139.761	92.360	298.702
CC	300.340	56.425	438.842
1974–CD	845.106	961.807	2822.913
REF	143.839	99.611	313.893
CC	297.409	56.337	439.545
1975–CD	862.287	1028.120	2928.987
REF	150.418	103.900	328.381
CC	299.218	54.683	439.655
1976–CD	926.031	1055.000	3050.500
REF	153.413	107.917	339.075
CC	305.980	53.520	450.000
1977–CD	954.417	1050.476	3107.770
REF	158.796	110.473	348.680
CC	307.746	51.795	455.196
1978–CD	981.807	1034.100	3207.867
REF	159.950	115.741	358.581
CC	310.460	56.420	465.514
1979–CD	1007.703	1021.650	3250.200
REF	164.409	112.245	362.987
CC	320.828	56.607	487.656
1980–CD	1015.920	1019.160	3288.300
REF	169.339	113.573	371.807
CC	332.804	61.420	505.639

1981-CD	1018.161	1006.249	3261.000
REF	166.423	111.474	377.053
CC	328.160	69.034	517.341
1982-CD	952.176	983.028	3112.500
REF	161.926	110.249	364.116
CC	308.063	67.256	502.026
1983-CD	890.922	867.615	2949.900
REF	151.953	100.015	346.950
CC	329.481	69.326	511.147

Sources:

- OPEC, 'Refinery Capacities 1960-1982, by process and region', (June 1983), unpublished reference. OPEC figures are based on *Oil & Gas Journal*, *Hydrocarbon Processing* and *Arab Oil and Gas*.
- *Oil & Gas Journal*, 'Worldwide refining', issues of last week of December, (USA, years 1970-1984).
- *BP statistical review of world energy*, years 1983 and 1984.

The figures are originally in bl/calendar day; the conversion factors bl/day to mt/year used here are as follows:

- CD Crude Distillation: 49.8 (basis Arabian Light crude, 7.33 bl/mt).

- REF Catalytic Reforming:

  North America - 41.95 (8.7 bl/mt of motor spirit/naphtha)

  Western Europe - 41.76 (8.74 bl/mt naphtha)

- CC Catalytic Cracking:

  North America - 52.9 (6.8997 bl/mt of heavy fuel oil).

  Western Europe - 55.13 (6.6208 bl/mt of heavy fuel oil).

  All other markets - 56.85 (6.42 bl/mt heavy fuel oil).

Table D-2. 1972-1984 catalytic cracking capacity, world excluding North America and CPEs.

Year	CC capacity th bl/d	CC capacity mmt/y	CC capacity changes th bl/d	CC capacity changes mmt/y
1972	1956.160	111.215		
1973	2018.674	114.769	62.514	3.554
1974	2345.489	133.349	326.815	18.581
1975	2395.791	136.209	50.302	2.860
1976	2429.920	138.150	34.129	1.940
1977	2432.677	138.306	2.756	0.157
1978	2482.346	141.130	49.670	2.824
1979	2585.825	147.013	103.478	5.883
1980	2807.391	159.610	221.566	12.597
1981	2889.728	164.291	82.337	4.681
1982	3235.042	183.924	345.314	19.632
1983	3308.197	188.083	73.155	4.159
1984	3989.838	226.836	681.641	38.754

Source: *Oil & Gas Journal*, 'Worldwide refining' section, issue of last week of December of each year.

CPEs.: Centrally Planned Economies.

Table D-3. 1970-1984 NWE hydroskimming and cracking capacities, mmt/y average processing feed.

Year	CD	REF	CC	HYC
1970	429.000	38.20	25.30	.500
1971	460.760	43.24	26.25	.727
1972	502.837	46.43	30.74	1.109
1973	514.414	54.17	31.37	2.194
1974	550.620	56.24	33.19	2.482
1975	567.416	60.20	33.90	2.600
1976	611.883	64.13	32.63	2.600
1977	632.614	66.79	33.27	1.929
1978	593.465	67.68	31.14	2.551
1979	592.243	72.10	34.24	3.707
1980	589.749	67.56	35.17	3.617
1981	592.485	68.23	36.77	5.605
1982	571.262	70.46	34.93	6.360
1983	502.069	63.69	35.74	5.190
1984	482.465	61.77	40.90	6.577

Source: *Oil & Gas Journal*, op.cit. Original figures in bl/calendar day; conversion factors applied:

CD Crude Distillation: 49.8 (34 API crude, 7.33 bl/mt crude),
REF Catalytic Reforming: 41.76 (8.74 bl/mt of naphtha),
CC Catalytic Cracking: 55.13 (6.6208 bl/mt hfo),
HYC Hydrocracking: 46.2 (7.9 bl/mt middle distillates).

Table D-4. 1972-1983 worldwide tanker fleet availability, million dwt.

1972	1973	1974	1975	1976	1977	
189.5	215.6	255.8	291.4	320.7	332.5	a)
149.7	170.3	202.0	211.0	232.5	258.3	b)
1978	**1979**	**1980**	**1981**	**1982**	**1983**	
328.5	327.9	324.8	320.2	303.7	283.2	a)
258.4	281.4	267.8	233.5	215.0	198.2	b)

Sources: *BP statistical review of world energy*, years 1983 and 1984 for available tanker fleet; for effective shipping capacity, years 1972-1975 estimated from supply figures of *BP statistical review of world energy*, op.cit.; and years 1976-1983 from *Lloyds Shipping Economist*.

Table D-5. 1972-1983 worldwide total and motor gasoline demands, million mt/year.

	total	%	motor
1972	564.00	84.	473.76
1973	593.20	84.	498.29
1974	582.40	84.50	492.13
1975	585.90	84.75	496.60
1976	612.70	85.	520.80
1977	624.70	85.	531.00
1978	656.30	85.	557.90
1979	653.10	84.	548.60
1980	628.40	86.	540.42
1981	608.80	87.	529.70
1982	609.80	88.	536.62
1983	616.30	88.	542.34

Source: total figures in *BP statistical review of world energy*, years 1983 and 1984. Percentages of motor gasoline from total demand are: estimated for 1972-1977, and BP direct communications for 1978-1983.

Table D-6. 1973-1983 North West Europe motor gasoline, gas oil and heavy fuel oil demands, mmt/y.

Year	Motor gasoline	Gas oil	Heavy fuel oil
1970	42.52	110.00	101.28
1971	46.69	116.71	125.73
1972	51.18	125.73	109.62
1973	54.16	135.25	115.39
1974	51.64	118.96	104.39
1975	58.92	127.83	100.95
1976	60.83	135.38	104.15
1977	63.51	136.00	97.93
1978	66.81	141.81	99.81
1979	67.32	142.99	100.96
1980	67.74	128.12	85.62
1981	65.34	115.83	68.42
1982	66.32	107.17	60.27
1983	68.60	118.21	46.94

Sources: *BP statistical review of world energy*, various years; CPDP, *Bulletin Mensuel*, respective years; OPEC, 'Demand of selected OECD countries', various years, unpublished reference; OECD, *Quarterly Statistics*, 4th quarter, 1984.

Table D-7. 1972-1983 Rotterdam spot prices, monthly nominal figures.

Month	PMS	HFO	NAPHTHA	JETKER	DFO	Arabian Light	
	-	US$ per metric tonne			-	US$/mt	US$/bl
Jan72	32.63	12.94	23.25	31.86	26.61	8.86	1.20
	32.54	14.03	22.08	30.69	24.02	8.86	1.20
	32.40	14.16	21.08	29.69	23.74	8.86	1.20
	32.21	14.21	20.64	29.25	25.25	8.86	1.20
	35.57	13.97	20.64	29.25	23.92	8.86	1.20
	37.50	13.43	18.14	26.75	23.46	8.86	1.20
	40.87	14.12	18.02	26.63	24.13	8.86	1.20
	43.57	14.69	19.73	28.34	25.68	9.59	1.30
	43.15	14.35	22.84	31.48	26.98	9.59	1.30
	43.57	13.95	25.75	34.86	28.81	9.59	1.30
	44.17	14.42	27.73	36.34	32.43	9.59	1.30
	47.86	15.29	31.23	39.84	35.94	9.59	1.30
Jan73	53.06	16.67	34.35	42.96	37.49	9.59	1.30
	55.05	18.46	38.41	47.02	41.88	9.59	1.30
	62.18	17.64	43.12	51.73	42.13	9.59	1.30
	73.68	16.33	42.75	55.50	43.92	12.55	1.70
	96.01	16.92	50.51	57.04	54.71	12.55	1.70
	111.95	18.90	60.22	64.38	66.56	12.55	1.70
	102.47	18.42	72.14	74.47	72.43	12.55	1.70
	83.88	16.73	68.00	78.74	71.39	12.55	1.70
	85.73	15.56	67.40	82.55	79.46	16.97	2.30
	102.02	21.64	83.67	123.69	121.21	16.97	2.30
	150.20	47.98	116.99	188.44	186.52	16.97	2.30
	186.77	119.60	145.34	222.42	185.91	36.90	5.00
Jan74	150.81	103.23	148.71	160.76	131.92	36.90	5.00
	153.35	76.25	142.30	108.59	103.49	79.70	10.80
	185.48	67.53	160.48	108.76	100.37	79.70	10.80
	181.05	65.10	163.00	103.34	83.62	86.35	11.70
	173.98	67.12	149.37	101.81	90.54	86.35	11.70
	148.28	64.23	127.10	98.30	88.54	86.35	11.70
	124.57	61.89	105.97	99.20	92.89	86.35	11.70
	115.25	61.03	97.35	97.94	92.28	86.35	11.70
	113.59	63.10	98.00	96.30	93.17	86.35	11.70
	109.45	65.23	97.74	97.92	92.29	87.08	11.80
	111.04	69.30	95.81	101.73	89.84	87.82	11.90
	115.62	69.77	93.13	104.95	92.01	87.82	11.90
Jan75	118.84	69.78	92.19	107.12	87.20	85.61	11.60
	120.28	69.98	92.59	104.86	81.68	87.82	11.90
	120.68	71.25	102.40	104.50	82.61	86.35	11.70
	127.27	68.69	108.47	104.50	88.41	86.35	11.70
	135.10	65.10	113.00	104.50	94.56	87.08	11.80
	141.56	61.02	109.05	106.85	106.13	86.35	11.70
	128.73	57.60	105.95	107.83	99.29	87.08	11.80
	126.45	57.06	109.17	112.48	109.48	87.08	11.80
	125.44	57.24	110.89	122.06	115.77	85.61	11.60
	128.63	53.47	116.11	127.88	117.76	86.35	11.70
	134.09	55.49	128.18	130.08	111.71	87.08	11.80
	138.04	57.89	128.58	125.19	105.47	87.08	11.80

Table D-7. (Cont.)

Jan76	137.88	64.45	130.65	121.72	103.41	84.80	11.49
	144.50	67.21	135.06	115.83	102.95	84.80	11.49
	150.36	66.65	140.24	116.37	104.22	85.09	11.53
	160.31	63.50	137.05	118.24	105.65	85.17	11.54
	164.23	64.14	134.07	119.61	105.16	85.31	11.56
	163.24	64.48	133.81	119.12	105.97	85.09	11.53
	157.04	64.39	134.39	116.73	104.07	85.02	11.52
	152.50	66.25	128.60	121.03	108.30	85.09	11.53
	147.42	70.00	125.38	122.53	111.06	86.64	11.74
	149.68	71.35	124.37	122.23	109.58	88.26	11.96
	148.44	71.10	120.58	121.32	105.91	89.22	12.09
	144.62	73.40	124.11	123.52	112.25	89.30	12.10
Jan77	143.27	81.24	124.67	127.63	116.16	90.85	12.31
	141.97	79.77	127.81	130.14	119.33	92.91	12.59
	141.93	74.12	129.91	128.23	115.62	92.69	12.56
	148.76	72.38	131.03	128.40	117.40	92.69	12.56
	147.71	71.61	127.70	129.73	118.50	93.21	12.63
	141.34	72.36	126.38	129.98	118.41	92.99	12.60
	141.21	75.44	126.54	130.81	119.71	93.50	12.67
	140.52	76.21	126.58	132.44	117.27	93.36	12.65
	139.29	76.08	120.44	129.94	115.92	93.06	12.61
	139.43	77.49	116.58	128.00	116.93	93.06	12.61
	139.30	77.59	117.78	128.38	118.06	93.43	12.66
	137.80	79.78	121.58	130.45	120.23	93.65	12.69
Jan78	137.48	78.38	120.80	129.41	117.85	93.43	12.66
	141.40	76.25	123.87	129.52	119.63	93.43	12.66
	148.80	75.50	128.54	131.82	122.56	93.50	12.67
	148.14	75.40	130.38	133.73	126.67	93.65	12.69
	151.36	74.10	125.95	132.02	122.64	93.87	12.72
	154.23	73.29	130.89	131.86	121.09	93.95	12.73
	160.28	73.21	141.20	137.44	122.50	93.87	12.72
	181.86	72.18	155.76	146.72	122.69	94.02	12.74
	188.39	71.64	156.79	149.89	126.92	94.24	12.77
	200.81	74.31	169.41	161.02	133.85	95.79	12.98
	220.77	83.87	189.97	184.25	155.50	106.49	14.43
	210.08	80.30	179.99	186.66	151.64	109.45	14.83
Jan79	219.26	84.03	199.08	206.76	194.31	119.85	16.24
	322.41	103.52	284.34	316.50	289.16	166.49	22.56
	295.02	106.68	264.62	311.53	251.20	165.16	22.38
	308.14	109.87	260.20	287.20	265.77	156.82	21.25
	368.67	128.63	294.50	324.84	319.94	213.58	28.94
	397.40	142.12	331.35	379.62	362.29	261.25	35.40
	380.34	140.65	342.96	382.75	356.66	244.50	33.13
	357.05	138.48	337.02	386.47	316.89	249.44	33.80
	345.54	139.67	327.60	383.97	325.78	258.30	35.00
	355.48	156.21	336.75	394.29	330.23	280.44	38.00
	391.94	174.90	368.04	411.63	361.39	302.58	41.00
	416.25	178.74	393.67	417.71	356.08	302.58	41.00
Jan80	400.57	164.18	383.46	392.45	344.18	281.40	38.13
	386.37	145.02	346.02	352.09	307.24	265.68	36.00
	379.90	143.94	341.97	345.87	297.05	265.24	35.94
	372.60	155.13	339.30	342.64	318.90	263.24	35.67
	376.11	161.29	325.39	339.16	320.95	267.30	36.22
	373.12	153.57	309.70	336.33	312.74	265.68	36.00

Table D-7. (Cont.)

Jul80	361.52	150.00	298.63	337.84	305.58	255.35	34.60
	326.63	152.60	266.79	328.10	272.45	235.72	31.94
	337.53	162.90	284.22	335.40	284.72	242.95	32.92
	357.00	202.42	315.84	352.62	300.84	280.44	38.00
	387.09	232.17	365.92	367.44	321.42	304.42	41.25
	377.99	219.81	341.78	361.26	299.65	299.63	40.60
Jan81	379.19	216.70	355.46	347.92	307.21	286.27	38.79
	370.66	216.26	343.92	340.87	303.88	274.24	37.16
	363.34	211.67	344.01	340.87	308.05	272.76	36.96
	359.03	203.19	332.56	333.82	288.65	268.12	36.33
	351.11	182.07	300.74	309.81	271.59	249.81	33.85
	363.34	167.64	315.08	301.58	271.59	236.60	32.06
	391.60	168.17	335.97	332.79	286.75	235.94	31.97
	384.45	161.08	331.07	334.29	294.10	238.60	32.33
	383.16	164.06	325.74	344.04	297.89	236.60	32.06
	382.21	171.48	321.28	343.88	311.00	245.83	33.31
	375.75	176.31	329.50	344.67	322.23	253.95	34.41
	350.42	167.84	317.87	350.37	322.07	252.84	34.26
Jan82	339.73	164.71	303.89	351.17	315.10	251.29	34.05
	316.72	165.90	284.22	345.39	280.38	223.91	30.34
	297.60	164.11	263.69	300.17	261.81	213.28	28.90
	324.13	167.09	286.15	293.75	277.81	234.76	31.81
	363.85	168.48	323.64	319.33	295.47	247.67	33.56
	368.94	167.22	324.25	316.56	284.86	242.58	32.87
	358.60	163.31	297.86	309.99	274.24	237.34	32.16
	347.83	154.92	304.15	320.66	285.84	232.47	31.50
	360.84	161.73	311.32	333.35	303.20	247.67	33.56
	360.75	172.45	301.44	339.85	313.81	246.57	33.41
	331.63	160.60	283.09	327.49	293.57	236.38	32.03
	317.24	162.52	284.75	309.75	289.86	227.30	30.80
Jan83	314.66	165.45	293.93	308.70	275.08	228.78	31.00
	285.62	154.86	271.11	281.85	242.86	218.15	29.56
	281.49	156.25	253.72	261.73	235.74	209.89	28.44
	303.55	162.28	280.12	265.85	247.03	214.39	29.05
	300.18	161.81	274.52	268.70	240.67	211.44	28.65
	307.65	158.00	286.00	270.00	245.00	213.87	28.98
	307.30	162.00	287.00	275.00	244.00	214.98	29.13
	307.40	168.55	294.70	281.00	254.00	213.87	28.98
	296.60	167.00	284.00	286.00	251.00	211.14	28.61
	292.85	167.00	275.00	277.00	248.00	210.77	28.56
	293.40	168.00	267.40	281.00	249.00	208.71	28.28
	278.30	170.00	255.00	275.00	245.00	208.56	28.26

Sources: *OPEC bulletin*, several years; OPEC, 'Oil and Product Statistics, 1976-1980', (January 1983), unpublished reference; OPEC figures are based on *Platts Oilgram* reports on Rotterdam spot prices.

Figures are reported in US$/bl, the following conversion factors (bl/mt) at Rotterdam have been used:

- Arabian Light: 7.38; Naphtha: 8.74
- Jet Kero: 7.9216; Premium Motor Spirit (PMS): 8.6161
- Gas Oil (DFO): 7.58; Heavy Fuel Oil (HFO): 6.6208.

Table D-8. 1972-1983 Rotterdam spot prices, annual nominal averages.

Year	PMS	HFO	NAPHTHA	JETKER	DFO	Arabian Light	
	–	US$ per	metric tonne	–		US$/t	Us$/bl
1972	38.84	14.13	22.59	31.25	26.75	9.16	1.24
1973	96.92	28.74	68.57	90.74	83.63	14.94	2.02
1974	140.21	69.48	123.25	106.63	95.91	81.43	11.03
1975	128.76	62.05	109.71	113.15	100.01	86.65	11.74
1976	151.68	67.24	130.69	119.85	106.54	86.15	11.67
1977	141.88	76.17	124.75	129.51	117.79	92.95	12.59
1978	170.30	75.70	146.13	146.19	128.63	96.31	13.05
1979	346.46	133.62	311.68	350.27	310.81	226.75	30.72
1980	369.70	170.25	326.58	349.27	307.14	268.92	36.44
1981	371.19	183.87	329.43	335.41	298.75	254.30	34.46
1982	340.65	164.42	297.37	322.28	289.66	236.77	32.08
1983	297.42	163.43	276.82	277.65	248.11	213.71	28.96

Source: Figures worked out from table D-7.

Table D-9. 1964-1984 US$ GDP/GNP deflators and selected nominal exchange rates.

Year	US$ GDP/GNP deflators 1980=100 (a)	1976=100 (b)	US$ exchange rates DM FRG	Pound UK	Guilder Nether.
1964	40.8	55.0	3.98	0.36	3.60
1965	41.7	56.2	4.00	0.36	3.61
1966	43.0	58.0	4.00	0.36	3.60
1967	44.3	59.7	3.99	0.36	3.60
1968	46.3	62.4	3.99	0.42	3.62
1969	48.6	65.5	3.93	0.42	3.62
1970	51.3	69.1	3.65	0.42	3.62
1971	53.8	72.5	3.48	0.41	3.50
1972	56.1	75.6	3.19	0.40	3.21
1973	59.3	79.9	2.67	0.41	2.79
1974	64.5	86.9	2.59	0.43	2.69
1975	70.5	95.0	2.46	0.45	2.53
1976	74.2	100.0	2.52	0.56	2.64
1977	78.5	105.8	2.32	0.57	2.45
1978	84.3	113.6	2.01	0.52	2.16
1979	91.6	123.5	1.83	0.47	2.00
1980	100.0	134.8	1.82	0.43	1.99
1981	109.4	147.4	2.26	0.49	2.50
1982	116.0	156.3	2.43	0.57	2.67
1983	120.9	162.9	2.55	0.66	2.85
1984	125.2	168.7	2.85	0.75	3.20

Sources: column (a), and exchange rates from OECD, *Main Economic Indicators*, several years, and *Main Economic Indicators, Hisorical Statistics 1964-1983*, (Paris, 1984), p.50., and *OECD Economic Outlook*, 27, (Paris, July 1980), p.145. Column (b) worked out from column (a).

Table D-10. 1972-1983 Rotterdam spot prices, monthly figures in real terms 1976 US$.

Year	PMS	HFO	NAPHTHA	JETKER	DFO	Arabian Light	
	–	US$ per metric tonne			–	US$/mt	US$/bl
Jan72	44.06	17.47	31.40	43.02	35.94	11.96	1.62
	43.79	18.88	29.71	41.30	32.32	11.92	1.61
	43.45	18.99	28.27	39.82	31.84	11.88	1.61
	43.05	18.99	27.58	39.09	33.75	11.84	1.60
	47.37	18.61	27.49	38.96	31.86	11.80	1.60
	49.77	17.83	24.08	35.50	31.14	11.76	1.59
	54.06	18.68	23.84	35.22	31.92	11.72	1.59
	57.44	19.37	26.01	37.36	33.85	12.64	1.71
	56.69	18.85	30.01	41.36	35.45	12.60	1.71
	57.05	18.27	33.72	45.64	37.72	12.56	1.70
	57.64	18.82	36.19	47.42	42.32	12.51	1.70
	62.24	19.89	40.62	51.81	46.74	12.47	1.69
Jan73	68.24	21.44	44.18	55.25	48.22	12.33	1.67
	70.48	23.63	49.18	60.20	53.62	12.28	1.66
	79.24	22.48	54.95	65.93	53.69	12.22	1.66
	93.47	20.72	54.23	70.41	55.72	15.92	2.16
	121.25	21.37	63.79	72.04	69.09	15.85	2.15
	140.74	23.76	75.71	80.94	83.68	15.78	2.14
	128.25	23.05	90.29	93.20	90.65	15.71	2.13
	104.51	20.85	84.73	98.11	88.95	15.64	2.12
	106.34	19.30	83.61	102.40	98.57	21.05	2.85
	125.99	26.72	103.33	152.75	149.69	20.96	2.84
	184.67	58.99	143.84	231.69	229.33	20.86	2.83
	228.63	146.40	177.91	272.27	227.58	45.17	6.12
Jan74	180.83	123.78	178.31	192.76	158.18	44.24	6.00
	182.60	90.79	169.44	129.30	123.23	94.90	12.86
	219.33	79.85	189.77	128.61	118.69	94.25	12.77
	212.62	76.45	191.43	121.36	98.20	101.41	13.74
	202.93	78.29	174.23	118.75	105.61	100.72	13.65
	171.79	74.41	147.25	113.88	102.58	100.04	13.55
	143.35	71.22	121.94	114.15	106.89	99.37	13.46
	131.74	69.76	111.28	111.95	105.48	98.70	13.37
	128.98	71.65	111.28	109.35	105.79	98.05	13.29
	123.46	73.58	110.25	110.46	104.11	98.23	13.31
	124.44	77.66	107.37	114.00	100.68	98.42	13.34
	128.73	77.68	103.69	116.85	102.44	97.78	13.25
Jan75	130.67	76.72	101.36	117.78	95.88	94.13	12.75
	131.27	76.38	101.05	114.44	89.15	95.85	12.99
	130.75	77.19	110.94	113.22	89.50	93.55	12.68
	136.89	73.88	116.67	112.40	95.09	92.87	12.58
	144.26	69.51	120.66	111.59	100.97	92.98	12.60
	150.08	64.69	115.61	113.28	112.52	91.55	12.40
	135.51	60.63	111.53	113.51	104.52	91.66	12.42
	132.17	59.64	114.11	117.56	114.43	91.02	12.33
	130.19	59.41	115.09	126.68	120.16	88.85	12.04
	132.57	55.11	119.67	131.80	121.37	89.00	12.06
	137.25	56.80	131.20	133.14	114.34	89.13	12.08
	140.32	58.85	130.70	127.26	107.21	88.52	11.99

Table D-10. (Cont.)

Jan76	141.42	66.10	134.00	124.84	106.06	86.97	11.78
	147.57	68.64	137.93	118.29	105.14	86.60	11.73
	152.91	67.78	142.62	118.34	105.99	86.53	11.73
	162.34	64.30	138.78	119.74	106.99	86.25	11.69
	165.61	64.68	135.20	120.62	106.04	86.03	11.66
	163.92	64.75	134.37	119.62	106.41	85.45	11.58
	157.04	64.39	134.39	116.73	104.07	85.02	11.52
	151.87	65.98	128.07	120.53	107.85	84.74	11.48
	146.20	69.42	124.34	121.52	110.14	85.92	11.64
	147.83	70.47	122.83	120.72	108.23	87.17	11.81
	146.01	69.93	118.60	119.33	104.17	87.76	11.89
	141.67	71.90	121.58	121.00	109.96	87.48	11.85
Jan77	139.23	78.95	121.16	124.03	112.89	88.29	11.96
	137.32	77.16	123.63	125.88	115.42	89.87	12.18
	136.65	71.36	125.07	123.46	111.32	89.24	12.09
	142.56	69.36	125.57	123.05	112.51	88.83	12.04
	140.90	68.31	121.81	123.75	113.04	88.91	12.05
	134.20	68.71	120.00	123.42	112.43	88.30	11.96
	133.47	71.30	119.60	123.64	113.15	88.37	11.98
	132.21	71.70	119.10	124.61	110.34	87.84	11.90
	130.46	71.26	112.81	121.70	108.57	87.16	11.81
	130.00	72.25	108.70	119.35	109.03	86.77	11.76
	129.30	72.02	109.33	119.16	109.59	86.72	11.75
	127.34	73.72	112.35	120.55	111.10	86.54	11.73
Jan78	125.32	71.45	110.12	117.97	107.43	85.17	11.54
	128.14	69.10	112.25	117.37	108.41	84.67	11.47
	134.05	68.02	115.80	118.76	110.41	84.23	11.41
	132.68	67.53	116.78	119.78	113.45	83.88	11.37
	134.78	65.98	112.15	117.56	109.21	83.59	11.33
	136.55	64.89	115.88	116.74	107.21	83.18	11.27
	141.09	64.45	124.30	120.99	107.83	82.63	11.20
	159.18	63.18	136.33	128.42	107.39	82.29	11.15
	163.96	62.35	136.46	130.45	110.46	82.02	11.11
	173.79	64.31	146.61	139.35	115.84	82.90	11.23
	189.99	72.18	163.49	158.56	133.82	91.64	12.42
	179.79	68.72	154.04	159.74	129.77	93.67	12.69
Jan79	184.95	70.88	167.93	174.41	163.91	101.10	13.70
	270.08	86.72	238.19	265.13	242.23	139.47	18.90
	245.44	88.75	220.15	259.18	208.99	137.40	18.62
	254.61	90.78	215.00	237.31	219.60	129.58	17.56
	302.56	105.56	241.69	266.59	262.57	175.28	23.75
	323.95	115.85	270.10	309.45	295.32	212.96	28.86
	307.97	113.89	277.70	309.92	288.79	197.98	26.83
	287.19	111.39	271.08	310.85	254.89	200.64	27.19
	276.10	111.60	261.77	306.81	260.31	206.39	27.97
	282.18	124.00	267.31	312.99	262.14	222.62	30.16
	309.10	137.93	290.25	324.63	285.01	238.63	32.33
	326.15	140.05	308.46	327.29	279.00	237.09	32.13
Jan80	310.16	127.12	296.91	303.87	266.50	217.89	29.52
	297.00	111.48	265.98	270.65	236.17	204.23	27.67
	289.93	109.85	260.98	263.96	226.70	202.42	27.43
	282.33	117.54	257.09	259.62	241.64	199.46	27.03
	282.97	121.35	244.81	255.17	241.47	201.10	27.25
	278.74	114.73	231.36	251.26	233.64	198.48	26.89

Table D-10. (Cont.)

Jul80	268.19	111.28	221.54	250.62	226.69	189.43	25.67
	240.63	112.42	196.54	241.71	200.71	173.65	23.53
	246.94	119.18	207.94	245.38	208.31	177.75	24.08
	259.40	147.08	229.49	256.22	218.59	203.77	27.61
	279.35	167.55	264.07	265.17	231.96	219.69	29.77
	270.94	157.56	244.99	258.95	214.79	214.78	29.10
Jan81	268.74	153.58	251.92	246.58	217.73	202.88	27.49
	260.75	152.14	241.94	239.80	213.77	192.92	26.14
	253.73	147.81	240.23	238.04	215.12	190.47	25.81
	248.89	140.86	230.54	231.42	200.10	185.87	25.19
	241.64	125.31	206.98	213.22	186.92	171.93	23.30
	248.27	114.55	215.29	206.07	185.58	161.67	21.91
	265.67	114.09	227.93	225.77	194.54	160.07	21.69
	258.98	108.51	223.02	225.19	198.11	160.73	21.78
	256.29	109.74	217.89	230.13	199.26	158.26	21.44
	253.88	113.90	213.40	228.42	206.58	163.29	22.13
	247.86	116.30	217.35	227.35	212.55	167.51	22.70
	229.56	109.95	208.23	229.52	210.99	165.63	22.44
Jan82	223.73	108.47	200.13	231.26	207.51	165.49	22.42
	207.56	108.72	186.26	226.35	183.75	146.74	19.88
	194.09	107.03	171.97	195.76	170.75	139.10	18.85
	210.37	108.45	185.72	190.65	180.31	152.37	20.65
	235.02	108.83	209.05	206.26	190.85	159.98	21.68
	237.17	107.50	208.44	203.50	183.12	155.94	21.13
	229.43	104.48	190.57	198.33	175.46	151.85	20.58
	221.49	98.65	193.67	204.19	182.02	148.03	20.06
	228.69	102.50	197.31	211.27	192.16	156.97	21.27
	227.57	108.78	190.15	214.38	197.96	155.54	21.08
	208.22	100.84	177.75	205.62	184.33	148.42	20.11
	198.26	101.57	177.96	193.58	181.15	142.06	19.25
Jan83	197.16	103.67	184.17	193.42	172.36	143.35	19.42
	178.35	96.70	169.28	175.99	151.65	136.22	18.46
	175.16	97.23	157.88	162.87	146.70	130.61	17.70
	188.25	100.64	173.72	164.87	153.20	132.96	18.02
	185.53	100.01	169.67	166.07	148.75	130.68	17.71
	189.50	97.32	176.16	166.31	150.91	131.73	17.85
	188.64	99.45	176.18	168.82	149.79	131.97	17.88
	188.07	103.12	180.30	171.92	155.40	130.85	17.73
	180.85	101.83	173.17	174.39	153.05	128.74	17.45
	177.97	101.49	167.12	168.34	150.71	128.09	17.36
	177.71	101.76	161.96	170.20	150.82	126.41	17.13
	168.00	102.63	153.94	166.01	147.90	125.90	17.06

Table D-11. 1972-1983 Rotterdam spot prices, monthly figures in real terms 1980 US$.

Year	PMS	HFO	NAPHTHA	JETKER	DFO	Arabian Light	
	–	US$ per metric tonne			–	US$/mt	US$/bl
Jan72	59.38	23.55	42.31	57.98	48.43	16.12	2.18
	59.01	25.44	40.04	55.66	43.56	16.07	2.18
	58.55	25.59	38.10	53.66	42.90	16.01	2.17
	58.01	25.59	37.17	52.68	45.48	15.96	2.16
	63.84	25.07	37.04	52.50	42.93	15.90	2.15
	67.07	24.02	32.45	47.85	41.96	15.85	2.15
	72.85	25.17	32.12	47.47	43.01	15.79	2.14
	77.40	26.10	35.05	50.34	45.62	17.04	2.31
	76.39	25.41	40.44	55.73	47.77	16.98	2.30
	76.88	24.61	45.43	61.51	50.83	16.92	2.29
	77.67	25.36	48.76	63.90	57.03	16.86	2.29
	83.88	26.80	54.73	69.82	62.99	16.81	2.28
Jan73	91.96	28.89	59.53	74.45	64.97	16.62	2.25
	94.97	31.85	66.26	81.12	72.25	16.54	2.24
	106.78	30.29	74.05	88.83	72.35	16.47	2.23
	125.95	27.91	73.08	94.87	75.08	21.45	2.91
	163.37	28.79	85.95	97.06	93.10	21.36	2.89
	189.64	32.02	102.01	109.06	112.75	21.26	2.88
	172.80	31.06	121.65	125.58	122.14	21.16	2.87
	140.82	28.09	114.16	132.19	119.85	21.07	2.85
	143.28	26.01	112.65	137.97	132.80	28.36	3.84
	169.75	36.01	139.22	205.81	201.68	28.24	3.83
	248.81	79.48	193.80	312.16	308.98	28.11	3.81
	308.03	197.25	239.70	366.83	306.61	60.86	8.25
Jan74	243.63	166.77	240.24	259.71	213.12	59.61	8.08
	246.02	122.33	228.29	174.21	166.03	127.86	17.33
	295.51	107.59	255.68	173.28	159.91	126.98	17.21
	286.47	103.01	257.91	163.51	132.31	136.63	18.51
	273.41	105.48	234.74	159.99	142.28	135.70	18.39
	231.45	100.25	198.39	153.43	138.20	134.78	18.26
	193.13	95.95	164.29	153.80	144.02	133.88	18.14
	177.49	93.99	149.92	150.83	142.11	132.98	18.02
	173.77	96.53	149.92	147.32	142.53	132.10	17.90
	166.34	99.13	148.54	148.81	140.26	132.34	17.93
	167.65	104.63	144.66	153.59	135.64	132.59	17.97
	173.43	104.65	139.69	157.42	138.01	131.73	17.85
Jan75	176.06	103.38	136.58	158.70	129.19	126.83	17.19
	176.88	102.91	136.16	154.21	120.12	129.15	17.50
	176.18	104.01	149.49	152.55	120.60	126.06	17.08
	184.45	99.55	157.20	151.45	128.13	125.14	16.96
	194.39	93.67	162.59	150.36	136.06	125.29	16.98
	202.23	87.17	155.79	152.64	151.61	123.36	16.71
	182.60	81.70	150.28	152.95	140.84	123.52	16.74
	178.10	80.37	153.76	158.42	154.20	122.65	16.62
	175.44	80.06	155.09	170.71	161.92	119.73	16.22
	178.65	74.26	161.26	177.61	163.56	119.93	16.25
	184.95	76.54	176.80	179.42	154.08	120.11	16.28
	189.10	79.30	176.14	171.49	144.48	119.29	16.16

Table D-11. (Cont.)

Jan76	190.57	89.08	180.58	168.24	142.93	117.21	15.88
	198.88	92.50	185.88	159.42	141.69	116.71	15.81
	206.07	91.34	192.20	159.48	142.83	116.61	15.80
	218.78	86.66	187.04	161.36	144.18	116.23	15.75
	223.19	87.17	182.20	162.55	142.91	115.94	15.71
	220.92	87.26	181.09	161.21	143.41	115.16	15.60
	211.64	86.78	181.12	157.32	140.26	114.58	15.53
	204.68	88.92	172.60	162.44	145.35	114.20	15.47
	197.04	93.56	167.58	163.77	148.44	115.80	15.69
	199.24	94.98	165.55	162.70	145.86	117.48	15.92
	196.78	94.26	159.85	160.83	140.40	118.28	16.03
	190.94	96.91	163.86	163.08	148.20	117.90	15.98
Jan77	187.65	106.40	163.29	167.16	152.14	118.99	16.12
	185.08	103.99	166.62	169.66	155.56	121.12	16.41
	184.17	96.18	168.57	166.39	150.03	120.27	16.30
	192.13	93.48	169.23	165.84	151.63	119.72	16.22
	189.90	92.06	164.17	166.78	152.35	119.83	16.24
	180.88	92.60	161.73	166.34	151.53	119.00	16.12
	179.89	96.10	161.20	166.64	152.50	119.11	16.14
	178.19	96.64	160.52	167.95	148.71	118.39	16.04
	175.83	96.04	152.04	164.03	146.33	117.48	15.92
	175.22	97.38	146.50	160.85	146.94	116.95	15.85
	174.27	97.07	147.35	160.61	147.70	116.88	15.84
	171.62	99.36	151.42	162.47	149.74	116.64	15.80
Jan78	168.89	96.29	148.40	158.98	144.78	114.78	15.55
	172.68	93.12	151.28	158.18	146.10	114.10	15.46
	180.66	91.66	156.06	160.04	148.80	113.52	15.38
	178.81	91.01	157.37	161.41	152.89	113.04	15.32
	181.63	88.92	151.14	158.42	147.17	112.64	15.26
	184.01	87.44	156.16	157.32	144.47	112.09	15.19
	190.13	86.84	167.50	163.04	145.31	111.35	15.09
	214.50	85.13	183.72	173.05	144.71	110.89	15.03
	220.94	84.02	183.88	175.79	148.85	110.52	14.98
	234.18	86.66	197.56	187.78	156.09	111.71	15.14
	256.01	97.26	220.30	213.66	180.32	123.49	16.73
	242.26	92.60	207.56	215.25	174.87	126.22	17.10
Jan79	249.30	95.54	226.36	235.09	220.93	136.27	18.47
	364.06	116.89	321.08	357.39	326.52	188.00	25.47
	330.86	119.64	296.77	349.38	281.72	185.23	25.10
	343.24	122.38	289.84	319.91	296.04	174.68	23.67
	407.90	142.32	325.83	359.40	353.98	236.30	32.02
	436.74	156.19	364.15	417.20	398.16	287.11	38.90
	415.22	153.55	374.41	417.85	389.37	266.92	36.17
	387.22	150.18	365.50	419.13	343.67	270.52	36.66
	372.28	150.48	352.95	413.69	350.99	278.29	37.71
	380.50	167.20	360.45	422.04	353.47	300.18	40.67
	416.81	186.00	391.39	437.75	384.32	321.78	43.60
	439.82	188.86	415.96	441.36	376.24	319.71	43.32
Jan80	418.13	171.38	400.27	409.66	359.27	293.74	39.80
	400.38	150.28	358.57	364.86	318.38	275.32	37.31
	390.84	148.09	351.82	355.83	305.61	272.88	36.98
	380.59	158.46	346.58	349.99	325.74	268.89	36.44
	381.45	163.58	330.01	343.98	325.51	271.09	36.73
	375.75	154.65	311.88	338.70	314.94	267.55	36.25

Table D-11. (Cont.)

	361.52	150.00	298.63	337.84	305.58	255.35	34.60
	324.36	151.54	264.94	325.82	270.56	234.08	31.72
	332.87	160.65	280.30	330.77	280.79	239.60	32.47
	349.66	198.26	309.34	345.37	294.65	274.67	37.22
	376.55	225.85	355.95	357.43	312.67	296.13	40.13
	365.21	212.38	330.22	349.04	289.52	289.50	39.23
Jan81	362.17	206.97	339.50	332.30	293.42	273.42	37.05
	351.39	205.02	326.04	323.15	288.08	259.98	35.23
	341.91	199.19	323.72	320.77	289.88	256.67	34.78
	335.39	189.81	310.66	311.84	269.64	250.46	33.94
	325.60	168.84	278.89	287.30	251.86	231.66	31.39
	334.52	154.34	290.08	277.66	250.04	217.83	29.52
	357.95	153.72	307.10	304.20	262.11	215.67	29.22
	348.92	146.19	300.47	303.39	266.92	216.55	29.34
	345.29	147.85	293.55	310.04	268.45	213.22	28.89
	342.02	153.45	287.50	307.72	278.30	219.98	29.81
	333.90	156.67	292.80	306.28	286.34	225.67	30.58
	309.24	148.12	280.51	309.20	284.22	223.13	30.23
Jan82	301.45	146.15	269.64	311.60	279.59	222.97	30.21
	279.66	146.49	250.97	304.98	247.58	197.71	26.79
	261.51	144.21	231.71	263.77	230.06	187.42	25.40
	283.45	146.12	250.24	256.89	242.95	205.30	27.82
	316.67	146.63	281.67	277.92	257.15	215.55	29.21
	319.57	144.84	280.86	274.20	246.74	210.12	28.47
	309.14	140.78	256.78	267.23	236.41	204.60	27.72
	298.44	132.92	260.96	275.13	245.25	199.46	27.03
	308.15	138.11	265.86	284.67	258.92	211.50	28.66
	306.63	146.58	256.22	288.86	266.73	209.58	28.40
	280.57	135.87	239.50	277.06	248.37	199.98	27.10
	267.15	136.86	239.79	260.84	244.09	191.41	25.94
Jan83	265.65	139.68	248.15	260.62	232.23	193.14	26.17
	240.30	130.29	228.09	237.13	204.33	183.54	24.87
	236.02	131.01	212.73	219.45	197.66	175.98	23.85
	253.65	135.60	234.07	222.14	206.42	179.14	24.27
	249.98	134.75	228.61	223.76	200.42	176.08	23.86
	255.33	131.13	237.36	224.08	203.33	177.50	24.05
	254.18	134.00	237.39	227.46	201.82	177.82	24.09
	253.40	138.94	242.93	231.64	209.38	176.30	23.89
	243.68	137.20	233.33	234.97	206.22	173.47	23.51
	239.80	136.75	225.18	226.82	203.07	172.59	23.39
	239.44	137.11	218.23	229.33	203.21	170.33	23.08
	226.37	138.28	207.42	223.68	199.28	169.64	22.99

Source: Figures on tables D-10 and D-11 were worked out from tables D-7 and D-9. The deflators in table D-9 were considered as midpoints (July) deflators of each year and for every month the correponding deflator was estimated on the basis of weighted averages between two succesive year (midpoint) deflators. This was done so because the quarterly deflators (also reported in OECD, *Main Economic Indicators*, op.cit.) did not show any cycle or seasonal effect, variations were completely random.

Table D-12. 1972-1983 Rotterdam spot prices, annual averages in real terms 1976 and 1980 US$.

Year	PMS	HFO	NAPHTHA	JETKER	DFO	Arabian Light	
	-	US$ per metric tonne			-	US$/mt	US$/bl
1976 = 100							
1972	51.38	18.69	29.88	41.34	35.38	12.12	1.64
1973	121.30	35.97	85.82	113.57	104.67	18.70	2.53
1974	161.35	79.95	141.83	122.70	110.37	93.71	12.69
1975	135.54	65.32	115.48	119.11	105.27	91.21	12.36
1976	151.68	67.24	130.69	119.85	106.54	86.15	11.67
1977	134.10	71.99	117.91	122.41	111.33	87.85	11.90
1978	149.91	66.64	128.64	128.69	113.23	84.78	11.49
1979	280.53	108.19	252.37	283.62	251.67	183.60	24.87
1980	274.26	126.30	242.27	259.10	227.85	199.50	27.03
1981	251.82	124.74	223.49	227.55	202.68	172.52	23.38
1982	217.95	105.20	190.26	206.19	185.32	151.48	20.52
1983	182.58	100.33	169.93	170.44	152.31	131.19	17.78
1980 = 100							
1972	69.23	25.19	40.27	55.70	47.68	16.33	2.21
1973	163.44	48.47	115.63	153.02	141.03	25.19	3.41
1974	217.38	107.72	191.09	165.32	148.70	126.25	17.10
1975	182.64	88.01	155.62	160.50	141.86	122.91	16.65
1976	204.42	90.62	176.13	161.52	143.58	116.11	15.73
1977	180.74	97.03	158.92	164.98	150.05	118.41	16.04
1978	202.02	89.80	173.35	173.42	152.59	114.25	15.48
1979	378.23	145.87	340.26	382.39	339.31	247.54	33.54
1980	369.70	170.25	326.58	349.27	307.14	268.92	36.44
1981	339.30	168.07	301.12	306.59	273.08	232.45	31.50
1982	293.66	141.74	256.35	277.83	249.71	204.11	27.66
1983	246.00	135.18	228.97	229.65	205.22	176.77	23.95

Source: Figures worked out from tables D-8 and D-9.

Table D-13. 1972-1983 average Rotterdam spot price exchange ratios.

	P/H	P/N	P/K	P/D	P/AL	N/K	N/D	D/H
1972	2.743	1.746	1.250	1.459	4.224	0.719	0.843	1.89
1973	4.274	1.468	1.192	1.303	6.669	0.803	0.880	3.40
1974	2.045	1.144	1.329	1.477	1.841	1.163	1.293	1.39
1975	2.101	1.180	1.144	1.303	1.486	0.970	1.105	1.65
1976	2.264	1.162	1.267	1.425	1.762	1.092	1.229	1.28
1977	1.866	1.138	1.096	1.205	1.527	0.963	1.059	1.22
1978	2.248	1.164	1.162	1.318	1.762	0.997	1.132	1.27
1979	2.630	1.116	0.996	1.118	1.586	0.892	1.004	1.68
1980	2.223	1.137	1.059	1.204	1.378	0.933	1.062	1.61
1981	2.045	1.128	1.108	1.246	1.467	0.983	1.105	1.39
1982	2.073	1.145	1.059	1.177	1.438	0.924	1.028	1.44
1983	1.821	1.075	1.072	1.199	1.392	0.998	1.116	1.31

Source: ratios are calculated from prices in Table D-7, where,

P - premium motor spirit
H - heavy fuel oil
D - gas oil
K - kerosine
N - naphtha

APPENDIX E. SINGLE REFINERY TECHNOLOGY MATRIX

SR Technology Matrix

		A1AL1J	AB3DAL	AB3EAL	AB3FAL	AB3GAL	AB3VAL	AB3WAL	AB3DHM	AB3EHM	AB3FHM	AB3GHM	AB3VHM	AB3WHM	AB3DNL	AB3ENL	AB3FNL
Cost Function	LR0000	-221.500															
Crude Balances at Refinery	1B1AL0	-1.0050	-1.0050	-1.0050	-1.0050	-1.0050	-1.0050										
	1B1HM0								-1.0050	-1.0050	-1.0050	-1.0050	-1.0050	-1.0050			
	1B1NL0														-1.0050	-1.0050	-1.0050
	1B1TJ0																
	1B1KT0																
	1B1FT0																
Refinery Processing Units Capacities	1B3BCC																
	1B3BCD		1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1B3BCH																
	1B3BCP																
	1B3BCV				.4830	.4830	.4830	.4830			.2910	.2910	.2910	.2910			.3640
Refinery Fuel	1B3RFL		-.0500	-.0500	-.0650	-.0650	-.0650	-.0650	-.0500	-.0500	-.0650	-.0650	-.0650	-.0650	-.0500	-.0500	-.0650
Process Restrictions and Ratios	1B3BH1		-.2020	-.2020			-.2490	-.2490									
	1B3BVH					-.2110		-.2110									
	1B3BVL										-.0886		-.0886				-.0970
Intermediate Product Balances	1B4AG0		.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010
	1B4GA0		.0140	.0140	.0140	.0140	.0140	.0140	.0350	.0350	.0350	.0350	.0350	.0350	.0154	.0154	.0154
	1B4SG0		.0260	.0260	.0260	.0260	.0260	.0260	.0610	.0610	.0610	.0610	.0610	.0610	.0430	.0430	.0430
	1B4SB0		.1320	.1320	.1320	.1320	.1320	.1320	.1820	.1820	.1820	.1820	.1820	.1820	.1480	.1480	.1480
	1B4NA0		.0590	.0590	.0590	.0590	.0590	.0590	.0898	.0898	.0898	.0898	.0898	.0898	.0650	.0650	.0650
	1B4KE0		.0830	.0830	.0830	.0830	.0830	.0830	.1169	.1169	.1169	.1169	.1169	.1169	.0990	.0990	.0990
	1B4DF0		.2020	.2020			.2490	.2490	.2231	.2231			.2687	.2687	.2645	.2645	
	1B4DH0				.2490	.2490											
	1B4DL0										.2687	.2687					.3135
	1B4LF0		.4830		.2110		.2110		.2910		.0886		.0886		.3640		.0970
	1B4HF0			.4830		.2110		.2110		.2910		.0886		.0886		.3640	
	1B4TD0										.1568	.1568	.1568	.1568			.2180
	1B4WD0				.2250	.2250	.2250	.2250									
	1B4UG0																
	1B4XS0																
	1B4YS0																
	1B4ZS0																
	1B4PM0																
	1B4RM0																
Product Quality Specifications	1B4DFS		.2222	.2222			.3162	.3162	.0268	.0268			.0403	.0403	.0317	.0317	
	1B4DFP		-.0005	-.0005			1.2206	1.2206	-.2856	-.2856			2.9173	2.9173	.7207	.7207	
	1B4DFF		-.0143	-.0143			-.0176	-.0176	-.0158	-.0158			-.0190	-.0190	-.0187	-.0187	
	1B4LFS		1.3862		.7807		.7807		.1048		.0390		.0390		.0946		.0378
	1B4LFV		13.3996		7.9746		7.9746		6.9928		3.0304		3.0304		8.7746		3.6087
	1B4LFP		20.1962		2.1026		2.1026		14.9912		3.5409		3.5409		58.8416		7.7566
	1B4HFS			1.3862		.7807		.7807		.1048		.0390		.0390		.0946	
	1B4HFV			13.3996		7.9746		7.9746		6.9928		3.0304		3.0304		8.7746	
	1B4HFP			20.1962		2.1026	2.1026		14.9912			3.5409		3.5409		58.8416	
	1B4RME																
	1B4RMD																
	1B4RML																
	1B4RMR																
	1B4RMB																
	1B4PME																
	1B4PMD																
	1B4PML																
	1B4PMR																
	1B4PMB																
Finished Product Balances	1B5AG0																
	1B5FS0																
	1B5RM0																
	1B5PM0																
	1B5KE0																
	1B5DF0																
	1B5LF0																
	1B5HF0																
	1B5BT0																

Crude & Vacuum Distillation

		AB3GNL	AB3VNL	AB3WNL	AB3DTJ	AB3ETJ	AB3FTJ	AB3GTJ	AB3VTJ	AB3WTJ	AB3DKT	AB3EKT	AB3FKT	AB3GKT	
Cost Function	LR0000														
Crude Balances at Refinery	1B1AL0														
	1B1HM0														
	1B1NL0	-1.0050	-1.0050	-1.0050											
	1B1TJ0				-1.0050	-1.0050	-1.0050	-1.0050	-1.0050	-1.0050					
	1B1KT0										-1.0050	-1.0050	-1.0050	-1.0050	-1.
	1B1FT0														
Refinery Processing Units Capacities	1B3BCC														
	1B3BCD	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0
	1B3BCH														
	1B3BCP														
	1B3BCV	.3640	.3640	.3640			.6205	.6205	.6205	.6205			.5545	.5545	.
Refinery Fuel	1B3RFL	-.0650	-.0650	-.0650	-.0500	-.0500	-.0650	-.0650	-.0650	-.0650	-.0500	-.0500	-.0650	-.0650	-.
Process Restrictions and Ratios	1B3BH1										-.1700	-.1700			-.2
	1B3BVH							-.3385		-.3385				-.3032	
	1B3BVL		-.0970												
Intermediate Product Balances	1B4AG0	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0
	1B4GA0	.0154	.0154	.0154	.0092	.0092	.0092	.0092	.0092	.0092	.0167	.0167	.0167	.0167	.0
	1B4SG0	.0430	.0430	.0430	.0330	.0330	.0330	.0330	.0330	.0330	.0420	.0420	.0420	.0420	.0
	1B4SB0	.1480	.1480	.1480	.0690	.0690	.0690	.0690	.0690	.0690	.0940	.0940	.0940	.0940	.0
	1B4NA0	.0650	.0650	.0650	.0410	.0410	.0410	.0410	.0410	.0410	.0547	.0547	.0547	.0547	.0
	1B4KE0	.0990	.0990	.0990	.0570	.0570	.0570	.0570	.0570	.0570	.0676	.0676	.0676	.0676	.0
	1B4DF0		.3135	.3135	.1690	.1690			.2130	.2130	.1699	.1699			.2
	1B4DH0												.2113	.2113	
	1B4DL0	.3135					.2130	.2130							
	1B4LF0		.0970		.6205		.3385		.3385		.5545		.3032		.3
	1B4HF0	.0970		.0970		.6205		.3385		.3385		.5545		.3032	
	1B4TD0	.2180	.2180	.2180											
	1B4WD0						.2380	.2380	.2380	.2380			.2099	.2099	.2
	1B4UG0														
	1B4XS0														
	1B4YS0														
	1B4ZS0														
	1B4PM0														
	1B4RM0														
Product Quality Specifications	1B4DFS		.0408	.0408	.1149	.1149			.1704	.1704	.2158	.2158			.3
	1B4DFP		2.6155	2.6155	-.2164	-.2164			.4401	.4401	.2445	.2445			1.3
	1B4DFF		-.0221	-.0221	-.0119	-.0119			-.0150	-.0150	-.0120	-.0120			-.0
	1B4LFS		.0378		1.4147		.9715		.9715		2.2291		1.5342		1.5
	1B4LFV		3.6087		19.9390		13.9873		13.9873		16.8777		11.7302		11.7
	1B4LFP		7.7566		20.4653		11.4972		11.4972		28.5656		3.0214		3.0
	1B4HFS	.0378		.0378		1.4147		.9715		.9715		2.2291		1.5342	
	1B4HFV	3.6087		3.6087		19.9390		13.9873		13.9783		16.8777		11.7302	
	1B4HFP	7.7566		7.7566		20.4653		11.4972		11.4972		28.5656		3.0214	
	1B4RME														
	1B4RMD														
	1B4RML														
	1B4RMR														
	1B4RMB														
	1B4PME														
	1B4PMD														
	1B4PML														
	1B4PMR														
	1B4PMB														
Finished Product Balances	1B5AG0														
	1B5FS0														
	1B5RM0														
	1B5PM0														
	1B5KE0														
	1B5DF0														
	1B5LF0														
	1B5HF0														
	1B5BT0														

											Catalytic Cracking								Hydrofining		Catalytic Reforming								Lead Reduction					
	AB3FKT	AB3GKT	AB3VKT	AB3WKT	AB3DFT	AB3EFT	AB3FFT	AB3GFT	AB3VFT	AB3WFT	AB3CH1	AB3CH2	AB3CH3	AB3CH4	AB3CH5	AB3CH6	AB3CH7	AB3CH8	AB3HO1	AB3HO2	AB3PP1	AB3PP3	AB3PP6	AB3PP7	AB3PR1	AB3PR3	AB3PR5	AB3PR6	AB3TP1	AB3TR1	AB43GR	AB43SR	AB43BR	AB43LR
											.3900	.3800	.4500	.4300	.5300	.5100	.4300	.4100	.1500	.1500	.5000	.8400	.5000	-.8400	.4200	.5000	.4100	.5000	-4.3840	-4.4050				
050	-1.0050	-1.0050	-1.0050	-1.0050																														
					-1.0050	-1.0050	-1.0050	-1.0050	-1.0050	-1.0050																								
											1.0000	1.0000	1.0000	1.0000	1.5000	1.5000	1.0000	1.0000																
000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000																								
											.3200	.1800	.5000	.3400	.2900	.1300	.2500	.2350	1.0000	1.0000														
																												1.0000						
	.5545	.5545	.5545	.5545			.4465	.4465	.4465	.4465																								
500	-.0650	-.0650	-.0650	-.0650	-.0500	-.0500	-.0650	-.0650	-.0650	-.0650	-.0700	-.0600	-.0900	-.0700	-.0900	-.0700	-.0700	-.0500	-.0200	-.0200	-.1100	-.1100	-.1100	-.1100	-.1100	-.1100	-.1100	-.1100			1.2000	1.1000	1.1000	1.00
700			-.2110	-.2110															1.0000															
		-.3032			-.3032																													
								-.1820		-.1820																								
010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0010	.0200	.0200	.0300	.0300	.0450	.0450	.0470	.0470			.0200	.0300	.0200	.0300	.0100	.0200	.0100	.0200						
167	.0167	.0167	.0167	.0167	.0254	.0254	.0254	.0254	.0254	.0254	.0750	.0750	.0350	.0200	.0600	.0650	.0750	.0750	.0050	.0050	.2200	.2600	.1900	.2100	.1500	.2200	.0900	.1900			-1.0000			
420	.0420	.0420	.0420	.0420	.0440	.0440	.0440	.0440	.0440	.0440																						-1.0000		
940	.0940	.0940	.0940	.0940	.1320	.1320	.1320	.1320	.1320	.1320											-1.0000	-1.0000	-.5000	-.5000	-1.0000	-1.0000	-.5000	-.5000					-1.0000	
547	.0547	.0547	.0547	.0547	.0600	.0600	.0600	.0600	.0600	.0600													-.5000	-.5000	-.5000	-.5000	-.5000	-.5000						
676	.0676	.0676	.0676	.0676	.0770	.0770	.0770	.0770	.0770	.0770																								
699			.2113	.2113	.2140	.2140			.2590	.2590	.2100	.1750	.3350	.3400	.1450	.1330	.2500	.2350	-.0150	-.0150														
	.2113	.2113															-1.0000																	
							.2590	.2590										-1.0000																
	.3032		.3032		.4465		.1820		.1820			.2000		.1300		.0620		.1500																-1.00
545		.3032		.3032		.4465		.1820		.1820	.1900		.1400		.1100		.1450																	
							.2195		.2195	.2195		-1.0000		-1.0000		-1.0000																		
	.2099	.2099	.2099	.2099							-1.0000		-1.0000		-1.0000																			
											.0470	.0470	.0700	.0700	.1050	.1050	.0900	.0900																
											.1700	.1800	.1350	.1500	.1950	.2200	.1450	.1500																
											.2250	.2500	.1850	.2050	.2650	.3100	.2000	.2100																
																					.7600	.7100	.7900	.7600										
																									.8400	.7600	.9000	.7900						
2158			.3148	.3148	.0278	.0278			.0440	.0440	.0840	.0088	.1340	.0170	.0580	.0067	.0125	.0024	-.8000	-.4000														
2445			1.3812	1.3812	-.0006	-.0006			1.3712	1.3718	.7938	.6615	1.2663	1.2852	.5481	.5027	.9450	.8883																
0120			-.0150	-.0150	-.0151	-.0151			-.0183	-.0183	-.0065	-.0065	-.0104	-.0125	-.0045	-.0050	-.0078	-.0087																
	1.5342		1.5342		.2545		.1420		.1420			.1800		.1170		.0558		.0450																-1.0
	11.7302		11.7302		11.6854		6.3635		6.3635			3.5020		2.2763		1.0856		2.6265																-27.1
	3.0214		3.0214		35.2244		14.5536		14.5536			61.3599		39.8840		19.0216		18.6300																-57.0
2291		1.5342		1.5342		.2545		.1420		.1420	.4940		.3640		.2860		.2175																	
8777		11.7302		11.7302		11.6854		6.3635		6.3635	3.3269		2.4514		1.9261		2.5390																	
5656		3.0214		3.0214		35.2244		14.5536		14.5536	41.6480		30.6880		24.1120		18.0090																	
																									-22.105	-19.487	-23.0769	-19.7500						
																									1.1053	-.974	-23.0769	-16.7875						
																									24.3158	16.5641	28.8461	20.7375						
																									3.4153	8.1944	3.5654	8.3049		-19.0000				
																														-1.0000				
																					-19.487	-17.975	-19.750	-18.8820										
																					-.9744	-1.797	-16.787	-15.1056										
																					16.5641	14.380	20.7375	16.9938										
																					3.2641	6.309	3.3081	6.6276					-15.0000					
																													-1.0000					

A B 3 P R 5	A B 3 P R 6	A B 3 T P 1	A B 3 T R 1	A B 4 3 G R	A B 4 3 S R	A B 4 3 B R	A B 4 3 L R	A B 4 3 H R	A B 4 4 A G	A B 4 4 G R	A B 4 4 G P	A B 4 4 S R	A B 4 4 S P	A B 4 4 N D	A B 4 4 K D	A B 4 4 K L	A B 4 4 K H	A B 4 4 D L	A B 4 4 D H	A B 4 4 L H	A B 4 4 H L	A B 4 4 T L	A B 4 4 T H	A B 4 4 W L	A B 4 4 W H	A B 4 4 U G	A B 4 4 X R	A B 4 4 X P	A B 4 4 Y R	A B 4 4 Y P	A B 4 4 Z R	A B 4 4 Z P	A B 4 5 G 1
		Lead Reduction									Blending: Intermediate Products to Finished Products																						
.4100	.5000	-4.3840	-4.4050																														
	1.0000																																
-.1100	-.1100			1.2000	1.1000	1.1000	1.0000	1.0000																									
																		1.0000	1.0000														
.0100	.0200								-1.0000																								
.0900	.1900			-1.0000					1.0000	-1.0000	-1.0000															1.0000							-1.0000
					-1.0000							-1.0000	-1.0000																				
-.5000	-.5000					-1.0000																											
-.5000	-.5000													-1.0000																			
															-1.0000	-1.0000	-1.0000																
														1.0000	1.0000			-1.0000	-1.0000														
							-1.0000									1.0000		1.0000		-1.0000	1.0000	1.0000		1.0000									
								-1.0000									1.0000		1.0000	1.0000	-1.0000		1.0000		1.0000								
																						-1.0000	-1.0000										
																								-1.0000	-1.0000								
																										-1.0000							
																											-1.0000	-1.0000					
																													-1.0000	-1.0000			
																															-1.0000	-1.0000	
											1.0000		1.0000															1.0000		1.0000		1.0000	
.9000	.7900									1.0000		1.0000															1.0000		1.0000		1.0000		
														.1000	.2000			-.8000	-.8000														
														-9.9200	-8.6200			-3.0800	-3.0800														
														.1630	-.0167			.0704	.0704														
							-1.0000									.2000		.8000		-1.0000	3.0000	.3000		2.5000									
							-27.1099											5.6400		-27.1100	29.5000	20.4300		21.4300									
							-57.0000									-103.7000		-69.6000		-57.0000	70.3000	306.800		200.7000									
								-3.0000									.2000		.8000	1.0000	-3.0000		.3000		2.5000								
								-29.5000											5.6400	27.1100	-29.500		20.4300		21.4300								
								-70.3000									-103.7000		-69.6000	47.0000	-70.300		306.8000		200.7000								
-23.0769	-19.7500									-34.4828		-30.3030															46.3768		18.9873		5.1948		
-23.0769	-16.7875									94.8276		83.3333															79.7101		-56.9620		19.4805		
28.8461	20.7375									-277.586		-30.3030															-30.4348		37.9747		-64.9350		
3.5654	8.3049		-19.0000							17.1034		-34.3030															10.4638		-6.3418		13.8701		
			-1.0000																														
											-34.4826		-30.3030															46.3768		18.9873		5.1948	
											94.8276		83.3333															79.7101		-56.9620		19.4805	
											-277.586		-30.3030															-30.4348		37.9747		-64.9350	
		-15.0000									6.7586		-43.3940															1.7681		-1.2532		6.0779	
		-1.0000																															
																																	1.0000

Finished Products to Demands

	AB45G1	AB45F1	AB45F2	AB45R1	AB45P1	AB45K1	AB45K2	AB45K3	AB45D1	AB45L1	AB45H1	AB45B1	AB45B2		RHS
				2.4228	2.4112									=	0.0000
															0.0000
															- 1.6200
														=	- 3.5000
															- .8200
															- 9.1100
															- 32.9400
															13.2900
															147.2900
														≤	24.5000
															18.4300
															80.0000
														=	0.0000
													1.0000	≤	0.0000
												1.0000			
															0.0000
	-1.0000														0.0000
		-.5000	-.2500												0.0000
		-.2500	-.7500												0.0000
		-.2500				-.2500	-.4000	-.1000							0.0000
						-.7500	-.6000	-.9000							0.0000
									-1.0000						0.0000
															0.0000
															0.0000
										-1.0000		-1.0000		=	0.0000
											-1.0000		-1.0000		0.0000
															0.0000
															0.0000
															0.0000
															0.0000
															0.0000
0															- 2.0000
0					-1.0000										0.0000
				-1.0000											0.0000
									-.5000						
									-1.4800						
														≤	0.0000
										-.8000		-1.0000			
										-22.2600		-36.2900		≥	0.0000
										-57.0000		-57.0000			
											-3.5000		-4.0000		
											-29.5000		-39.0600	≤	0.0000
											-70.3000		-70.3000		
														≥	0.0000
				.5500										≤	0.0000
8															
5														≥	0.0000
0															
9														≤	0.0000
					.5500										
	1.0000														1.6600
		1.0000	1.0000												5.3600
				1.0000											4.1800
					1.0000										12.3300
						1.0000	1.0000	1.0000						=	7.7000
									1.0000						25.5800
										1.0000					15.0000
											1.0000				28.0500
												1.0000	1.0000		5.4200

BIBLIOGRAPHY

Books

Adelman, M.A., *The World Petroleum Market*, (USA: The Johns Hopkins University Press, 1972).

Banks, F.E., *The Political Economy of Oil*, (USA and Canada: D.C. Heath and Company, 1980).

Bannock, G., Baxter, R.E., and Rees, R., *The Penguin Dictionary of Economics*, (Great Britain: Penguin Books, 1980).

Baumol, W.J., *Economic Theory and Operations Analysis*, (London: Prentice-Hall International, Inc., 1977).

Beckenstein, A.R., Grayson, L.E., Overholt, S.H., and Sutherland, T.F., *Performance Measurement of the Petroleum Industry*, (USA: Lexington Books, D.C. Heath & Company, 1979).

Bending, R., Eden, R., Posner, M., Stanislaw, J., and Crouch, E., *Energy Economics*, (Cambridge: Cambridge University Press, 1981).

Boulding, K.E., and Spivey, W.A., *Linear Programming and the Theory of the Firm*, (New York: The Macmillan Company, 1960).

Cellier, F.E. (ed.), *Progress in Modelling and Simulation*, (London: Academic Press Inc. Ltd., 1982).

Chenery, H.B., and Clark, P.G., *Interindustry Economics*, (New York: John Wiley and Sons, 1959).

Chevalier, J-M., *The New Oil Stakes*, (London: Penguin Books Ltd., 1975).

Cohen, K.J., and Cyert, R.M., *Theory of the Firm: Resource Allocation in a Market Economy*, (New Jersey: Prentice-Hall Inc., Englewood Cliffs, 1975).

Daniel, C., and Wood, F.S., *Fitting equations to data, Computer Analysis of Multifactor data for scientists and engineers*, (New York: John Wiley & Sons, Inc., 1971).

Daniel, C., *Applications of Statistics to Industrial Experimentation*, (USA: John Wiley & Sons, Inc., 1976).

Dantzig, G.B., *Linear Programming and Extensions*, (New Jersey: Princenton University Press, 1963).

Davies, O.L., *The Design and Analysis of Industrial Experiments*, (London: Oliver and Boyd, 1963).

Dorfman, R., Samuelson, P. and Solow, R., *Linear Programming and Economic Analysis*, (Tokyo: McGraw-Hill Kogakusha Ltd., 1958).

Draper, N.R., and Smith, H., *Applied Regression Analysis*, (USA: John Wiley & Sons, 1981).

Dunkerley, J., (ed.), *International Energy Strategies, Proceedings of the 1979 IAEE/RFF Conference*, (Cambridge, Massachusetts: Oelgeschlager, Glunn & Hain, Publishers, Inc., 1980).

Dutch, R.A., *Roget's Thesaurus of English Words and Phrases*, (England: Penguin Books, 1983).

Fine, B., *Marx's Capital*, Macmillan Studies in Economics, (London: The Macmillan Press Ltd., 1979).

Fisher, R.A., *The Design of Experiments*, (Edinburgh: Oliver & Boyd, 1960).

Friedman, F.L., and Koffman, E.B., *Problem Solving and Structured Programming in Fortran*, (USA and Canada: Addison-Wesley Publishing Company, Inc., 1981).

Gal, T., *Postoptimal Analyses, Parametric Programming, and Related Topics*, (Great Britain: McGraw-Hill Inc., 1979).

Gass, S.I., *Linear Programming*, (USA: McGraw-Hill Book Company, 1975, 4 ed.).

Ginsburgh, V.A., and Waelbroeck, J.L., *Activity Analysis and General Equilibrium Modelling*, (The Netherlands: North-Holland Publishing Company, 1981).

Hadley, G., *Linear Programming*, (Massachusetts: Addison-Publishing Co., 1962).

Hanks, P., editor, *Dictionary of the English Language*, (London: William Collins Sons & Co. Ltd., 1982).

Intriligator, M.D., *Mathematical Optimization and Economic Theory*, (USA: Prentice-Hall, INC., Englewood Cliffs, N.J., 1971).

Johnston, J., *Econometric Methods*, (Tokyo: McGraw-Hill, Kogakusha Ltd., 1972).

Jorgenson, D.W., (ed.), *Econometric Studies of U.S. Energy Policy*, (Amsterdam: North-Holland Publishing Company, 1976).

Kleijnen, J.P.C., *Statistical Techniques in Simulation*, Part I and Part II, (New York: Marcel Dekker, Inc., 1974).

Koopmans, T.C., *Three Essays on THE STATE OF ECONOMIC SCIENCE*, (USA: McGraw-Hill Book Company, Inc., 1957).

Koutsoyiannis, A., *Theory of Econometrics*, (London: The Macmillan Press Ltd., 1977).

____________, *Modern Microeconomics*, (London: The Macmillan Press Ltd., 1979).

Kavrakoglu, I. (ed.), et al., *Mathematical Modelling of Energy Systems*, (1981).

Lawley, D.N., and Maxwell, A.E., *Factor Analysis as Statistical Method*, (London: Butterworths, 1963).

Leffler, W.L., *Petroleum Refining for the Non-technical person* , (Oklahoma, The Petroleum Publishing Co., 1979).

Leftwich, R.H., *The Price system and resource allocation*, (New York: Holt, Rinehart & Winston, 1961).

Lipsey, R., *An Introduction to Positive Economics*, (London: Weindenfeld & Nicolson, 1963).

Manne, A.S., *Scheduling of Petroleum Refinery Operations*, (Cambridge, Massachusetts: Harvard University Press, 1956).

Manne, A.S. and Markowitz, H.M., *Studies in Process Analysis : ECONOMY-WIDE PRODUCTION CAPABILITIES*, (USA: John Wiley & Sons, Inc., 1963).

Mendenhall, W., *Introduction to Linear Models and the Design and Analysis of Experiments*, (California: Wadsworth Publishing Co., Inc., 1968).

Myers, R.H., *Response Surface Methodology*, (Boston: Allyn & Bacon, Inc., 1971).

Nelson, W.L., *Petroleum Refinery Engineering*, (USA: McGraw-Hill Book Company, INC., 1958).

____________, *Guide to Refinery Operating Costs (Process Costimating)*, (Tulsa, USA: The Petroleum Publishing Company, 1976).

Odell, P.R., *Oil and World Power*, (Great Britain: Penguin Books, 7th. edition, 1983).

Ogawa, J., *Statistical Theory of the Analysis of Experimental Designs*, (New York: Marcel Dekker Inc., 1974).

Pasinetti, L.L., *Essays on The Theory of Joint Production*, (London: The Macmillan Press Ltd., 1980).

Pearce, D.W., *The Dictionary of Modern Economics*, (London: Macmillan Press Ltd., 1983).

Peng, K.C., *The Design and Analysis of Scientific Experiments*, (USA: Addison-Wisley Publishing Co., Inc., 1967).

Raktoe, B.L., Hedayat, A., and Federer, W.T., *Factorial Designs*, (USA: John Wiley & Sons, Inc., 1981).

Robinson, J., and Eatwell, J., *An Introduction to Modern Economics*, (UK: McGraw-Hill Book Company (UK) Limited, 1973).

Robinson, J., *The Economics of Imperfect Competition*, (London: The Macmillan Press Ltd., 1969, reprinted 1976).

Sage, A.P., *Economic Systems Analysis, Microeconomics for Systems Engineering, Egineering Management and Project Selection*, (New York: North-Holland, 1983).

Salter, W.E.G., and Reddaway, W.B., *Productivity and Technical Change*, (Cambridge: Cambridge University Press, 1966).

Schneider, S.A., *The Oil Price Revolution*, (USA: The Johns Hopkins University Press, 1983).

Seber, G.A.F., *Linear Regression Analysis*, (New York: John Wiley & Sons, Inc., 1977).

Shuman, R.B., *The Petroleum Industry, An Economic Survey*, (Norman: University of Oklahoma Press, 1940).

Slesser, M., *Energy in the Economy*, (Hong Kong: The Macmillan Press Ltd., 1978).

Smith, D., *Linear Programming Models in Business*, (London: Polytech Publishers Ltd., 1973).

Sraffa, P., *Production of Commodities by Means of Commodities*, (Cambridge: Cambridge University Press, 1960).

Stigler, G.J., *The Theory of Price*, (New York: Macmillan Publishing Co., Inc., 1966).

Taffler, R., *Using Operational Research*, (Great Britain: Prentice-Hall International, Inc., 1979).

Taha, H.A., *Operations Research, An Introduction*, (New York: Macmillan Publishing Co., Inc., 1971).

Tempest, P., *Energy Economics in Britain*, (London: Graham & Trotman, 1983).

Thomas, J.A.G., *Energy Analysis*, (Surrey, England: IPC Science and Technology, 1977).

Towsend, H., *Price Theory*, (Penguin Modern Economics Readings, 1980).

Varian, H.R., *Microeconomic Analysis*, (USA: W.W. Norton & Company, Inc., 1978).

Wagner, H.M., *Principles of Operations Research*, (New Jersey: Prentice-Hall, Inc., Englewood Cliffs, 1975).

Williams, H.P., *Model Building in Mathematical Programming*, (John Wiley & Sons, 1978).

Yates, F., *Experimental Design: selected papers*, (London: Charles Griffing & Company Limited, 1970).

Zeigler, B., Elzas, M.S., Klir, G.J., and Oren, T.I. (eds.), *Methodology in Systems Modelling and Simulation*, (Amsterdam: North-Holland Publishing Co., 1979).

Articles

Arrow, K.J., 'Limited Knowledge and Economic Analysis', *The American Economic Review*, LXIV, 1, (March 1974).

Aucamp, D. C., and Steinberg, D. I., 'The Computation of Shadow Prices in Linear Programming', *The Journal of the Operational Research Society*, XXXIII, 6, (June 1982).

Charnes, A., Cooper, W.W., and Mellon, B., 'Blending Aviation Gasoline - A study in programming interdependent activities in an integrated oil company', *Econometrica*, XX, 2, (April 1952).

Chevalier, J-M., 'Elements Theoriques D'introduction a l'Economie du Petrole', *Revue d'Economie Politique*, (1975).

Choucri, N., 'Analytical Specifications of the World Oil Market, A review and comparison of twelve models', *Journal of Conflict Resolution*, XXIII, 2, (June 1979), pp. 346-372.

Deam, R.J., 'Linear Programming in Management Accounting', *The Accountant*, (23rd September 1967).

———————, et al., 'World Energy Modelling: the Development of Western European Oil Prices', *Energy Policy*, I, 1, (June 1973).

———————, et al., 'World Energy Modelling: Part I. Concepts and Methods', *A Special Energy Policy Publication on Energy Modelling*, (UK, 1974).

———————, et al., 'World Energy Modelling: Part II. Preliminary results from the petroleum-natural gas model', *A Special Energy Policy Publication on Energy Modelling*, (UK, 1974).

Gibson, B., and McLeod, D., 'Non-produced means of production in Sraffa's system: basics, non-basics and quasi-basics', *Cambridge Journal of Economics*, VII, (1983).

Griffin, J.M., 'The Process Analysis Alternative to Statistical Cost Functions', *The American Economic Review*, LXII, (1972).

Hoffman, K.C., and Wood, D. O., 'Energy System Modeling and Forecasting', *Annual Review of Energy*, I, (1976).

Howe, M., 'Marginal Analysis in Accounting', *Yorkshire Bulletin of Economic and Social Research*, (Nov. 1962)., pp. 81-89.

Huntington, H.G., Weyant, J.P., and Sweeney, J.L., 'Modeling for Insights, not Numbers: the Experiences of the Energy Modeling Forum', *OMEGA*, X, 5, (1982), pp. 449-462.

Keneddy, M., 'An Economic Model of the World Oil Market', *The Bell Journal of Economics and Management Science*, V, 2, (Autumn 1974).

Kuenne, R.E., 'The GENESYS model of OPEC, 1974-1980, Structural insights from a non-forecasting model', *Energy Policy*, (July 1982).

Kurz, H.D., 'Rent Theory in a Multisectoral Model', *Oxford Economic Papers*, XXX, (1978).

Machlup, F., 'Marginal Analysis and Empirical Research', *The American Economic Review*, (September 1946).

Manne, A., et al., 'Energy Policy Modeling: A Survey', *Operations Research, Journal of ORSA*, XXVII, 1, (Jan-Feb 1979), Feature Article.

Meriam, R.S., 'Quasi-Rent', in: *Explorations in Economics: Notes & Essays in Honor of Frank W. Taussing*, (New York: Kelley, reprinted 1936 edition).

Montani, G., 'La teoria ricardiana della rendita', *L'industria*, (Milano, 1972).

Ogilvie, F.W., 'Marshall On Rent', *The Economic Journal*, XL, 157, (March 1930).

Pantin, D.A., 'Resource Depletion Theories and Planning in Mineral Exporting Economies with particular reference to Petroleum Exporters', *OPEC Review*, IV, 4, (Winter 1980).

Rath-Nagel, S., and Voss, A., 'Energy models for planning and policy assessment', *European Journal of Operational Research*, VIII, (1981), Invited Review, pp.99-114.

Robinson, J., 'Capital Theory up to date', *Canadian Journal of Economics*, III, 2, (May 1970), Review Article.

Solow, R.M., 'Richard T. Ely Lecture: The Economics of Resources or the Resources of Economics', *American Economic Association*, LXIV, 2, (May 1974).

Worcester, D.A. Jr., 'A Reconsideration of the Theory of Rent', *The American Economic Review*, (June 1946).

Yates, F., 'The design and analysis of factorial experiments', *Imp.Bur.Soil.Sci.Tech.Commun.*, XXXV, (1937).

Periodicals and Other Publications

The British Petroleum Company Ltd., *Our Industry Petroleum*, (London, 1977).

The British Petroleum Company p.l.c., *BP statistical review of world energy*, (London, years 1982, 1983 and 1984).

CPDP, *Bulletin Mensuel*, (France, years 1973 to 1984).

CDC, *APEX-III Reference Manual*, Version 1.2, (USA, 1979).

Deam, R.J., and Laughton, M.A., *Oil Producer and Refiner Revenues, A possible solution to the Problem of Decreased Profits*, paper presented at the 1982 Annual BIEE/IAEE Conference on International Energy Markets - The Changing Structure, (June 1982).

Esso Magazin, 1/82.

Eurostat (Statistical Office of the European Community), *Energy Statistical Yearbook 1983*, (Brussels, 1985).

'1982 Refining Process Handbook', *Hydrocarbon Processing*, (September 1982).

IBM, *Document Composition Facility - Introduction to the Generalized Markup Language: Using the Starter Set*, Program Number 5748-XX9, First Edition, (USA, February 1980).

——, *Document Composition Facility Generalized Markup Language: Starter Set Reference*, Program Number 5748-XX9, First Edition, (USA, April 1980).

——, *Document Composition Facility: User's Guide*, Program Number 5748-XX9, Release 2, Fourth Edition, (USA, 1982).

——, *IBM Mathematical Programming System Extended/370 (MPSX/370) Messages*, Second Edition, (USA, November 1977).

——, *IBM Mathematical Programming System Extended/370 (MPSX/370) Program Reference Manual*, Third Edition, (USA, December 1977).

IEA, *Energy Prices and Taxes*, second quarter 1984, (Paris).

IIASA,

Charpentier, J.P., *A review of energy models No. 1*, (Laxenburg, Austria, 1974).

————————, *A review of energy models No. 2*, (Laxenburg, Austria, 1975).

————————, and Beaujean, J.M., *A review of energy models No. 3*, (Laxenburg, Austria, 1976).

Lebanon, A., and Singler, S. F., 'Optimum oil pricing by OPEC hard core', A Report from *The Energy Policy Studies Center*, (The University of Virginia, January 1981).

Lloyds Shipping Economist, (London, various years).

MIT,

Paddock, J.L., 'M.I.T. World Oil Project Model Simulations' for International Energy Workshop Stanford University, December 10 and 11, 1981.

The International Petroleum Exchange Model, (brochure without date).

OECD, *Main Economic Indicators*, (Paris, several years).

——, *Main Economic Indicators, Historical Statistics 1964-1983* , (Paris, 1984).

——, *OECD Economic Outlook*, 27, (Paris, July 1982).

——, *Quarterly Statistics*, 4th quarter 1984, (Paris).

Oil & Gas Journal, *Oil & Gas Journal*, 'Worldwide refining', issues of last week of December, (USA, years 1970-1984).

OPEC, *Basic Oil Industry Information*, (Vienna, 1983).

——, *Oil Industry Conversion Factors*, (Vienna, 1983).

——, *OPEC bulletin*, XV, 1, (Feb 1984) and 6, (July/August 1984), and XVI, 7, (September 1985).

Petroleum Economist, issues of June 1984, and September and October 1985.

Petroleum Intelligence Weekly, March 8, 1982.

Rhoe, A., et al., 'Meeting the refiner's upgrading needs', *The Lummus Company*, paper presented at the National Petroleum Refiners Association 81st. Annual Meeting, (California: March 20-22, 1983).

Salama, S., and Greene, J., 'EXECUTIVE SUMMARY WOIL80 World Energy Model', (Arlington, Virginia: Energy and Environmental Analysis, Inc., May 18, 1981).

United Nations, *Techniques of Petroleum Development*, (New York, 1962).

World Energy Models Ltd., (former WEM), *Seven Area Short Term World Energy Model, A Technical Description*, (London, 1976).

——————————, *The World Energy Model - A Technical Description*, (London, 1976).

Unpublished References

Deam, R.J., 'The Integrated Programming of Refinery Supply and Procurement', mimeograph, (April 1958).

—————, 'Coal cost caps Crude Price', (Queen Mary College, University of London, September 1985).

————— and Giesecke, C., 'The price of crude can be controlled by the cost of methanol', (Juelich, FRG: Programme Group on Systems Research and Technological Development, April 1983).

Giesecke, C., 'World computer model of oil markets: OPEC pricing strategy model in the short and long term', Ph.D thesis, (Queen Mary College, University of London, December 1984).

Land, A.H., 'Notes on Basic Mathematical Programming', (Notes for a Mathematical Programming Post-graduate Course, London School of Economics, 1977).

Longley, R.C.M., 'Price Cost Assignment and Back-up Notes' (London: Chem Systems International Ltd., 1983).

Martin, R.J., 'Spatial models with applications in sampling and experimental design', Ph.D thesis, (London School of Economics, University of London, 1977).

Murfin A., 'Monopoly and competition: a theoretical consideration and an empirical application to the U.K. car industry', Ph.D thesis, (Berbick College, University of London, 1983).

O'Carroll, F.M., 'Price Determination and Economic Mechanisms in the European Oil Industry 1964-71', Ph.D thesis, (Queen Mary College, University of London, 1974).

OPEC

'Demand of selected OECD countries', (various years).

'Petroleum refining and products marketing in Western Europe', (Paris: Enerfinance; direct communication of OPEC News Agency; article referenced in *OPEC bulletin*, May 1985, p.59, in: 'Difficult future for European refiners').

'Oil and Product Statistics, 1976-1980', (OPEC DSD/Statistics Unit, January 1983).

'Refinery Capacities 1960-1982, by process and region', (June 1983).

Poleo, V., 'On Energy System and Modelling with reference to the World Energy Model, Part I - A One-Area Case', (Energy Research Unit, Queen Mary College, 1979).

————, 'Interactive Netbacking System', (London: Petroleos de Venezuela (UK) S.A., 1982).

————, 'OPEC Crude Oil Price Differentials: A flexible approach', (London: Petroleos de Venezuela (UK), S.A., 1983).

Sanchez, A., 'On World Energy Modelling: A One Area Single Refinery Model', (Queen Mary College, Univeristy of London, 1982).

'World Energy Model: 22-area version data base', (Queen Mary College, University of London, 1976).

Reihe Angewandte Systemanalyse

der Programmgruppe Systemforschung und Technologische Entwicklung Kernforschungsanlage Jülich GmbH - KFA/STE

Nr. 1: Schmitz, K., Niehaus, F., Rath-Nagel, St., Voß, A.
Die Entwicklungsmöglichkeiten der Energiewirtschaft in der Bundesrepublik Deutschland - Untersuchung mit Hilfe eines dynamischen Simulationsmodells,
Jül-Spez-1, Bd. I, Nov. 1977

Schmitz, K., et al.
Die Entwicklungsmöglichkeiten der Energiewirtschaft in der Bundesrepublik Deutschland - Untersuchung mit Hilfe eines dynamischen Simulationsmodells,
Jül-Spez-1, Bd. II, Nov. 1977

Nr. 2: Bohn, Th., Eich, P., Hansen, U., Jehle, B.
Künftige Stromgestehungskosten von Großkraftwerken,
Jül-Spez-2, Nov. 1977

Nr. 3: Wibbe, H.-B.
Probleme der kurzfristigen Markteinführung eines nuklearen Fernenergiesystems - eine Betrachtung aus Abnehmersicht für einen räumlich begrenzten Markt,
Jül-1486, Feb. 1978

Nr. 4: Jehle, B.
Einsatzmöglichkeiten und Einsatzbeispiele der Kernenergie in NRW unter besonderer Berücksichtigung ihres Beitrags zur Strukturverbesserung,
Jül-Spez-11, Juni 1978

Nr. 5: Hensel, W.
Beitrag zur Standardisierung der Standortbestimmung von Kernkraftwerken,
Jül-Spez-12, Juni 1978

Nr. 6: v. Lojewski, D.
Wärme-, Strom- und Synthesegasversorgung von Ballungsräumen durch Fernenergie - aufgezeigt am Großraum Köln,
Jül-1516, Juni 1978

Nr. 7: Lenhardt, W.
Entwicklung des Strombedarfs in der Bundesrepublik Deutschland und alternative Möglichkeiten seiner Deckung,
Jül-Spez-18, Juli 1978

Nr. 8: Kolb, G. (Redaktion)
gemeinsam mit KFA/IRE, KFA/PTH, HRB, GHT
Studie über die Wirtschaftlichkeit der Stromerzeugung mit Hochtemperatur-Reaktoren,
Jül-1527, Aug. 1978

Nr. 9: Meliß, M.
Möglichkeiten und Grenzen der Sonnenenergienutzung in der Bundesrepublik Deutschland mit Hilfe von Niedertemperaturkollektoren - Grundlagen, Technische Systeme, Wirtschaftlichkeit;
Jül-Spez-25, Dez. 1978

Nr. 10: Wagner, H.J.
Der Energieaufwand zum Bau und Betrieb ausgewählter Energieversorgungstechnologien - eine nettoenergetische Analyse,
Jül-1561, Dez. 1978

Nr. 11: Bartholdi, J.
Einführung der Nuklearen Fernenergie - umweltbedeutsame Konsequenzen und deren Bewertung,
Jül-1573, Jan. 1979

Nr. 12: Egberts, G.
Kostenoptimale Entwicklungsperspektiven des Raumheizungssektors im Energiesystem der Bundesrepublik Deutschland,
Jül-Spez-41, Juni 1979

Nr. 13: Düring, K.
Transport- und Verteilungskosten konventioneller Systeme zur Wärmeversorgung der Bundesrepublik Deutschland,
Jül-Spez-57, Okt. 1979

Nr. 14: Leimkühler, K.
Metallische Rohstoffe - Rezyklierung - Energieeinsatz. Untersuchung mit Hilfe eines Simulationsmodells,
Jül-Spez-44, Juli 1979

Nr. 15: Lenhardt, W., Schwefel, H.P., Sievert, D., et al.
Ein Energieversorgungsmodell zur Langfristprognose der Umwandlungskapazitäten,
Jül-Spez-63, Dez. 1979

Nr. 16: Kollmann, H.
Die räumliche Wärmebedarfsverteilung der Haushalte und Kleinverbraucher in der Bundesrepublik Deutschland,
Jül-Spez-64, Dez. 1979

Nr. 17: Orth, D.
Niedertemperatur-Wärmeversorgung unter besonderer Berücksichtigung ausgewählter neuer Technologien,
Jül-Spez-65, Dez. 1979

Nr. 18: Manthey, Ch. (editor)
Energy Technology Data Handbook - Conversion Technologies,
Jül-Spez-70, Vol. I, Jan. 1980

Nr. 19: Manthey, Ch., Tosato, G.C.
Energy Technology Data Handbook - End Use Technologies. Insulation, Space- and Water Heating Systems,
Jül-Spez-70, Vol. II, Okt. 1980

Nr. 20: Hildebrandt, T.
Die nächsten 50 Jahre,
Analyse und Szenarium der wirtschaftlichen Evolution,
Jül-Spez-81, Juni 1980

Nr. 21: Finnis, M.W.
Phase II - Final Report of MARKAL Studies for the United Kingdom,
Jül-Spez-92, Okt. 1980

Nr. 22: Scharff, E., Walbeck, M. (Federführung)
Planstudie: Halboffene Fernenergieversorgung, insbesondere für den Raum Frankfurt a.M. mit Heißwasser-Fernwärmeversorgung für den Raum Köln,
JÜL-Spez-95, Dez. 1980

Nr. 23: Costa, J.O., Gil Sordo, V., Blasco, M., Jara, A.
Energy Scenarios and Implementation of New Technologies for Spain,
JÜL-Spez-110, April 1981

Nr. 24: Jaek, W.
Mögliche Entwicklung des weltweiten Ausbaus der Kernenergie unter Berücksichtigung wirtschaftlicher, gesellschaftlicher und politischer Aspekte,
JÜL-Spez- 118, Juli 1981

Nr. 25: Schmitz, J.
Abschätzung des energiesparenden Innovationspotentials der Industrie Steine und Erden,
JÜL-1729, Aug. 1981

Nr. 26: Bansal, N.K., Uhlemann, R., Boettcher, A.
Plastic Solar Air Heaters of a Novel Design - Testing and Performance,
JÜL-1783, April 1982

Nr. 27: Höpfinger, E., Drepper, F., Heckler, R., Schwefel, H.P.
unter Mitarbeit von Hermes, U., Horst, H.
Zur Einbettung der Energiewirtschaft in die Volkswirtschaft - Vier dynamische Modelle -
JÜL-Spez-166, Juli 1982

Nr. 28: Grüter, J.W., Wolff, M.
Die Energieversorgung der Gemeinde Seedorf im Landkreis Lüchow-Dannenberg, Niedersachsen - Eine Endenergieanalyse
JÜL-Spez-170, August 1982

Nr. 29: Huber, W., Allhorn, H., Birnbaum, U.
Emissionen von Kohlenutzungsanlagen und mögliche Minderungsmaßnahmen,
JÜL-1815, Nov. 1982

Nr. 30: Müller, M., Maher, K.J., with contributions of Rath-Nagel, St., Tosato, G.C.
Summary Report on Technology Characterizations,
International Energy Agency: Energy Technology Systems Analysis Project,
JÜL-Spez-185, Dez. 1982

Nr. 31: Hildebrandt, T.
Wechselwirkungen zwischen Energiewirtschaft und Gesamtwirtschaft
JÜL-Spez-192, Dez. 1982

Nr. 32: Höpfinger, E., Huber, W.
EFOM 12C CASE STUDIES - Candidate Technologies to Relieve the EUROPEAN Energy System, Germany,
JÜL-Spez-193, EUR 8428 EN, Jan. 1983

Nr. 33: Späte, F.
Bestimmung des thermischen Verhaltens von zwei Thermosiphonanlagen,
JÜL-Spez-194, Febr. 1983

Nr. 34: Uhlemann, R.
Performance and Cost Analysis of Pressurized and Non-pressurized Domestic Solar Water Heating Systems with Thermosyphonic Flow,
JÜL-1882, Dez. 1983

Nr. 35: Kollmann, H., Plewnia, M., Schmitz, J., Huber, W.
Stand und Entwicklungstendenzen der Raumwärmeversorgung der Haushalte -
Bd. 1: Determinanten des Heizwärmebedarfs von Gebäuden,
Bd. 2: Kosten für Energieeinsparung durch Wärmedämmung von Gebäuden und heizungstechnische Maßnahmen,
Bd. 3: Vergleich ausgewählter Heizungssysteme unter energetischen, ökonomischen und ökologischen Gesichtspunkten,
JÜL-Spez-234, Dez.1983

Nr. 36: Heckler, R., Pohlmann, M., Schwefel, H.P.
Ein Modell zur Analyse der Umwandlungskapazitäten im Energiesektor der Bundesrepublik Deutschland,
JÜL-Spez-237, Dez. 1983

Nr. 37: Aringhoff, R., Bezerra, S., Hymmen, H.A., Kolb, G., Pimentel, R. (Working Group KFA/STE, CAEEB/Brasilien, SETEC/Bras.)
Program of Research and Development on the Utilization of Brazilian Coal and on Energy Systems Analysis and Planning for Brazil,
JÜL-Spez-261, June 1984

Nr. 38: Correa-Guzman, E.J.
Erweiterung des unscharfen linearen Programmierens bei Mehrfachzielsetzungen. Beschreibung und Anwendung auf Energiesystemanalysen,
JÜL-Spez-262, Juli 1984

Nr. 39: Allhorn, H.
Modellmäßige Analyse der Energie- und Massenströme einer Kohleverflüssigungsanlage zur Bestimmung der Wirtschaftlichkeit unter besonderer Berücksichtigung von Maßnahmen zur Emissionsminderung;
JÜL-1937, Juli 1984

Nr. 40: Romahn, B.,
mit Beiträgen von Wilde, D.,
Redaktion der gekürzten Fassung: Kolb, G.
Energiestudie für die Provinz Guangdong/Volksrepublik China; gekürzte Fassung des Bandes V: Energieversorgungsstrategien für die Provinz Guangdong bis zum Jahr 2000,
JÜL-Spez-264, Aug. 1984

Nr. 41: Wensierski, P.W., unter Mitarbeit von Scheller, W.
Analytische Untersuchungen zur Leistungsfähigkeit von Solaranlagen zur Warmwasserbereitung,
JÜL-Spez-268, Aug. 1984

Nr. 42: Wensierski, P.W.
Analyse und Optimierung solarer Warmwasser- und Raumheizungssysteme von Wohnbauten mit verschiedenem Wärmeschutz;
JÜL-Spez-301, Febr. 1985

Nr. 43: Riemer, H.
Analyse der Einsatzmöglichkeiten solarthermischer Heizsysteme zur zentralen Niedertemperatur-Wärmeversorgung in der Bundesrepublik Deutschland;
JÜL-Spez-307, Apr. 1985

Nr. 44: Jobsky, Th.
Elektrizität im industriellen Energiemarkt. Eine sektorale Analyse der Gründe und Ursachen der Elektrizitätsanwendung unter Berücksichtigung der Kraft-Wärme-Kopplung;
JÜL-Spez-308, Apr. 1985

Nr. 45: Wensierski, P.W.
SOLARYS - ein interaktives Modell zur Simulation von Solarsystemen.
Teil I und Teil II,
JÜL-Spez-346, Febr. 1986

Nr. 46: Giesecke Sara-Lafosse, C.R.
World Computer Model of Oil Markets: OPEC Pricing Strategy Model in the Short and the Long Run,
JÜL-Spez-352, March 1986

Nr. 47: Scheller, W.
Solaranlagen zur Warmwasserbereitung. Analytische Untersuchungen von Thermosiphon- und Pumpenanlagen unter mitteleuropäischen Klimabedingungen,
JÜL-2047, März 1986

Nr. 48: Sanchez Buelga, A.
Mathematical Programming Studies of Short Run Oil Refinery Rents,
JÜL-Spez-377, Nov. 1986